Failure statistics in disordered complex systems

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Pavia

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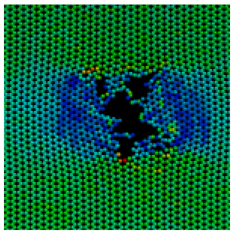
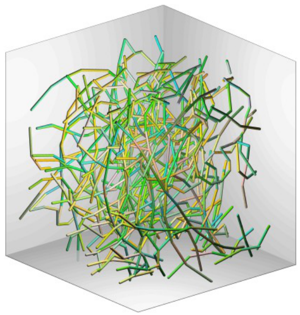
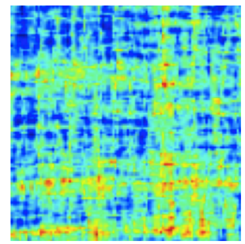
Failure across scales



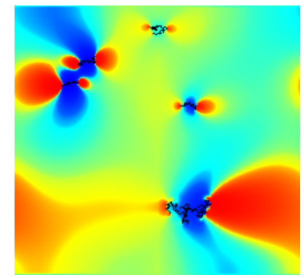
Atomic scale cracks



Dislocations

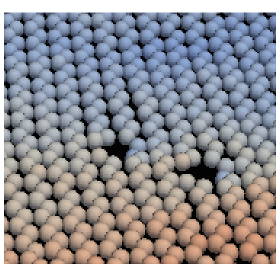


Shear bands

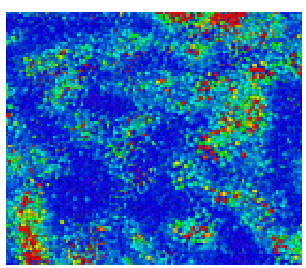


Interacting cracks

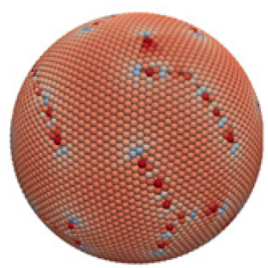
Disorder & structural features



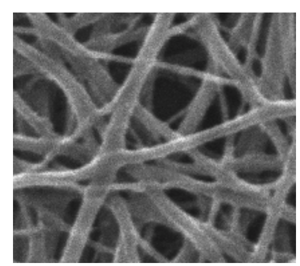
Crystals with atomic defects



Amorphous media



Topological disorder



Fiber networks

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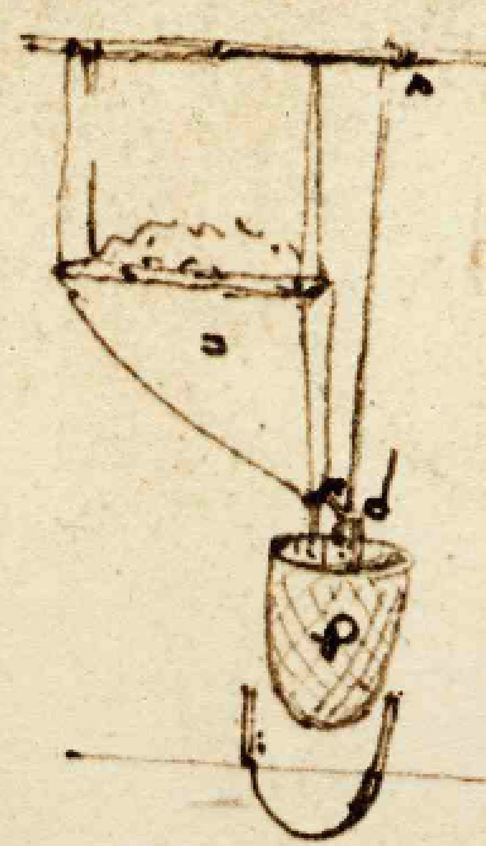
SIZE EFFECTS



..an old problem

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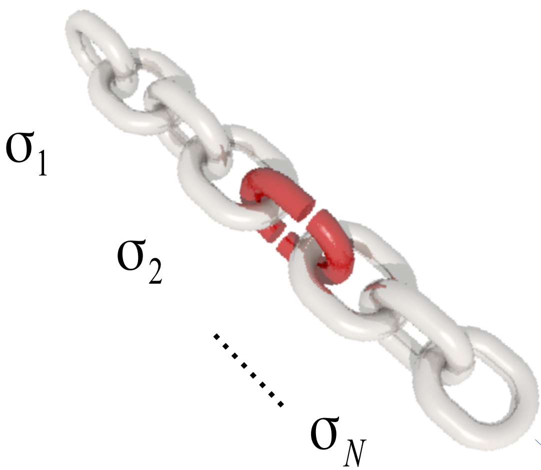
The weakest link hypothesis

The strength of corresponds to the strength of the weakest link, the minimum of σ_i



$$S_1(\sigma) \equiv \int_\sigma^\infty dx\, p(x)$$ Survival Distribution

$$S_N(\sigma) = S_1(\sigma)^N$$ for N links

If the low stress tail scales as: $$S_1(\sigma) \simeq 1 - (\sigma/\sigma_0)^\mu$$

$$S_N(\sigma) \simeq e^{-N(\sigma/\sigma_0)^\mu}$$ Weibull distribution

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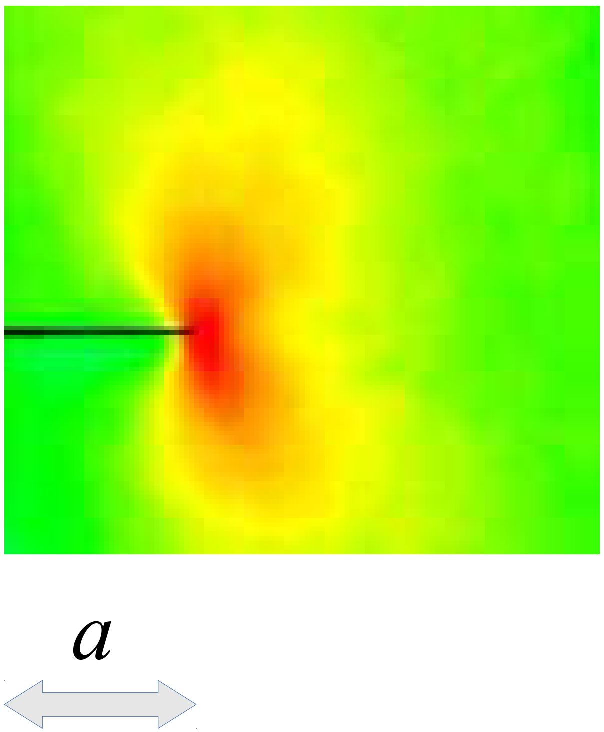
Stress concentration

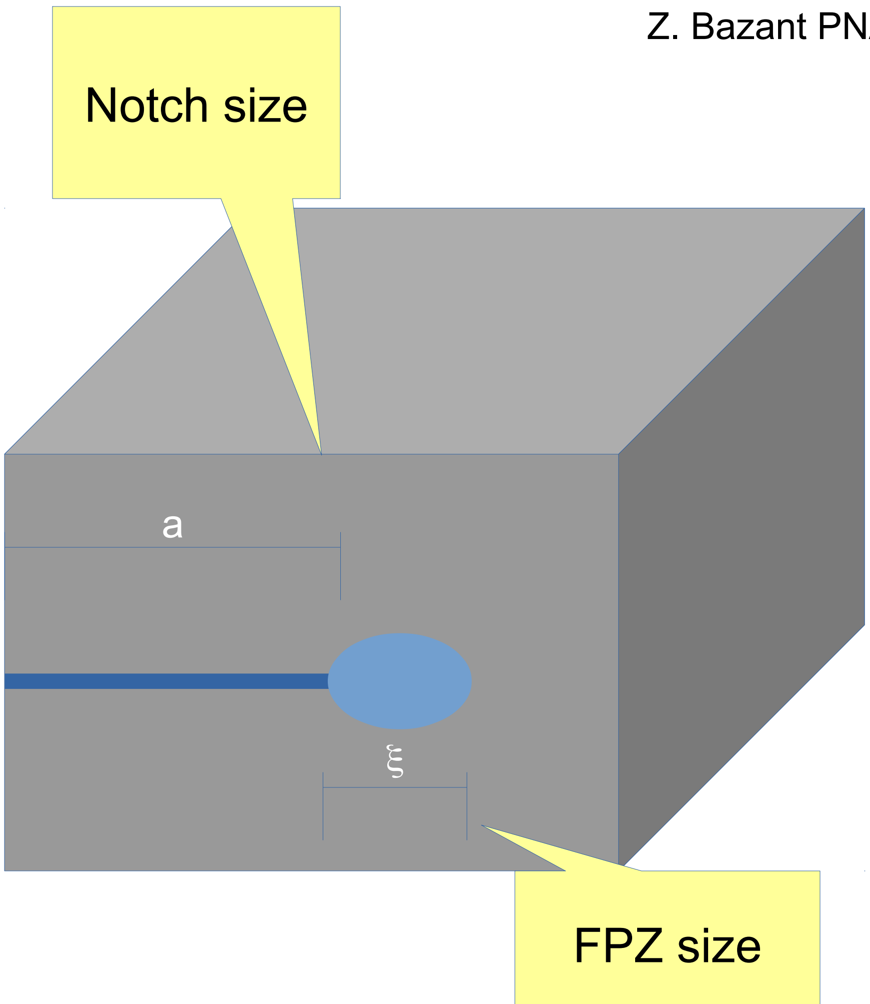


According to Griffith theory a crack of length a is unstable at stress:

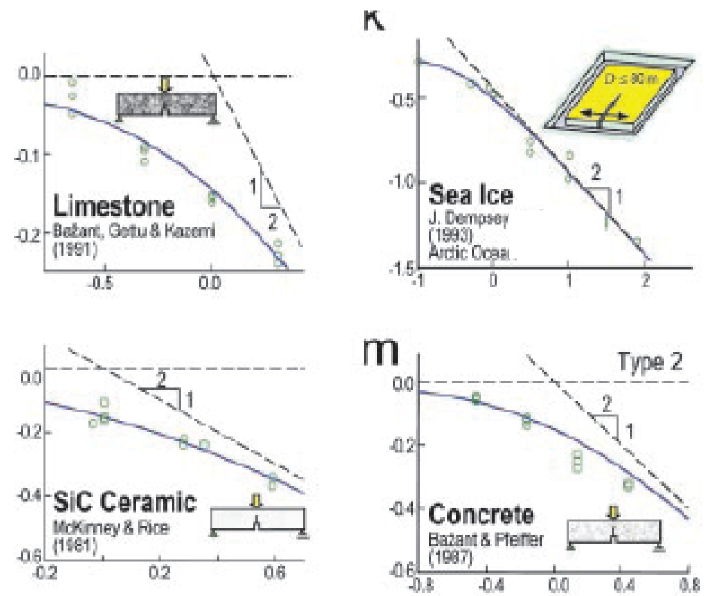
$$\sigma_c = \frac{K_c}{\sqrt{a}}$$

Energetic size effect

Z. Bazant PNAS 101, 13400, 2004



$$a \sim L \quad \Rightarrow \quad \sigma_c = \frac{K_c}{\sqrt{L+\xi}}$$



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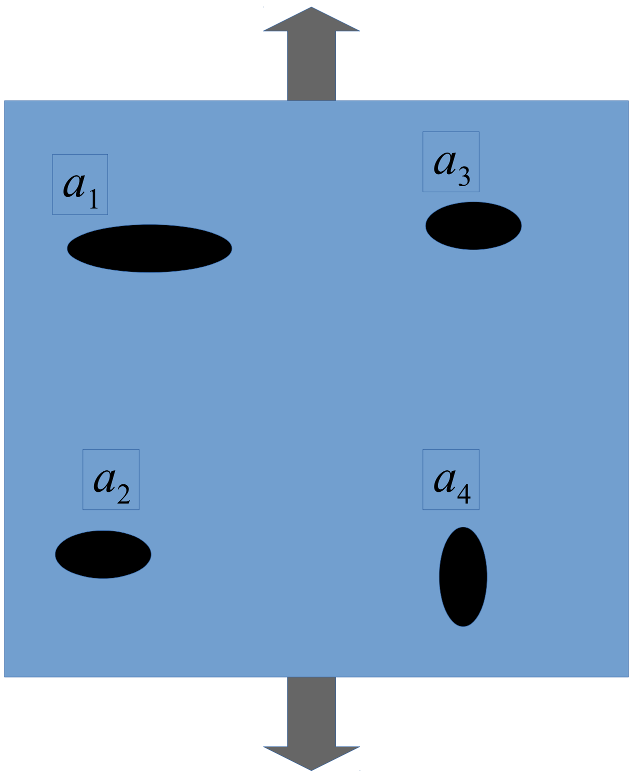
Crack nucleation (largest crack)



With a random micro-crack population, the material fails when the largest crack becomes unstable. The distribution of the maximum over N cracks:

$$P_N(a) \simeq 1 - e^{-NP(a)}$$

1) For an **exponential** tail we obtain asymptotically

$$S_N(\sigma) \simeq \exp[-N e^{-(\sigma_c/\sigma)^2}]$$

Duxbury-Leath-Beale (**DLB**) distribution.

2) For **power law** crack distribution we obtain

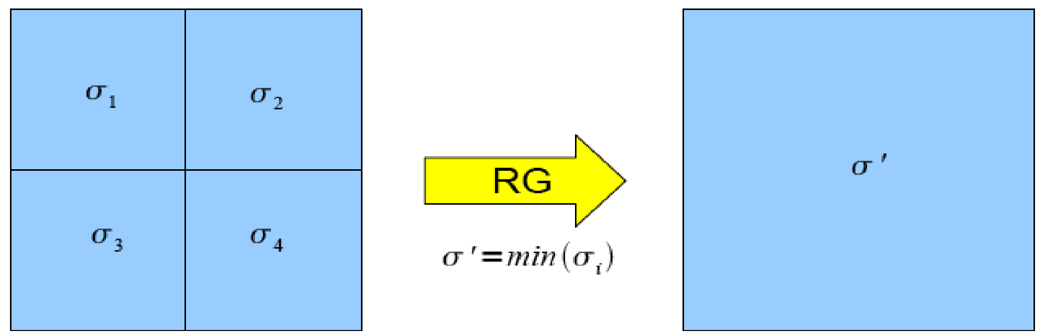
$$S_N(\sigma) \simeq \exp[-N c \sigma^{\alpha}]$$ **Weibull** distribution

Duxbury-Leath-Beale (DLB) 1987, Ray-Chakrabarti 1985, Freudenthal (1967)

Renormalization Group for fracture

(i) Coarse graining:

$$S_L(\sigma) = [S_{L/2}(\sigma)]^4$$



(ii) Rescaling:

$$\sigma = \quad A_L\sigma + B_L$$

(iii) Fixed point:

$$S^*(\sigma) = \mathcal{R}[S^*(\sigma)] = [S^*(a\sigma + b)]^4$$

Weibull

Gumbel

Extreme Value Theory is the “free” RG fixed point

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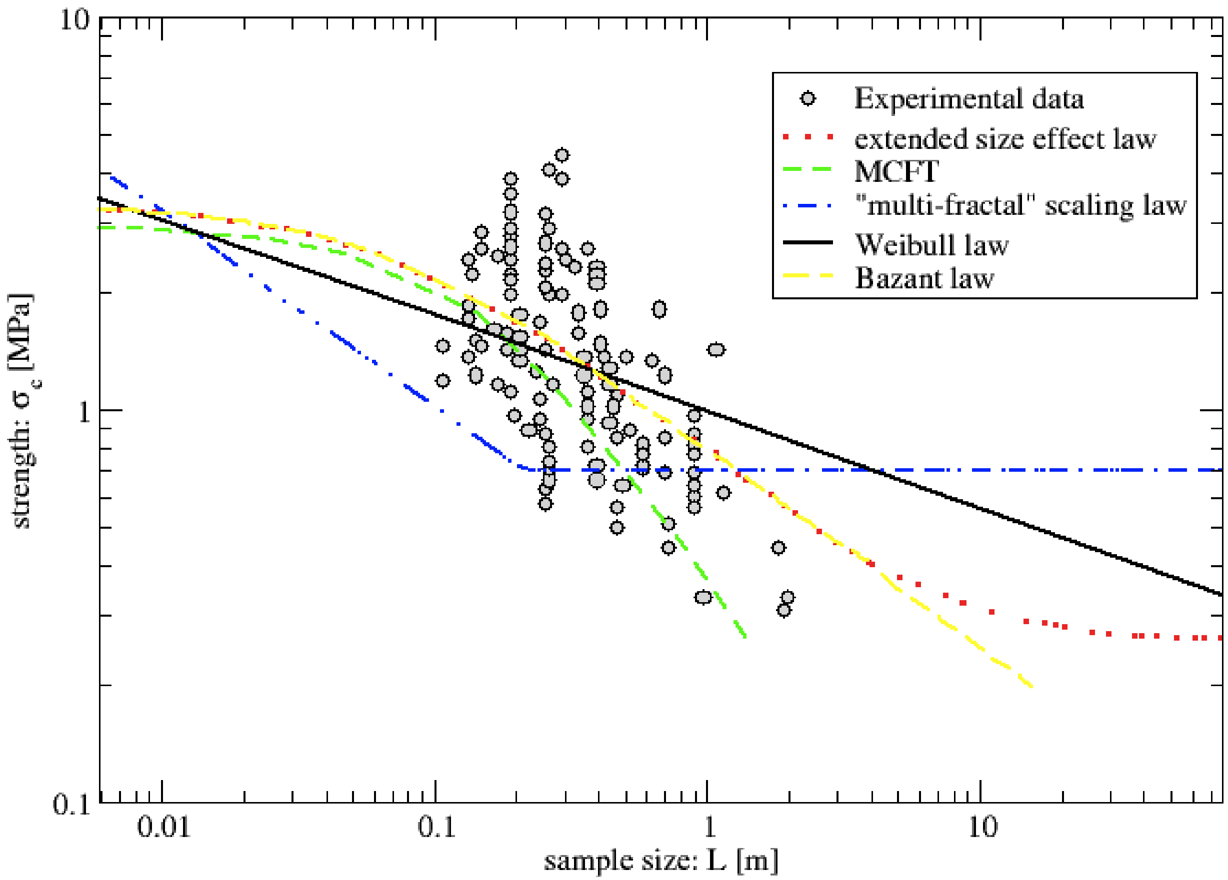
C. Manzato, A. Shekhawat, P. K. V. V. Nukala, M. J. Alava, J. P. Sethna, SZ, PRL 2012

Testing the theories



Z. P. Bazant and A. Yavari, Engineering Fracture Mechanics 72 1 (2005).

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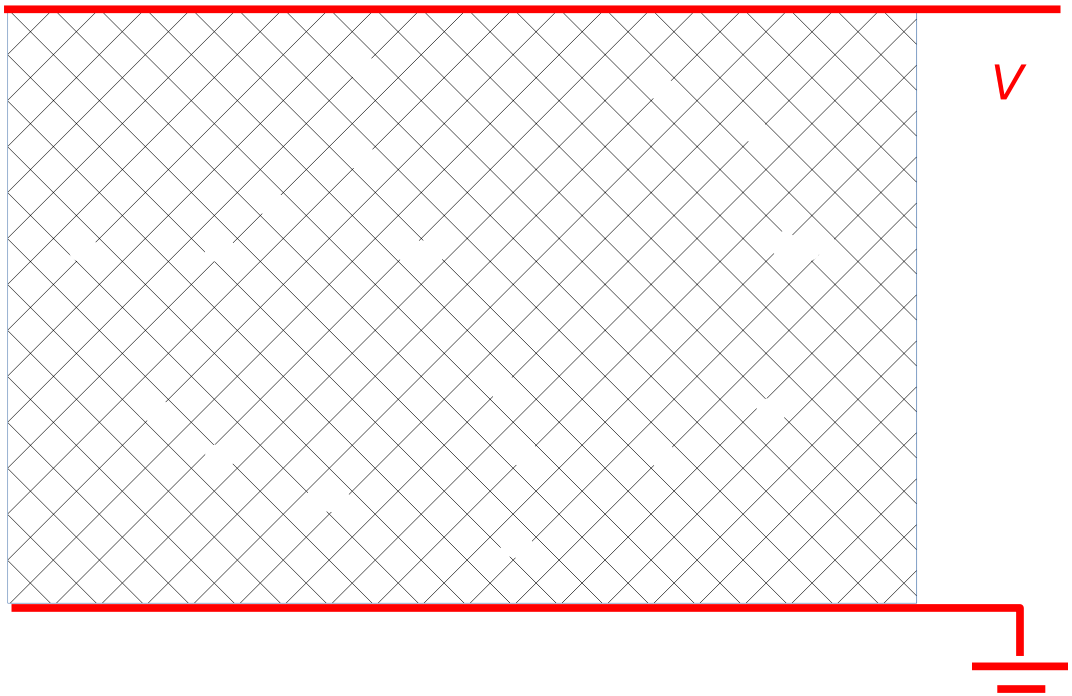
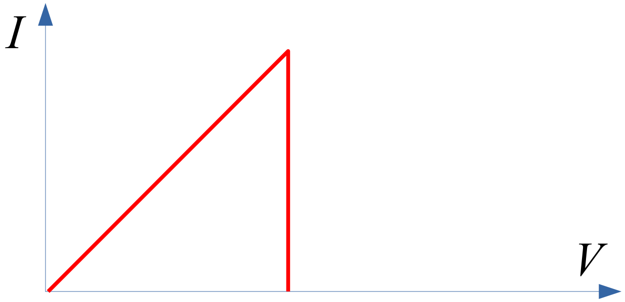
Modeling fracture with fuse models



- External current is applied through the bus bars to a a resistor network
- Local current are obtained solving Kirchhoff equations

$$\sum_i \sigma_{ij}(V_i - V_j) = 0$$

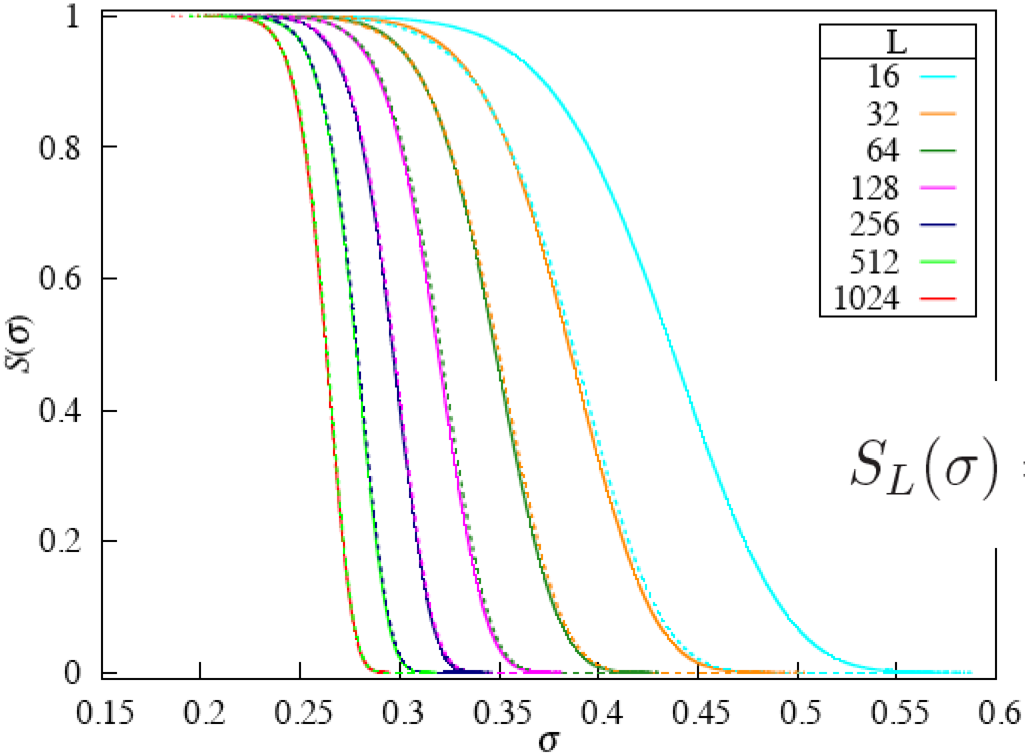
- Fuses have unit conductivity and disordered thresholds.



As the current *I* is raised, fuses burn until a spanning crack is nucleated

de Arcangelis, Redner & Herrmann
J. Phys (Paris) Lett 46, 585 (1985)

Testing renormalization group



$$S_L(\sigma) = \left[S_{L/2}(\sigma)\right]^4$$

No visible corrections from interactions: EVT works!

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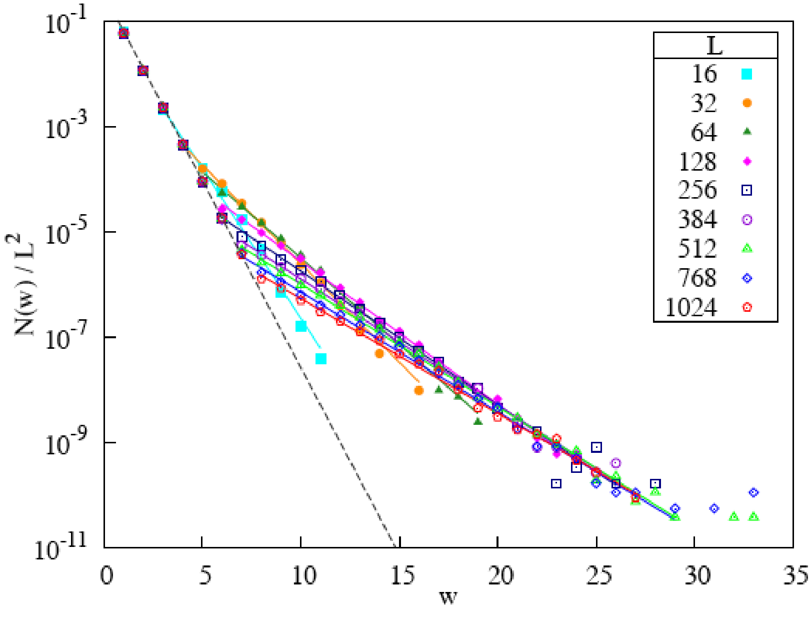
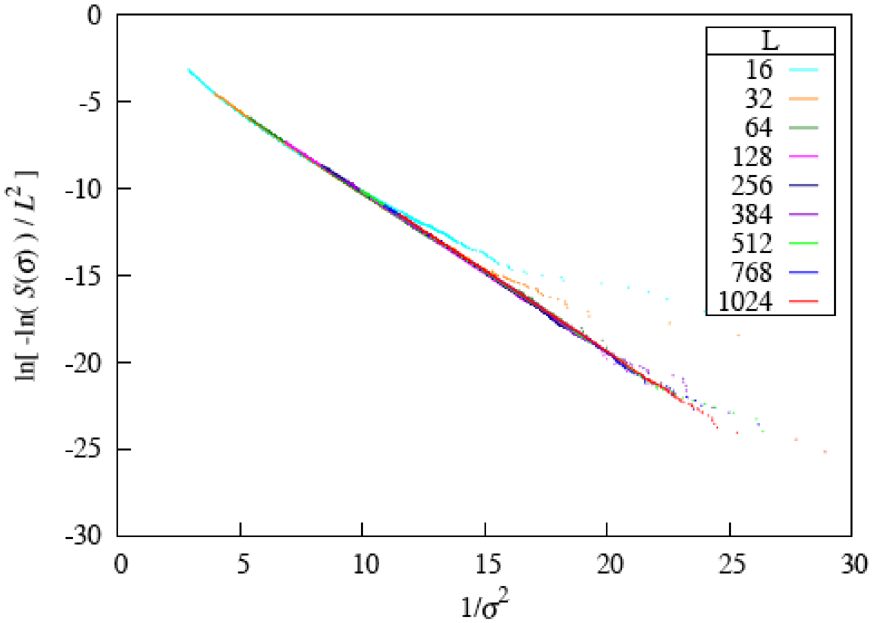
Testing the largest crack model

Random dilution: exponential crack distribution





Cracks distribution is exponential: **DLB failure distribution**

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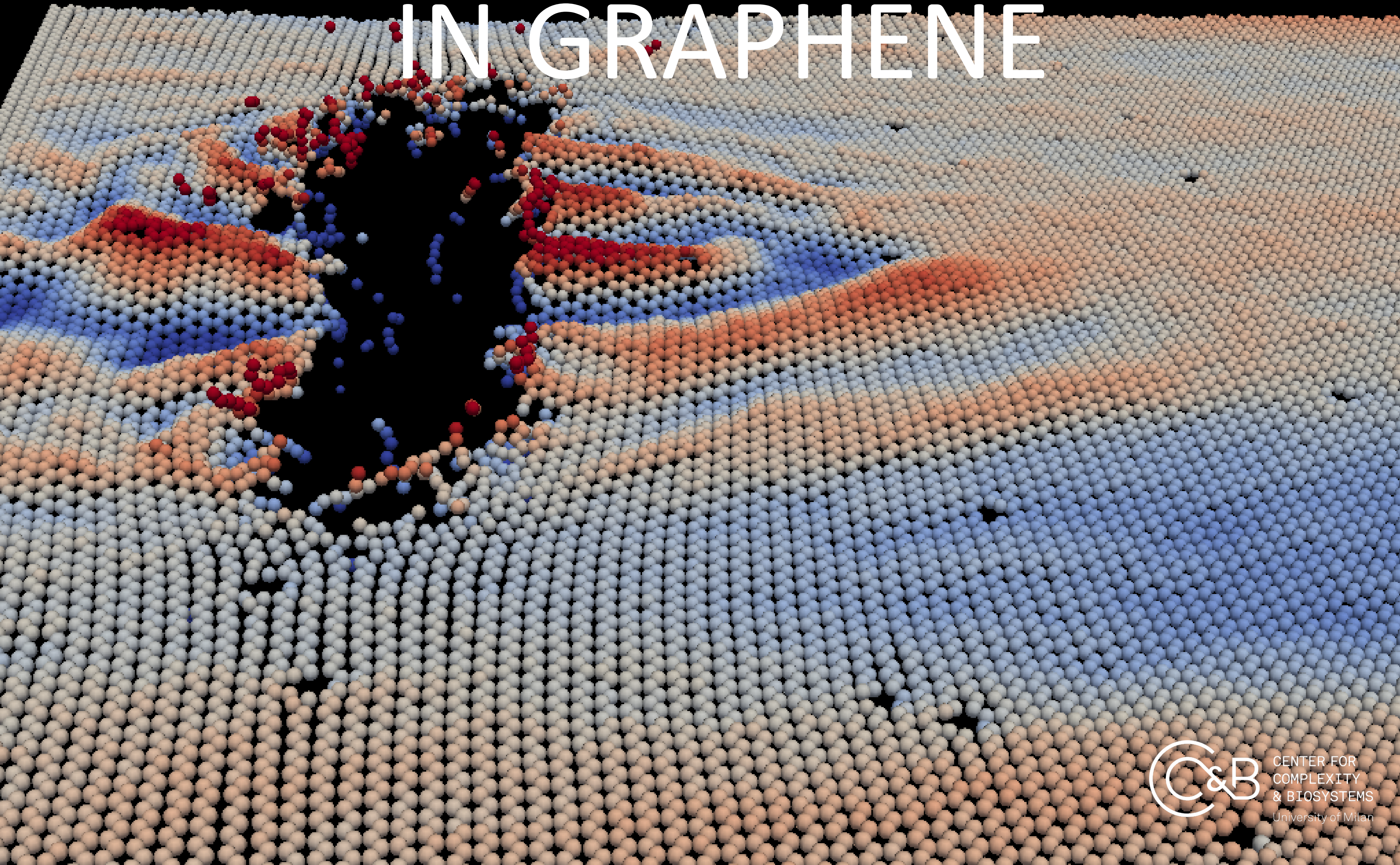
FRACTURE SIZE EFFECTS IN GRAPHENE



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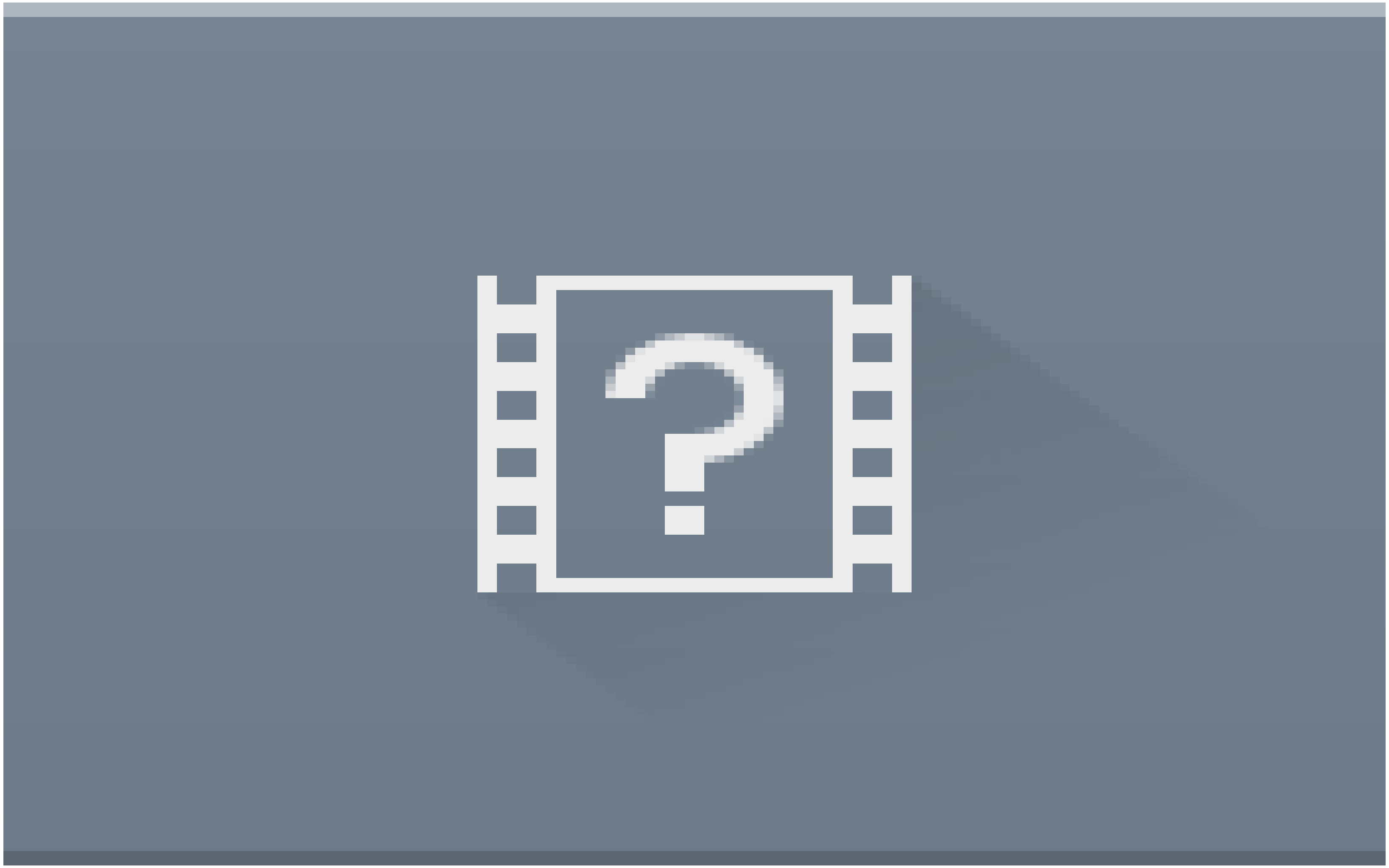
Fracture of defected graphene



A. L. Sellerio, A. Taloni and SZ. PR Applied (2015)

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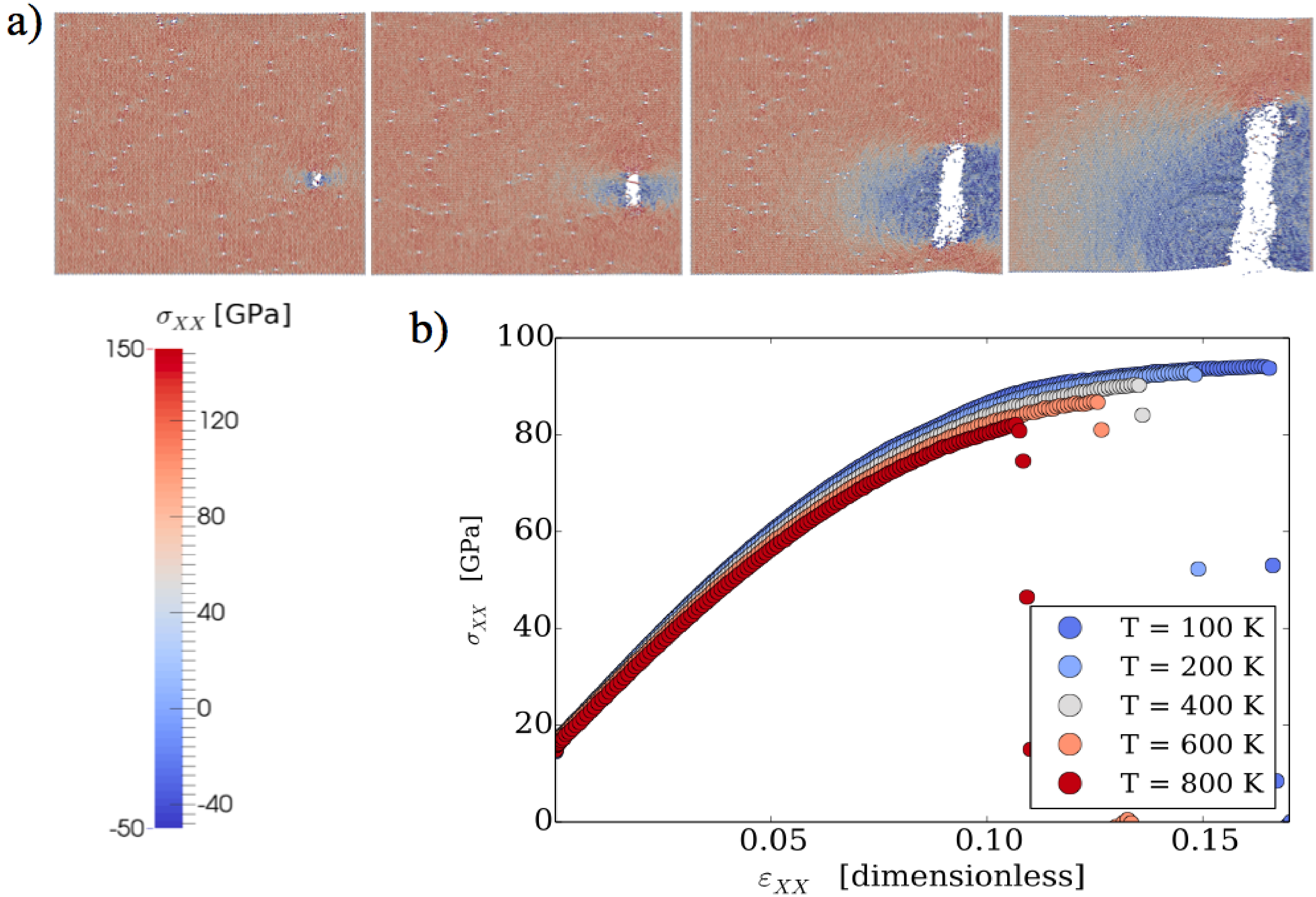
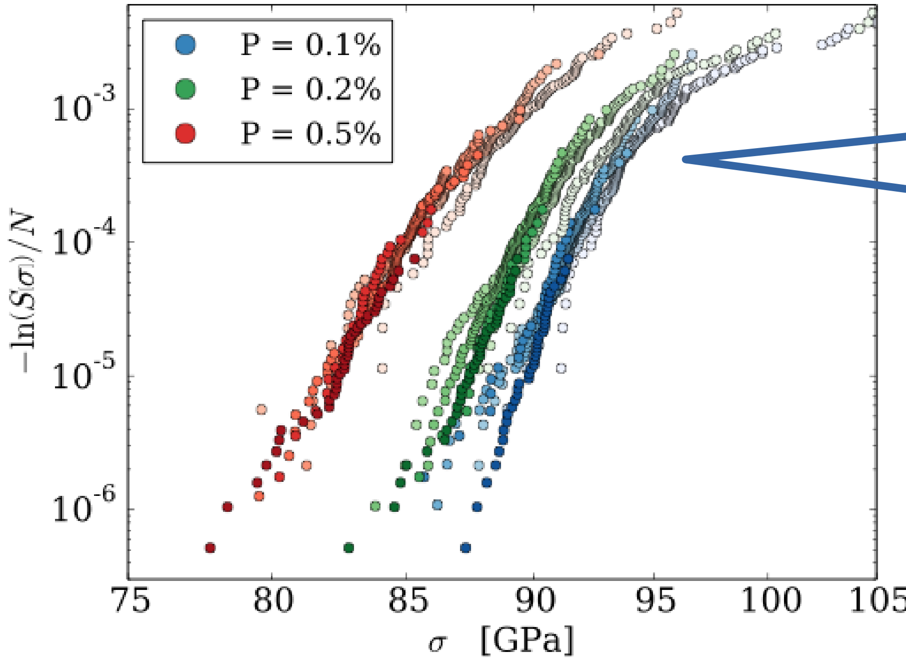
Fracture of defected graphene



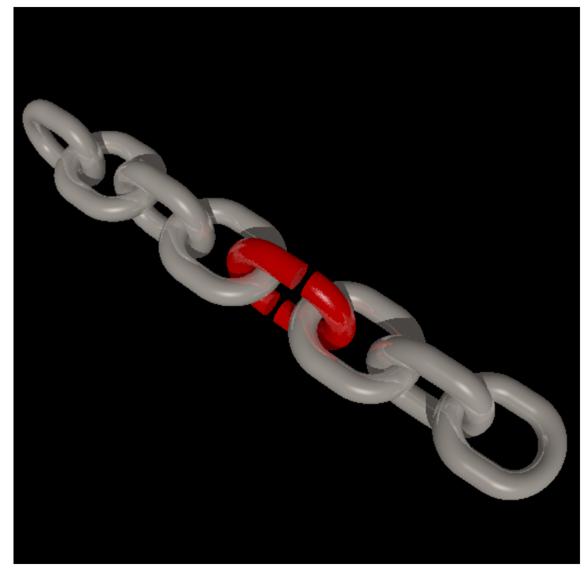
A. L. Sellerio, A. Taloni and SZ. PR Applied (2015)

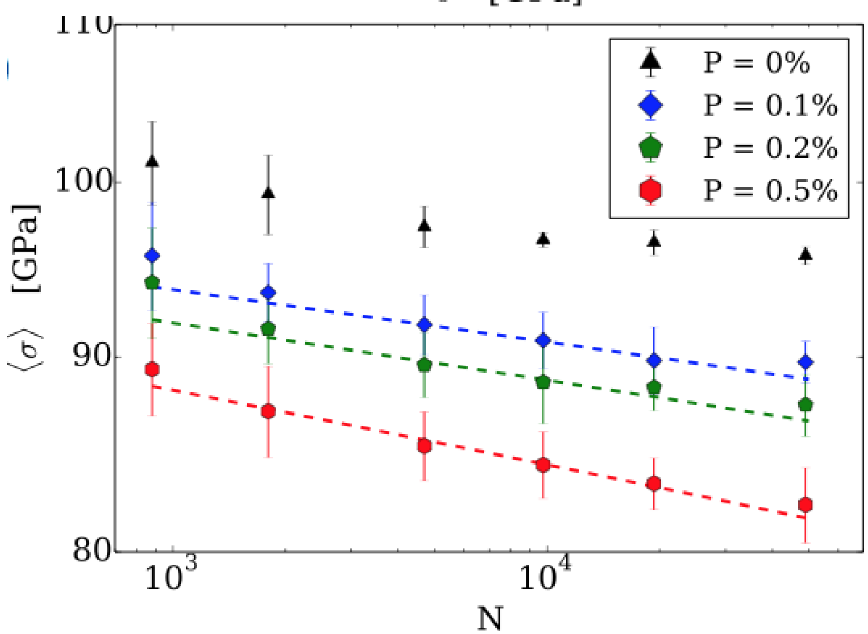
Failure distribution: size effects



Extreme value statistics?

$$S(\sigma) = S_0(\sigma)^{V/V_0}$$





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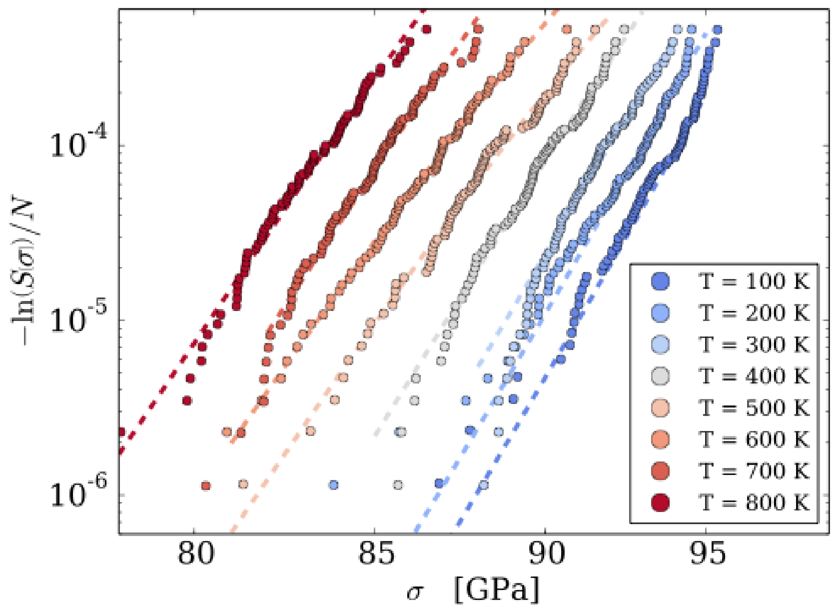
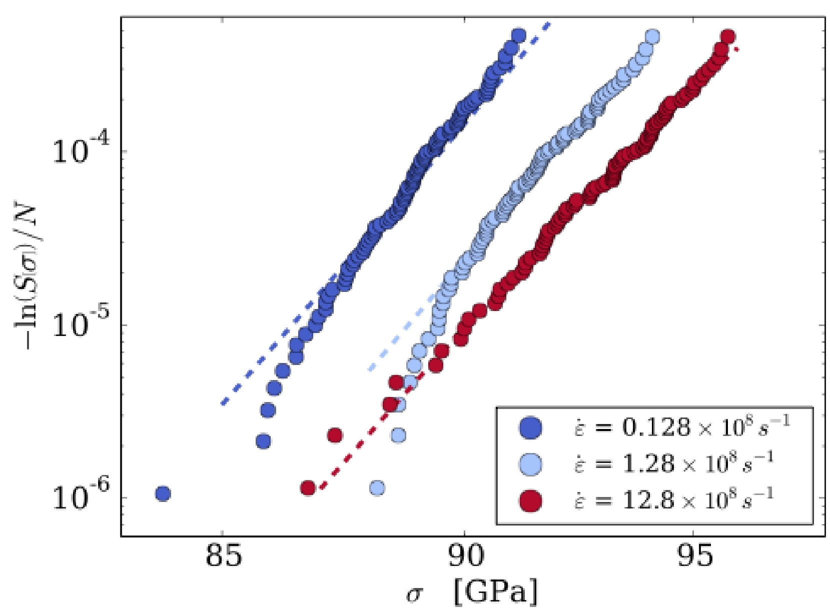
Failure distribution: temperature and rate effects





Theory

Effective potential for n RVEs:

$$U_0(\varepsilon, \varepsilon_c) = \begin{cases} V_0 \frac{E\varepsilon^2}{2} & \varepsilon \leq \varepsilon_c \\ -\infty & \varepsilon > \varepsilon_c \end{cases}$$

Survival of a single RVEs:

$$S_0(\sigma|T, \dot{\varepsilon}) = \int_{\sigma/E}^{\infty} d\varepsilon_c \rho(\varepsilon_c) \Sigma_0(\sigma|\varepsilon_c, T, \dot{\varepsilon})$$

Survival for n RVEs:

$$S_n(\sigma|T, \dot{\varepsilon}) \doteq [S_0(\sigma|T, \dot{\varepsilon})]^n$$

$$S_n(\sigma|T, \dot{\varepsilon}) = \Bigg(\int_{\sigma/E}^{\infty} d\varepsilon_c \rho(\varepsilon_c) \times \exp -\frac{\omega_0}{\dot{\varepsilon}} \sqrt{\frac{V_0 E}{\pi k_B T}} \left[e^{-(\varepsilon_c - \sigma/E)^2 \frac{V_0 E}{k_B T}} - e^{-\varepsilon_c^2 \frac{V_0 E}{k_B T}} \right] \Bigg)^n$$

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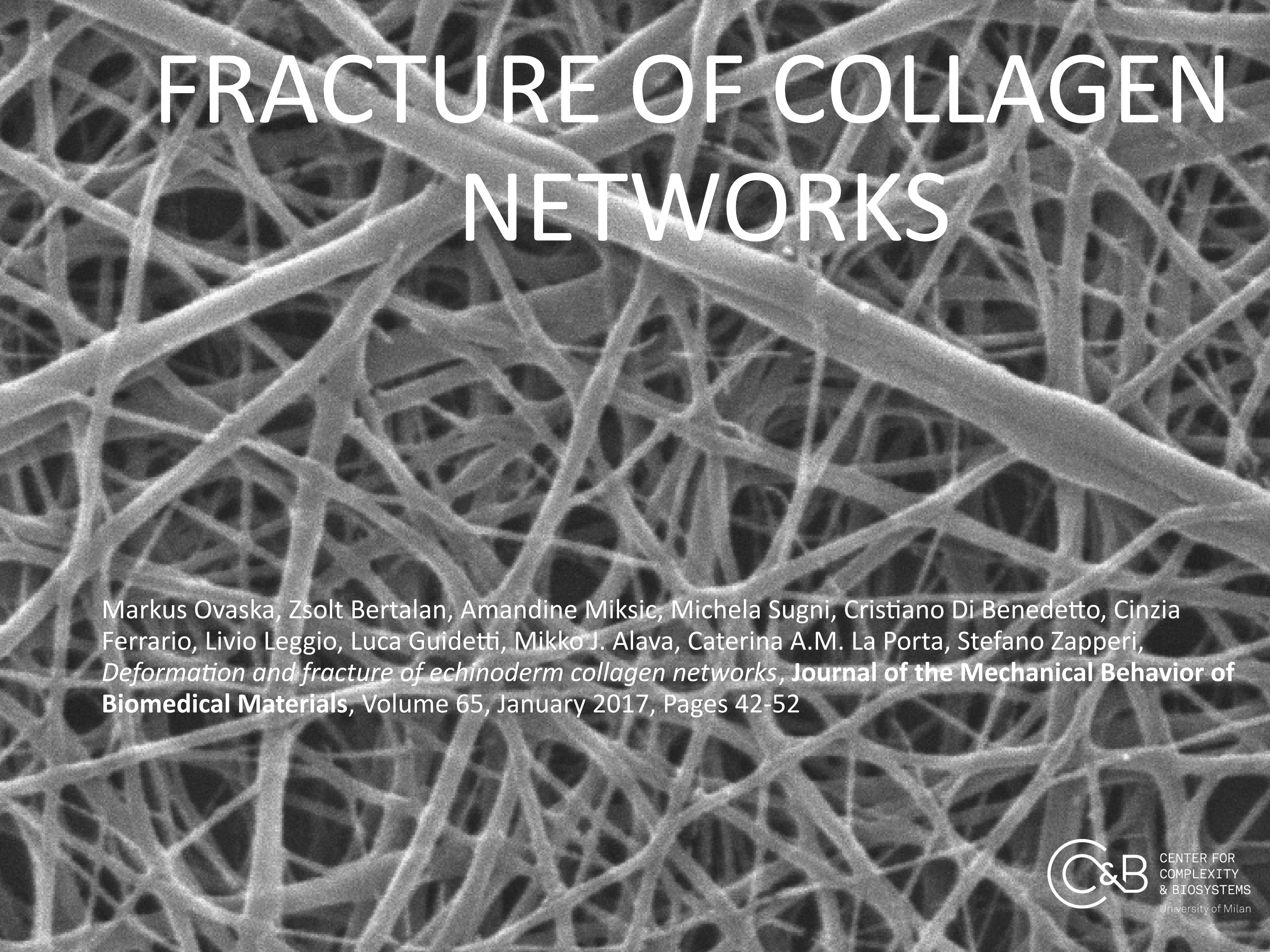

FRACTURE OF COLLAGEN NETWORKS

Markus Ovaska, Zsolt Bertalan, Amandine Miksic, Michela Sugni, Cristiano Di Benedetto, Cinzia Ferrario, Livio Leggio, Luca Guidetti, Mikko J. Alava, Caterina A.M. La Porta, Stefano Zapperi, *Deformation and fracture of echinoderm collagen networks*, **Journal of the Mechanical Behavior of Biomedical Materials**, Volume 65, January 2017, Pages 42-52

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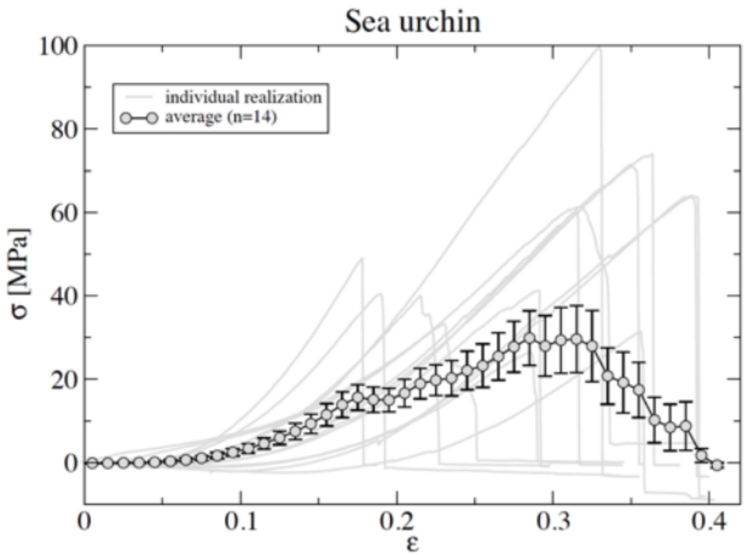
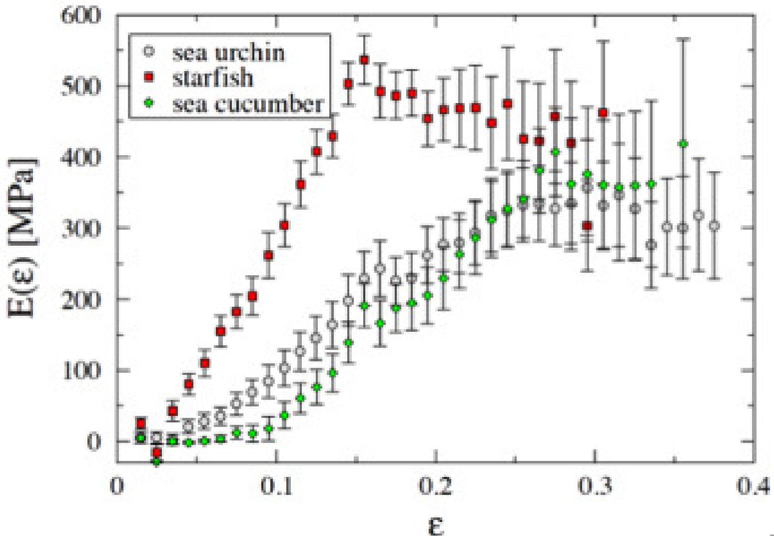
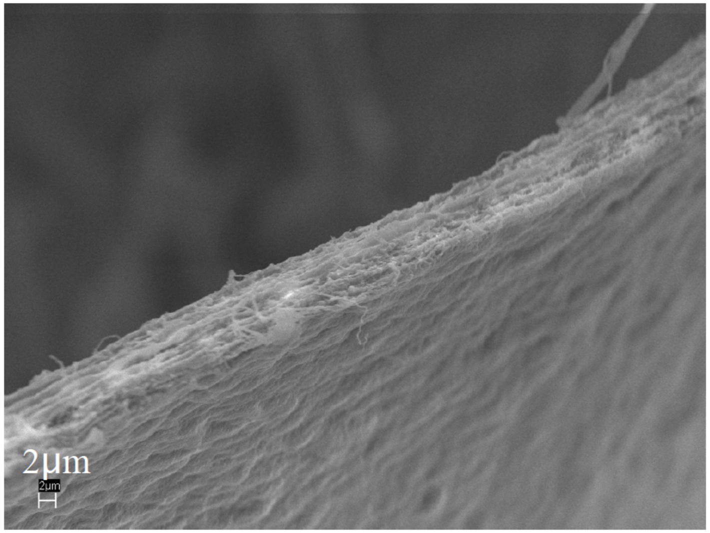
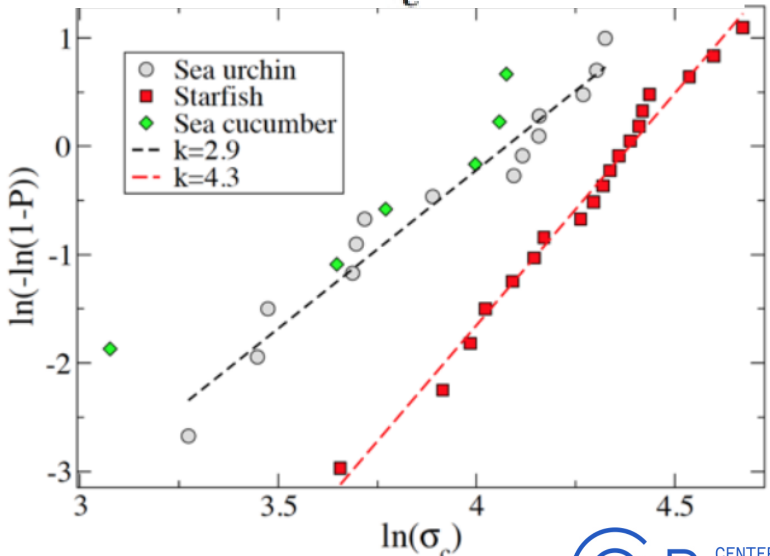
Sea Urchin
Starfish
Sea cucumber

Non-linear stiffening and fracture









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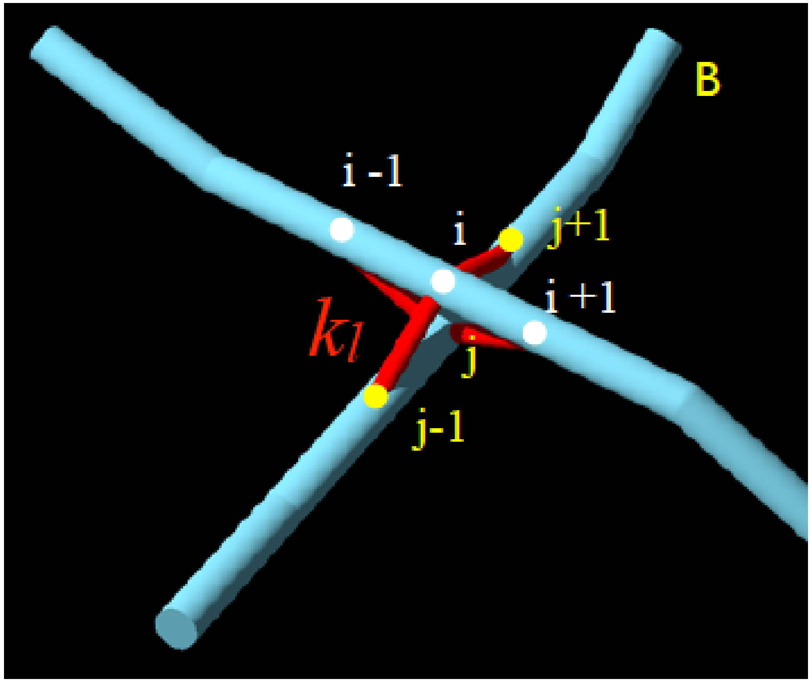
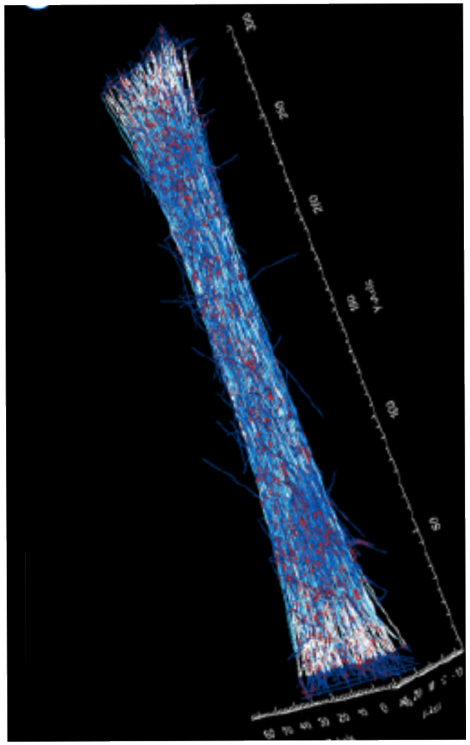
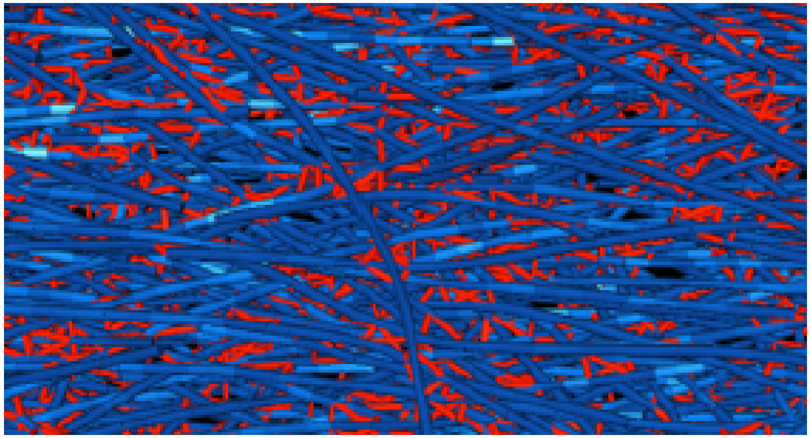
3D computational model



$$E_{\mathrm{f}} = \frac{1}{2}\sum_{i=2}^{L} k_{\mathrm{f}}(|\mathbf{r}_i - \mathbf{r}_{i-1}| - a)^2 + K\sum_{j=1}^{L-1}\cos(\theta_j),$$





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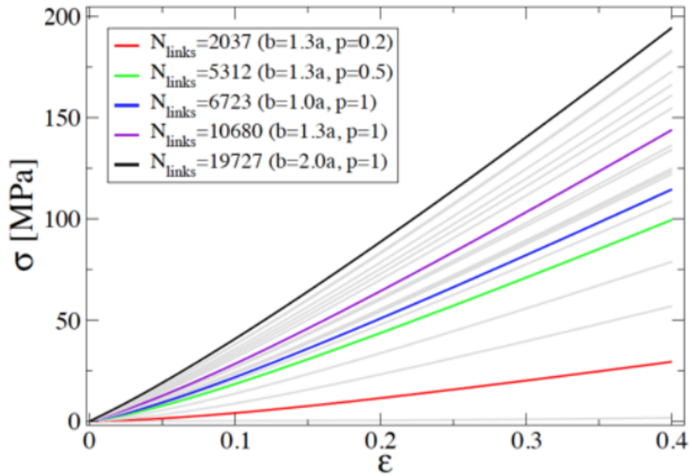
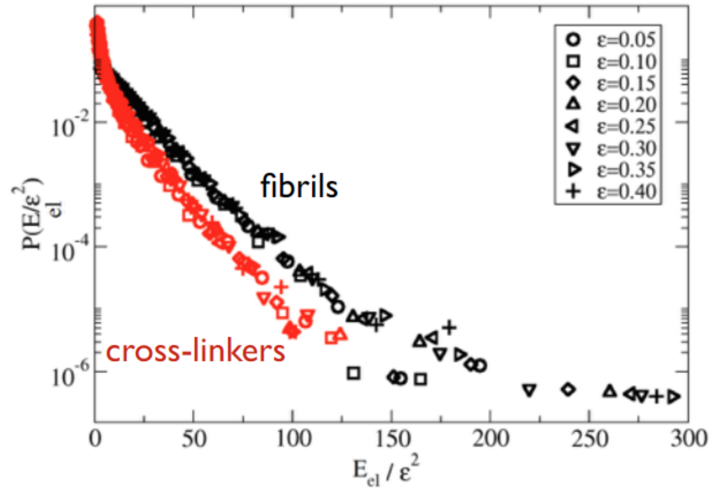
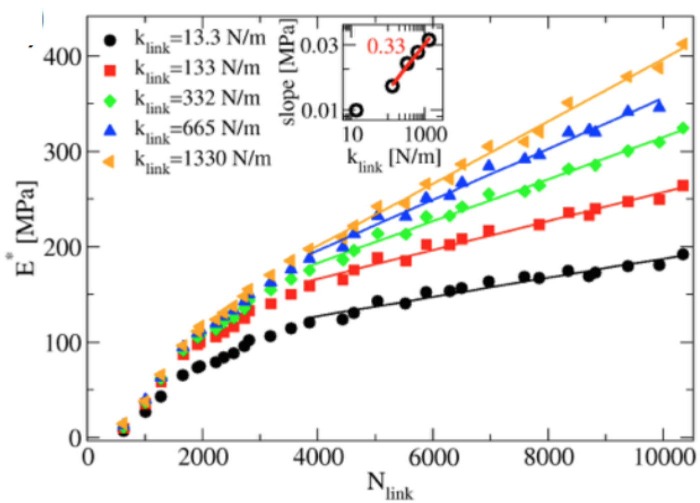
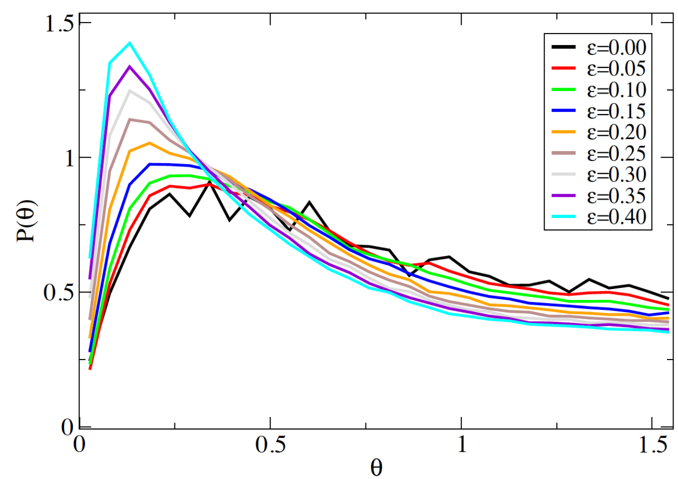
3D computational model









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FRACTURE AND PLASTICITY IN CURVED SPACE

Carlotta Negri, Alessandro L. Sellerio, Stefano Zapperi, M. Carmen Miguel
Deformation and failure of curved colloidal crystal shells, PNAS 112 14545 (2015)

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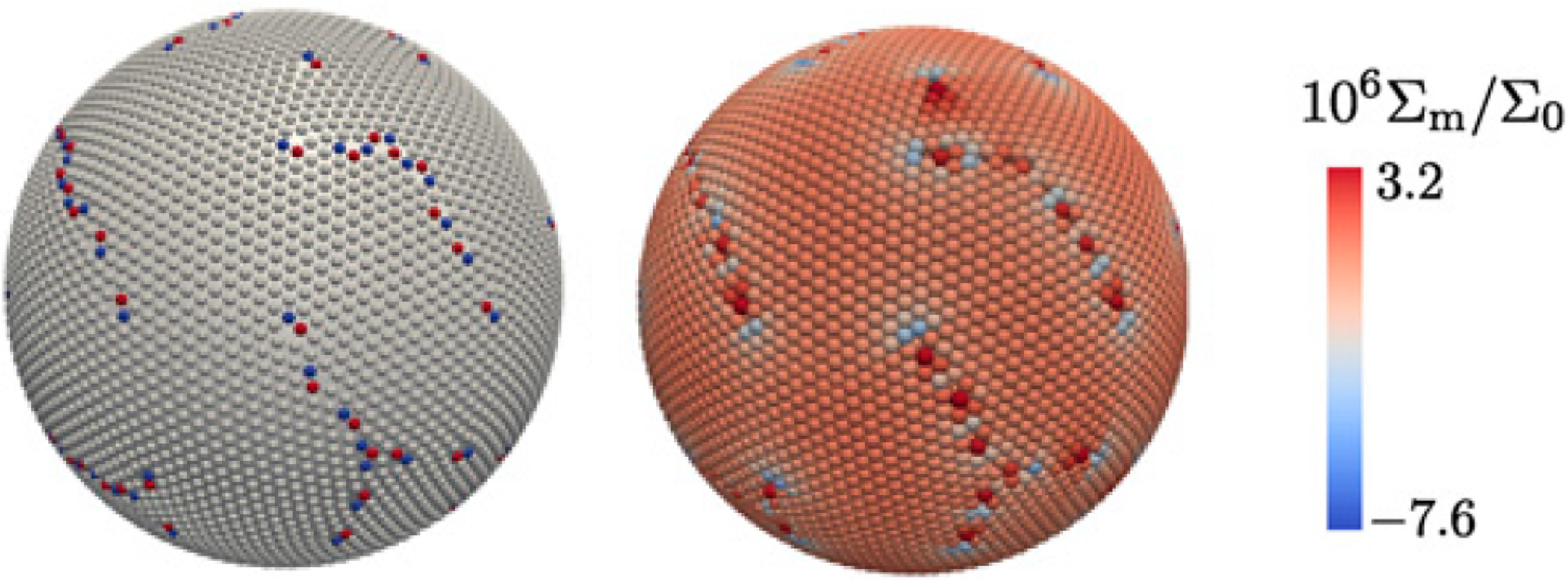
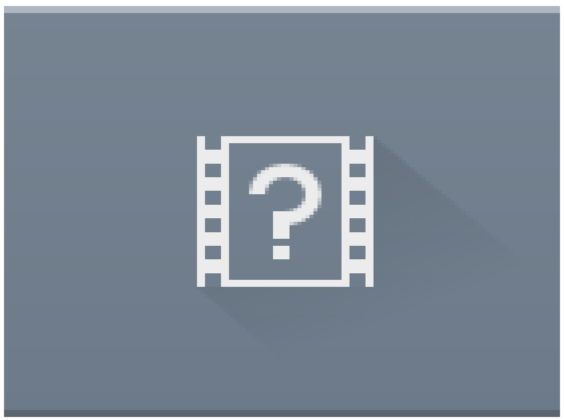
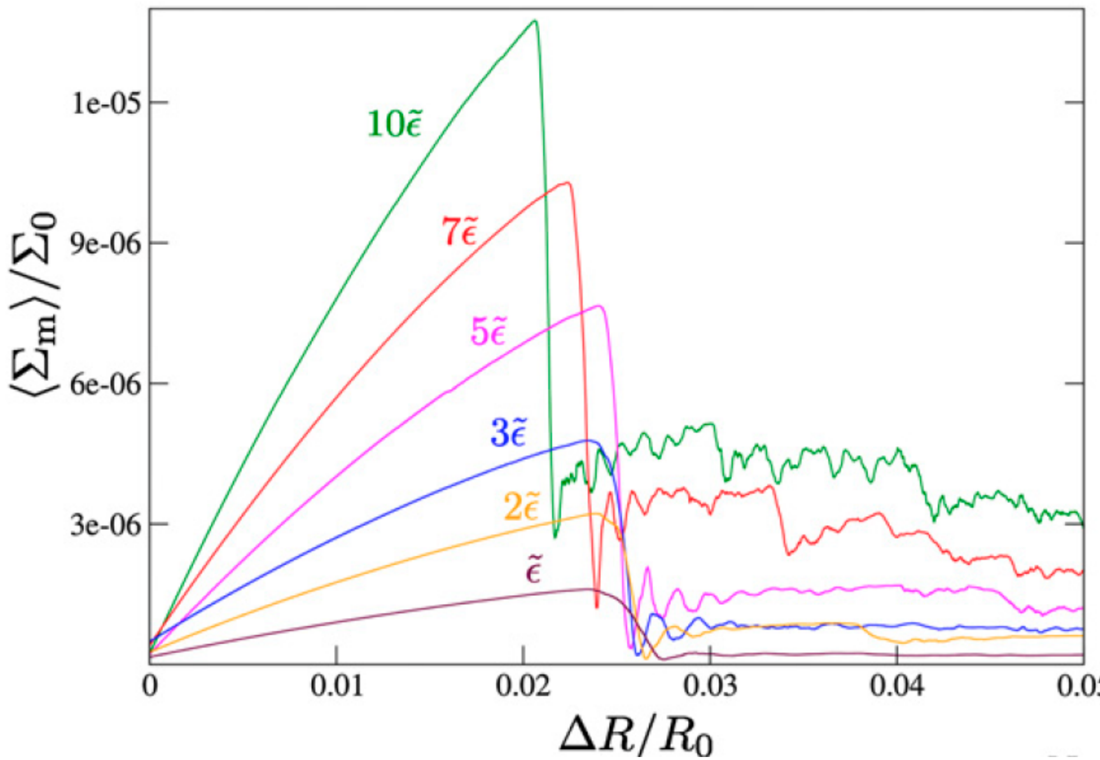
Colloidal crystals on spherical shells



Method: MD simulations of LJ interacting particles with confining potential

Curved crystals: Fracture





$$\mathbf{F} = -K(r - R(t))\frac{\mathbf{r}}{r},$$

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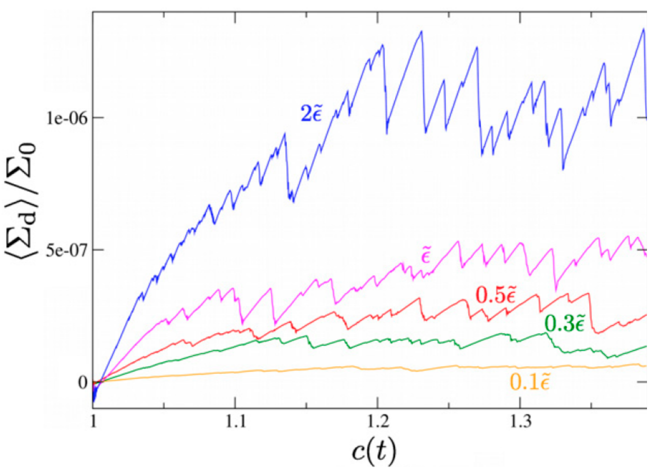
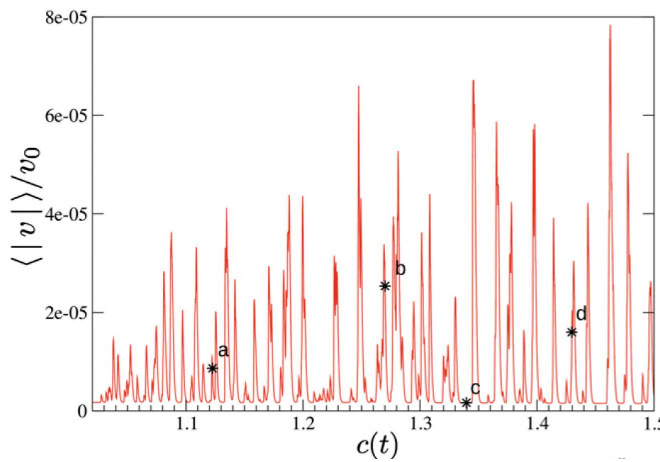
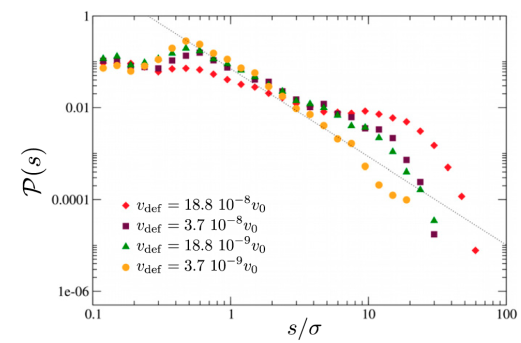
Curved crystal plasticity







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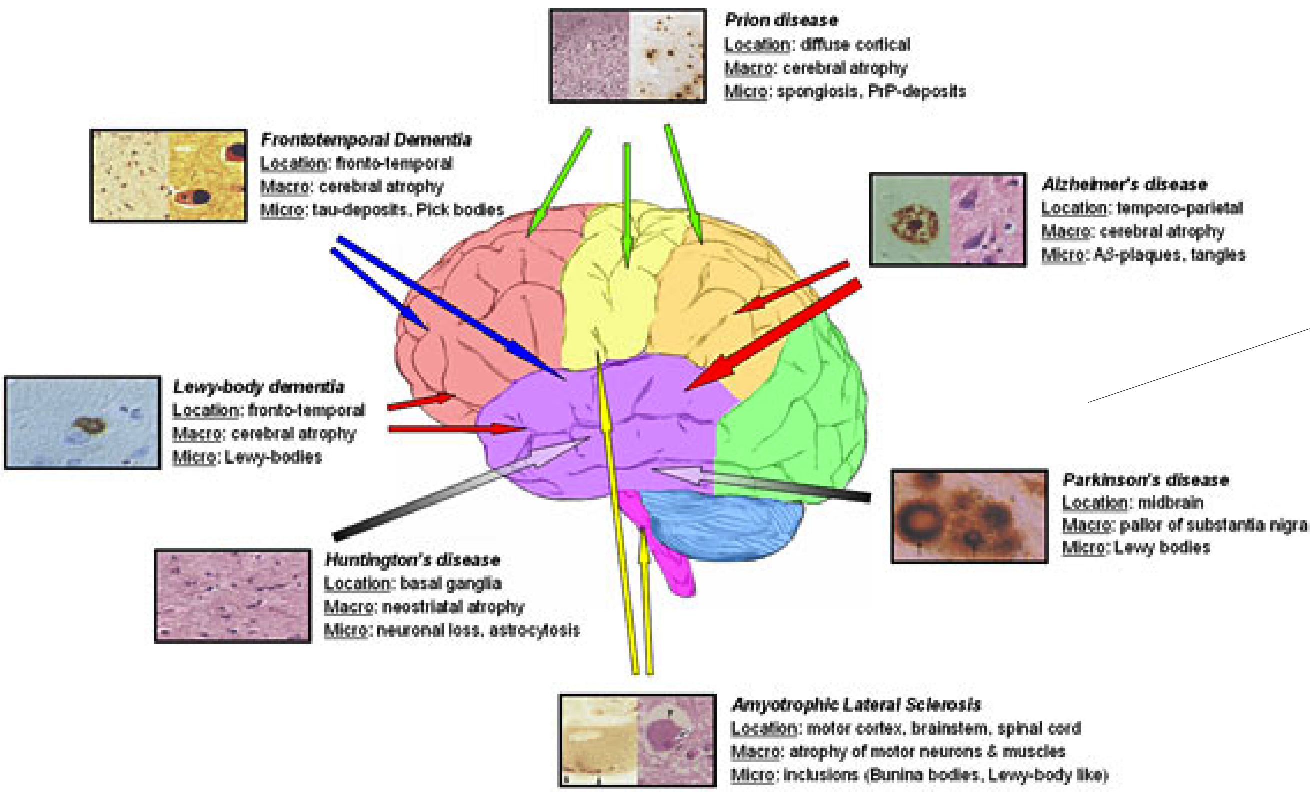
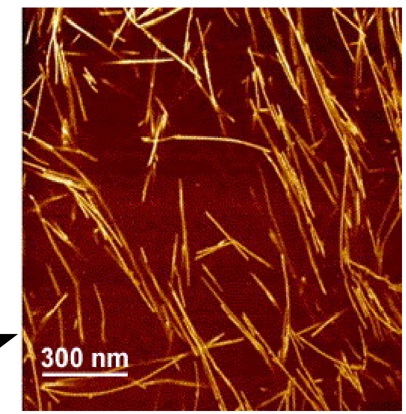
$$\mathbf{F} = -K(d(t) - R)\frac{\mathbf{r}}{d(t)} \qquad d = \sqrt{(x^2+y^2)/a^2 + z^2/c^2} \qquad c(t) = 1 + t\, v_{\mathrm{def}}$$

Extreme events in neurodegenerative diseases





α-synuclein

Bertram and Tanzi, *JCI*, 2005

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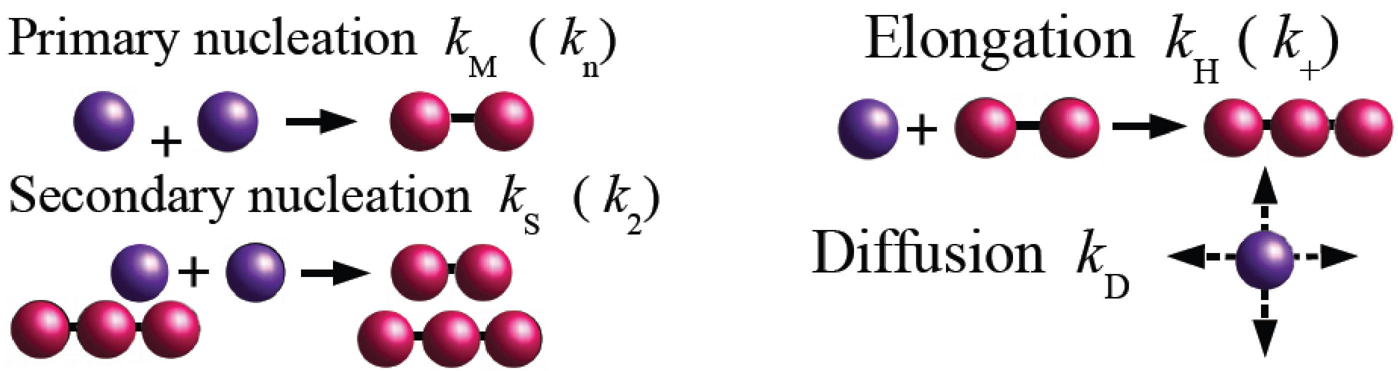
Secondary nucleation speeds up aggregation



Mean-field kinetics

$$\dot{P}(t) = k_2 M(t) m(t)^{n_2} + k_n m(t)^{n_c},$$

$$\dot{M}(t) = 2k_+ m(t) P(t) + n_2 k_2 m(t)^{n_2} + n_c k_n m(t)^{n_c},$$

Number concentration

Mass concentration

S. I. A. Cohen, S. Linse, L. M. Luheshi, E. Hellstrand, D. A. White, L. Rajah, D. E. Otzen, M. Vendruscolo, C. M. Dobson, and T. P. J. Knowles, Proceedings of the National Academy of Sciences **110**, 9758 (2013).

A. K. Buell, C. Galvagnion, R. Gaspar, E. Sparr, M. Vendruscolo, T. P. J. Knowles, S. Linse, and C. M. Dobson, Proc Natl Acad Sci U S A **111**, 7671 (2014).

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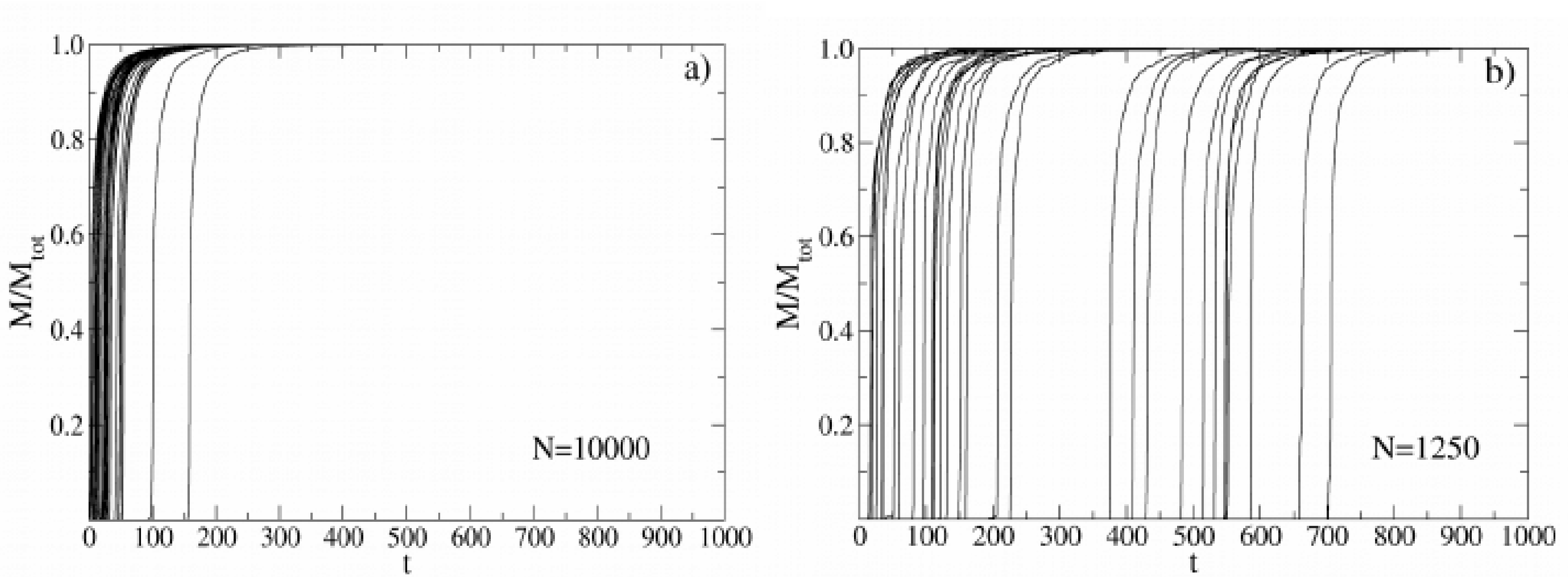
Fluctuations and size-effects



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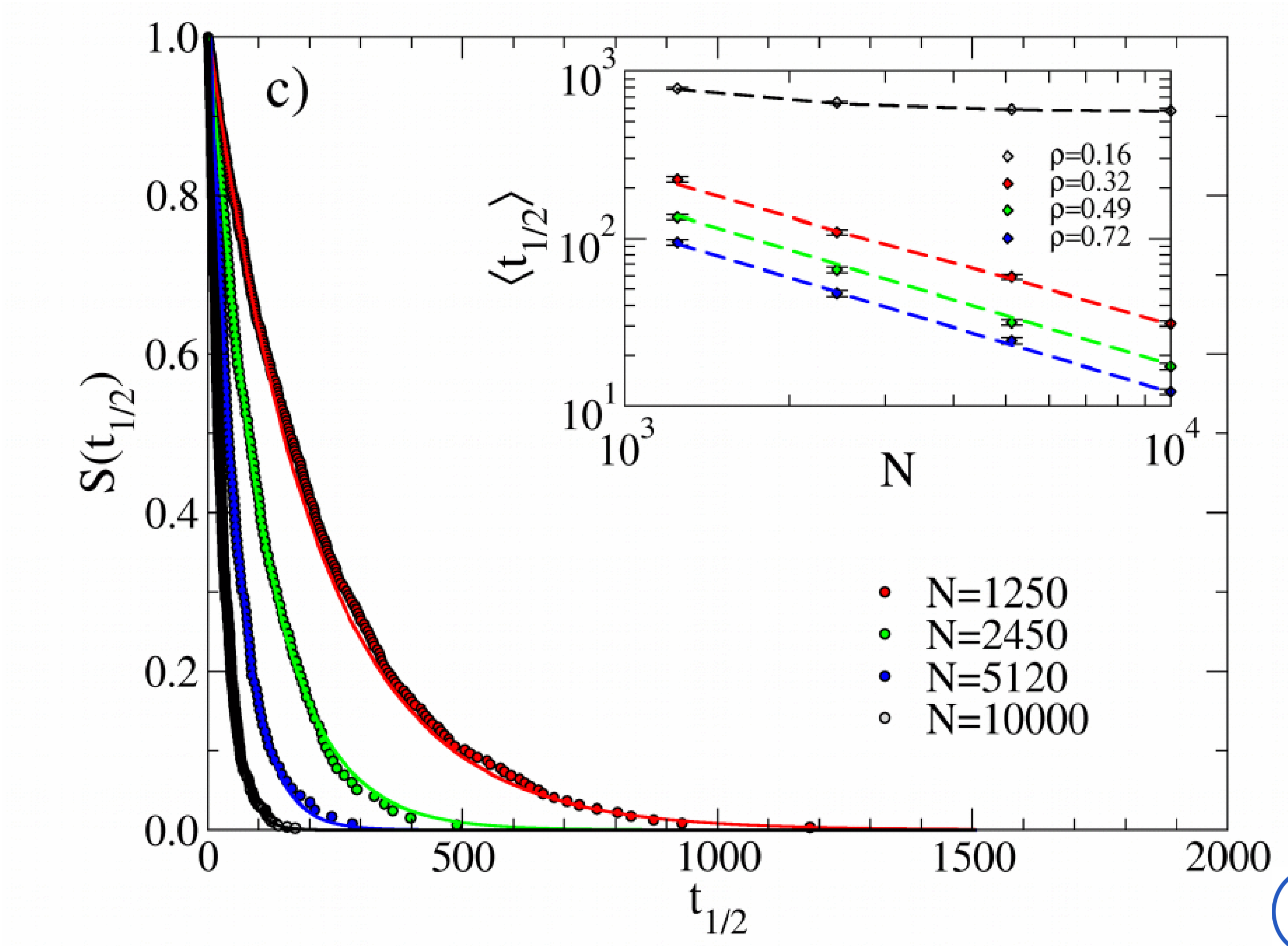
Half-time (cumulative) distribution



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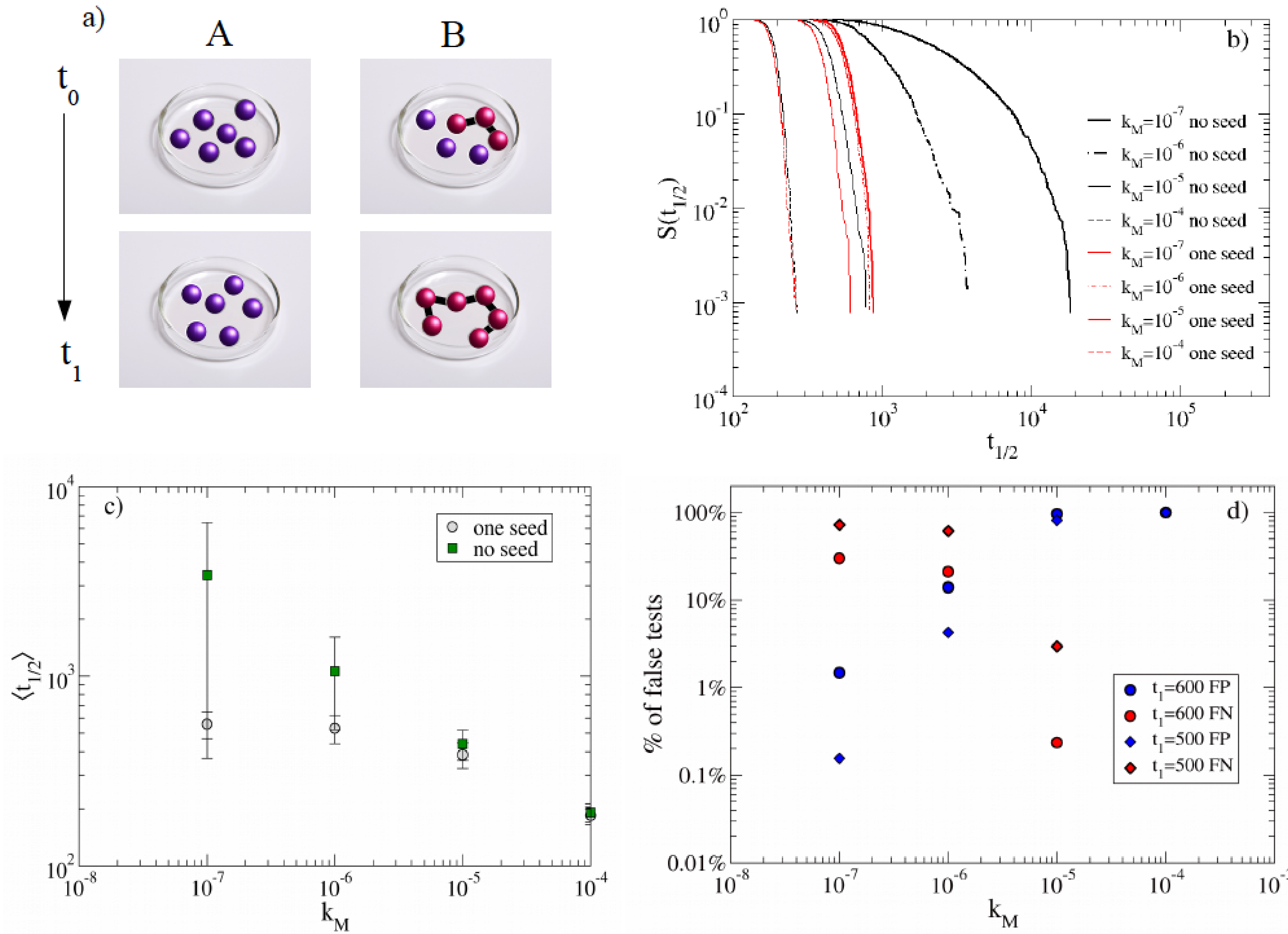
Seed aggregation testing (in silico)



Theory: first passage time

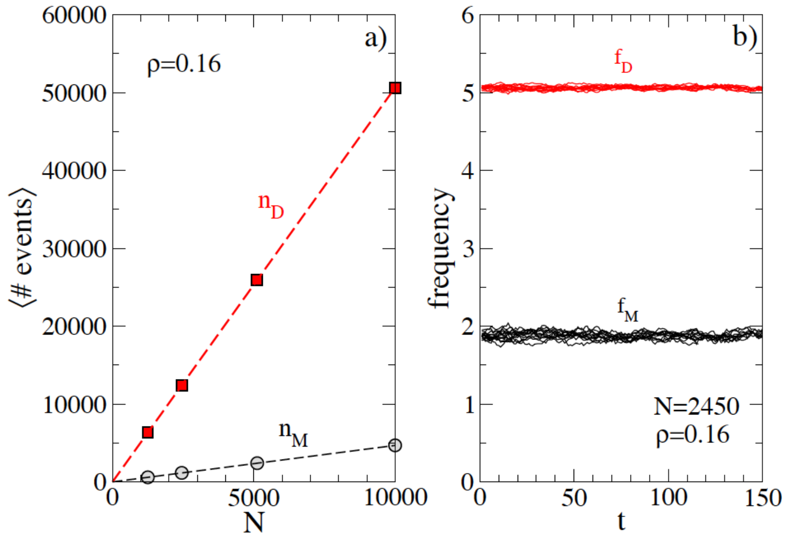
$$\tilde{P}_0(t_0) = \lim_{n\to\infty}\left(1 - n_M k_M \frac{t_0}{n}\right)^{n-1} n_M k_M = n_M k_M e^{-n_M k_M t_0}.$$

Nucleation time

$$P_0(t_0) \simeq \langle \tilde{P}_0(t_0)\rangle \simeq \langle n_M \rangle k_M e^{-f_M k_M N t_0}$$

$$S_0(t_0) = \int_{t_0}^{\infty} d\tau P_0(\tau) = e^{-f_M k_M N t_0},$$

$$S(t_{1/2}) = \begin{cases} 1 & t_{1/2} \leq \langle\tau\rangle \\ S_0(t_{1/2} - \langle\tau\rangle) & t_{1/2} > \langle\tau\rangle. \end{cases}$$



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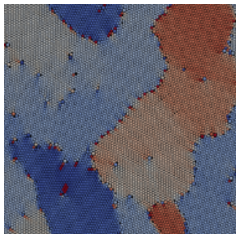
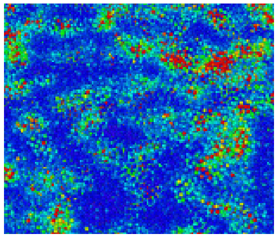
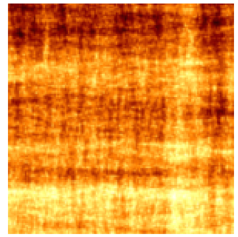
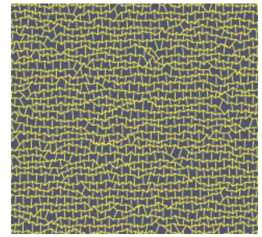
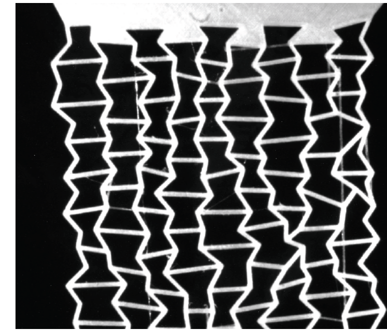
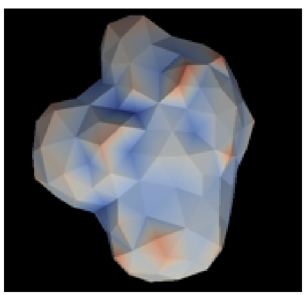
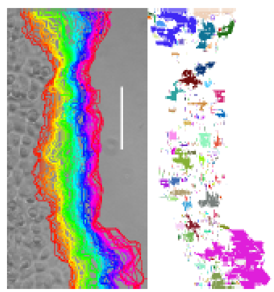
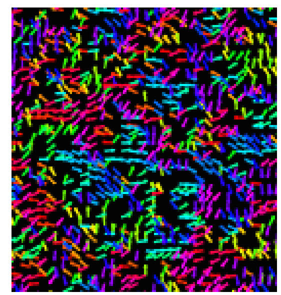
Present and future directions

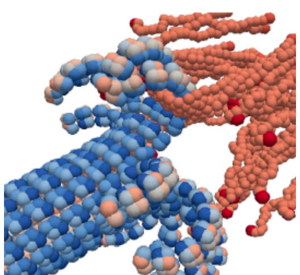
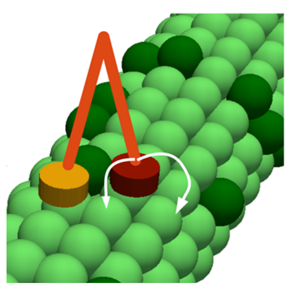
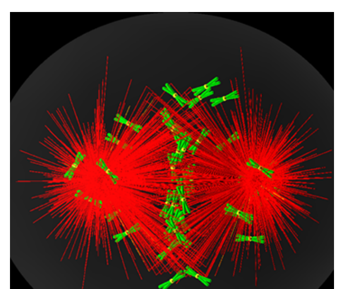
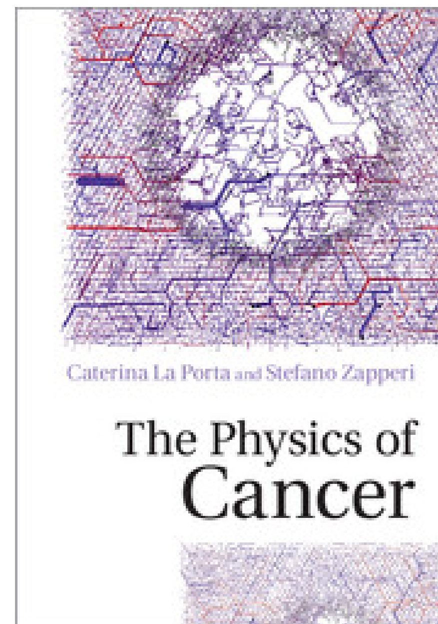
From disordered materials to meta-materials

From individual to collective cell migration

Cell division and the physics of cancer

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Thanks

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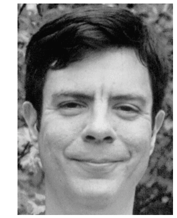

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