Is quantum theory informationally complete?

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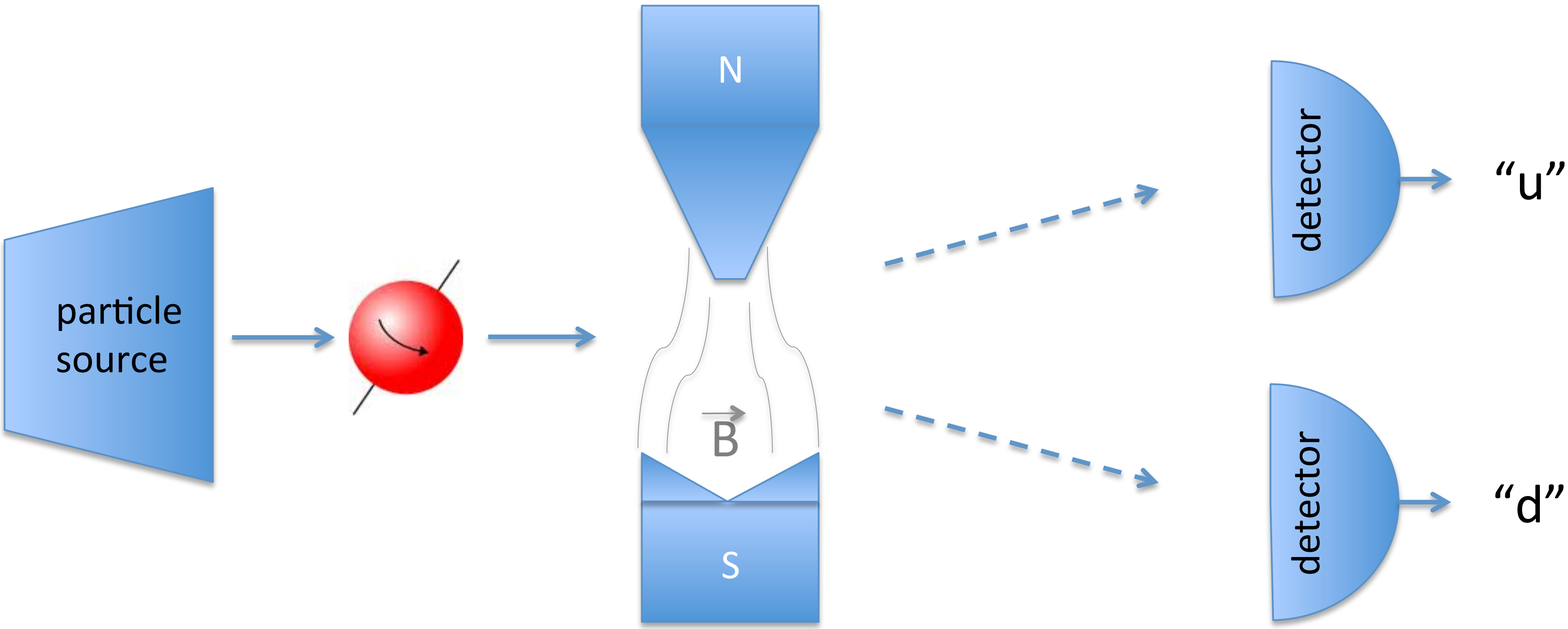
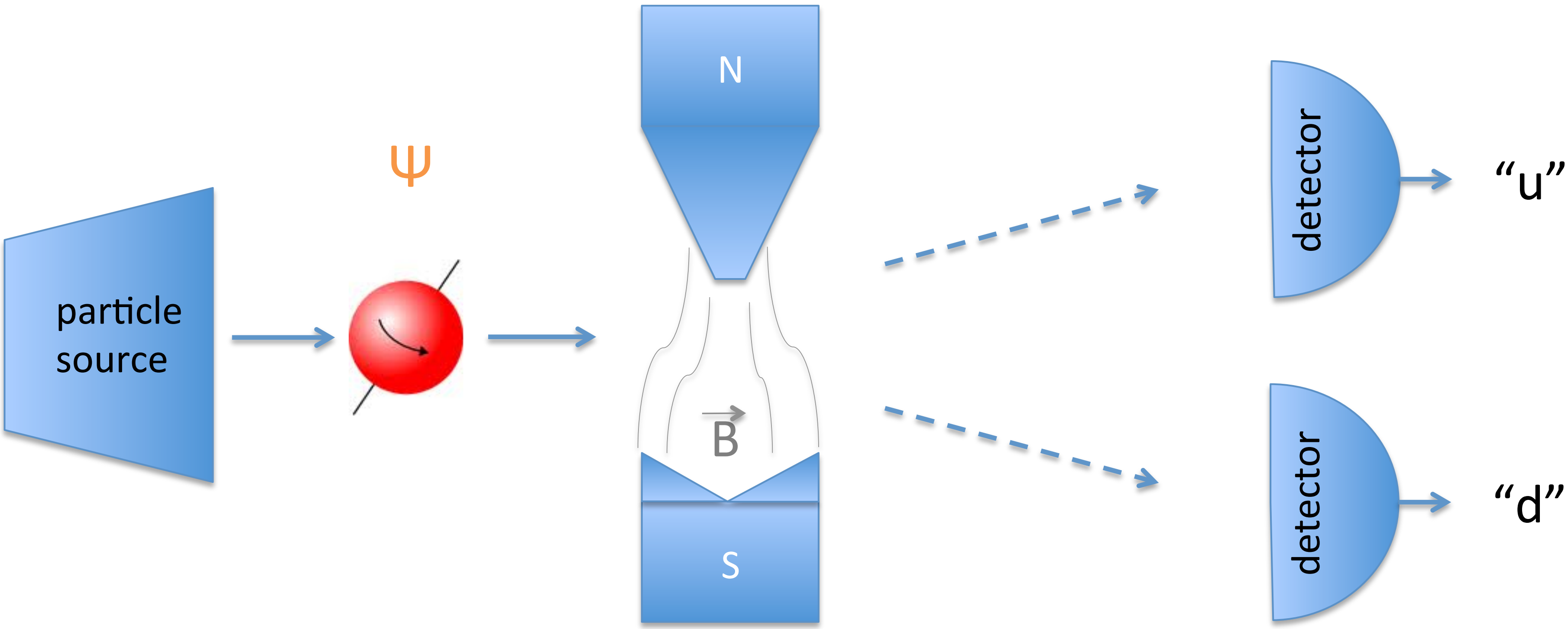
Predictions by quantum theory



Stern-Gerlach experiment (1921)

Predictions by quantum theory



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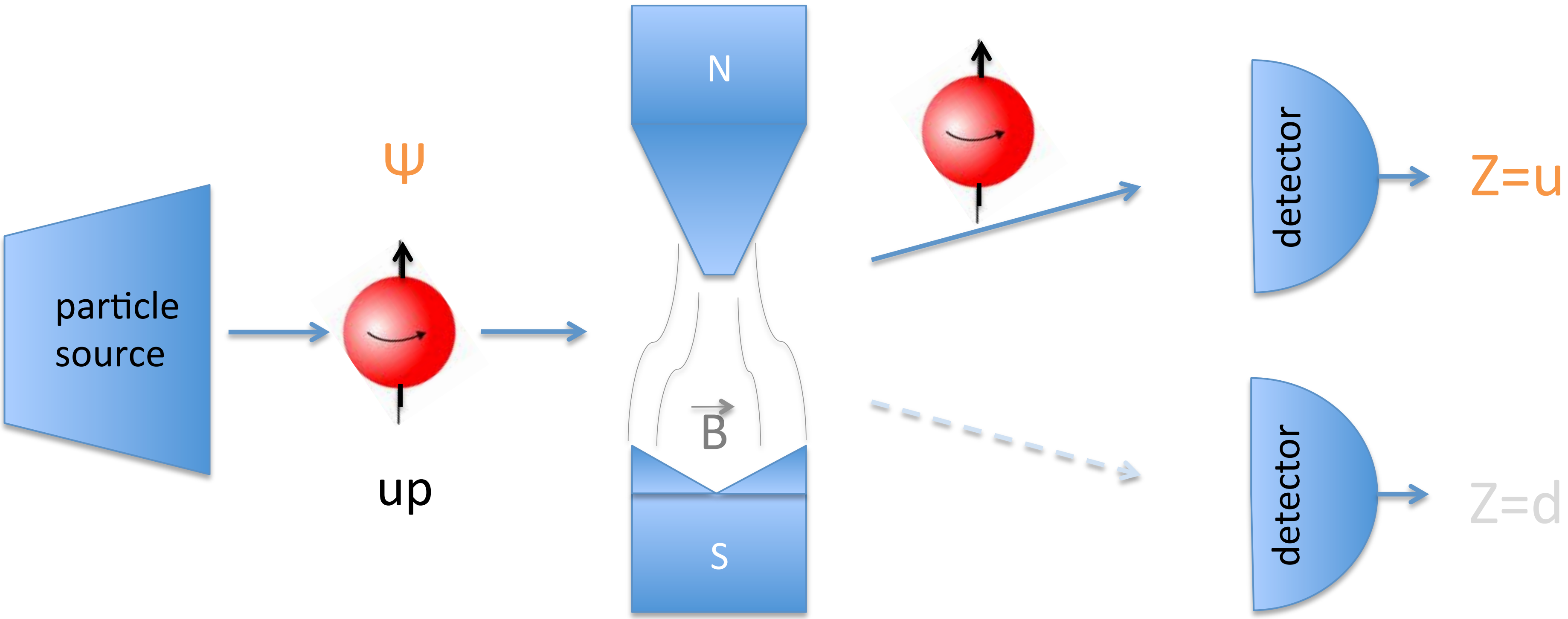
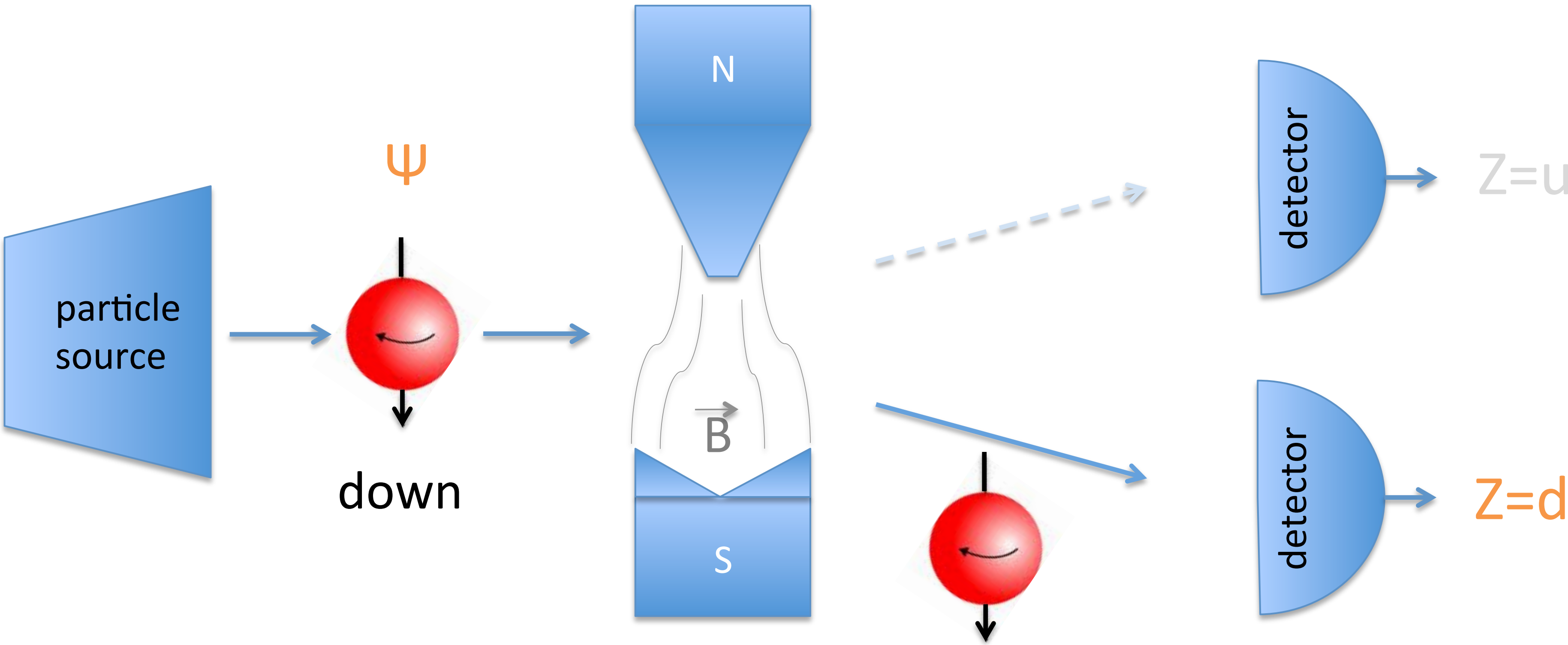


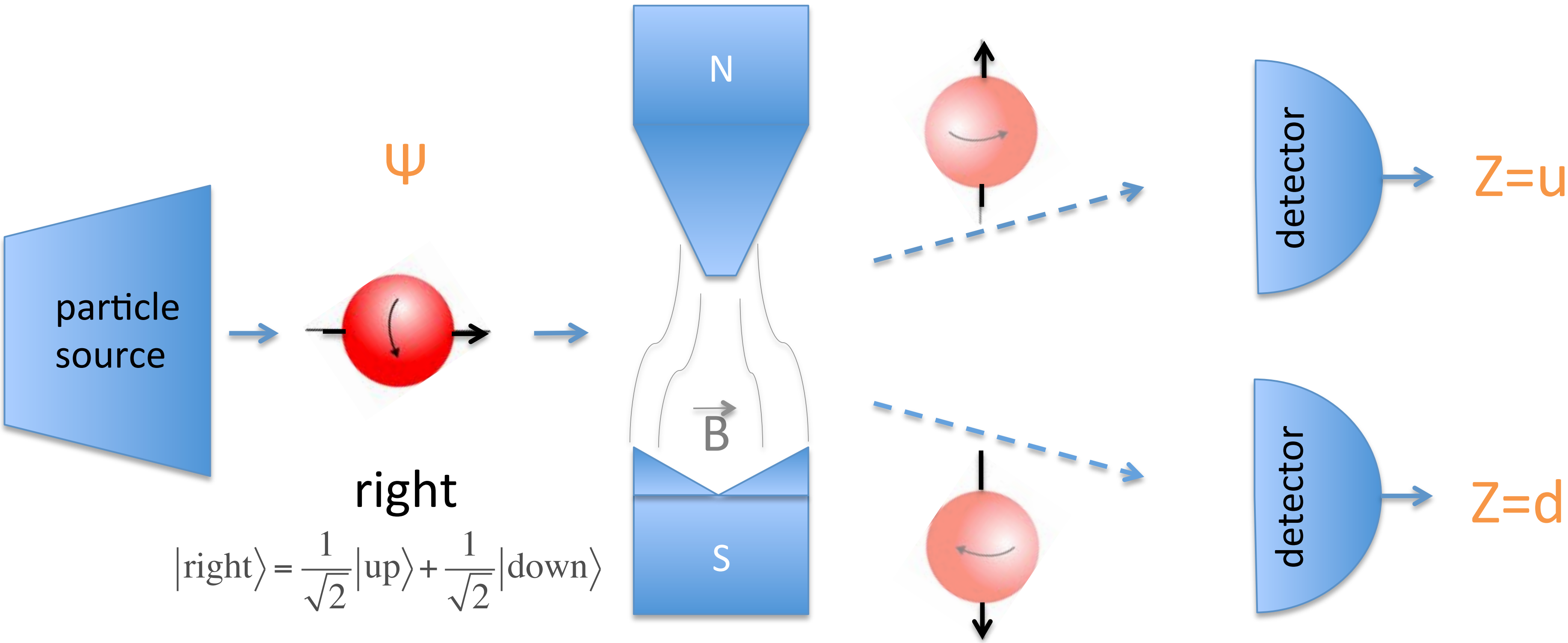
Probability of outcome “u”: Pr[Z=u | Ψ = up] = 1

Predictions by quantum theory



Probability of outcome “d”: Pr[Z=u | Ψ = down] = 0

Predictions by quantum theory



Probability of outcome “u”: Pr[Z=u | Ψ = right] = ½

Both outcomes are equally likely!

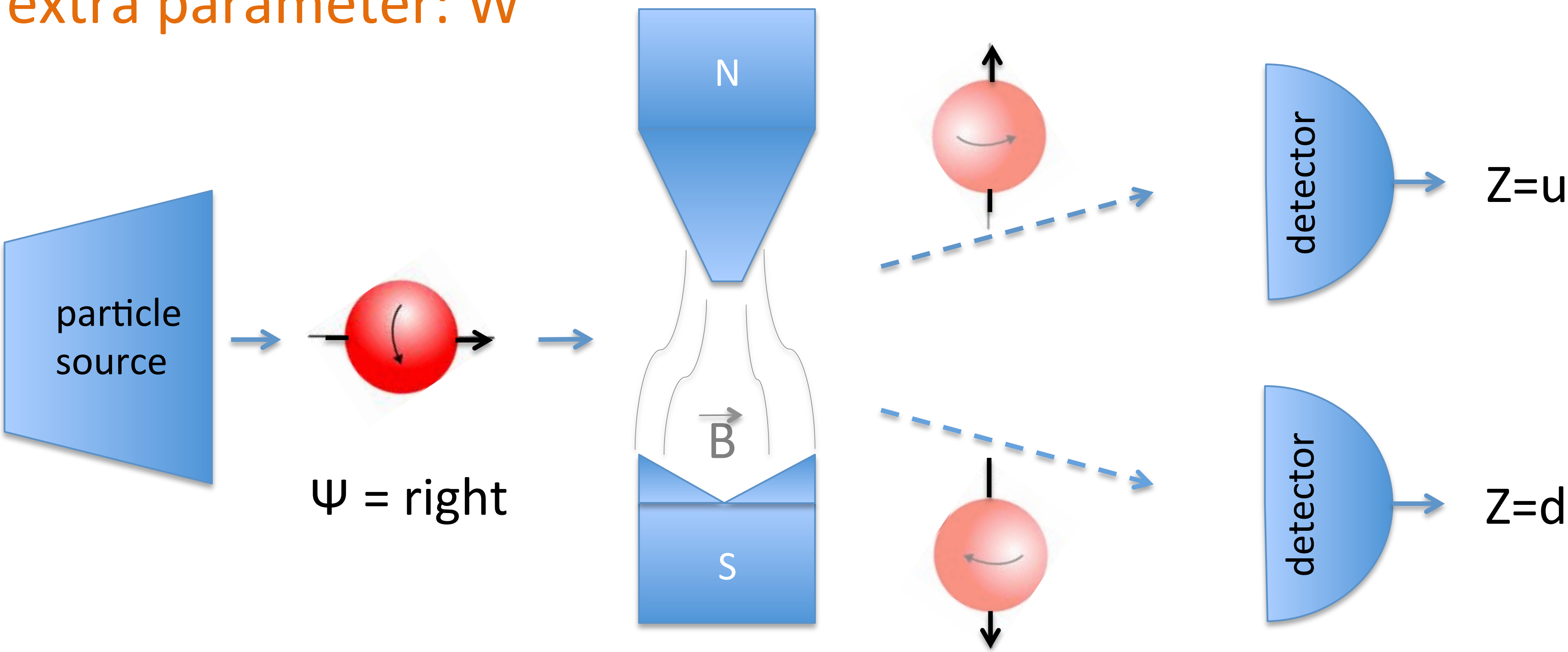
Is this the best possible prediction?

Is this the best possible prediction?

Let us have a look at a (toy) example for an extension of quantum theory ...

An extended theory

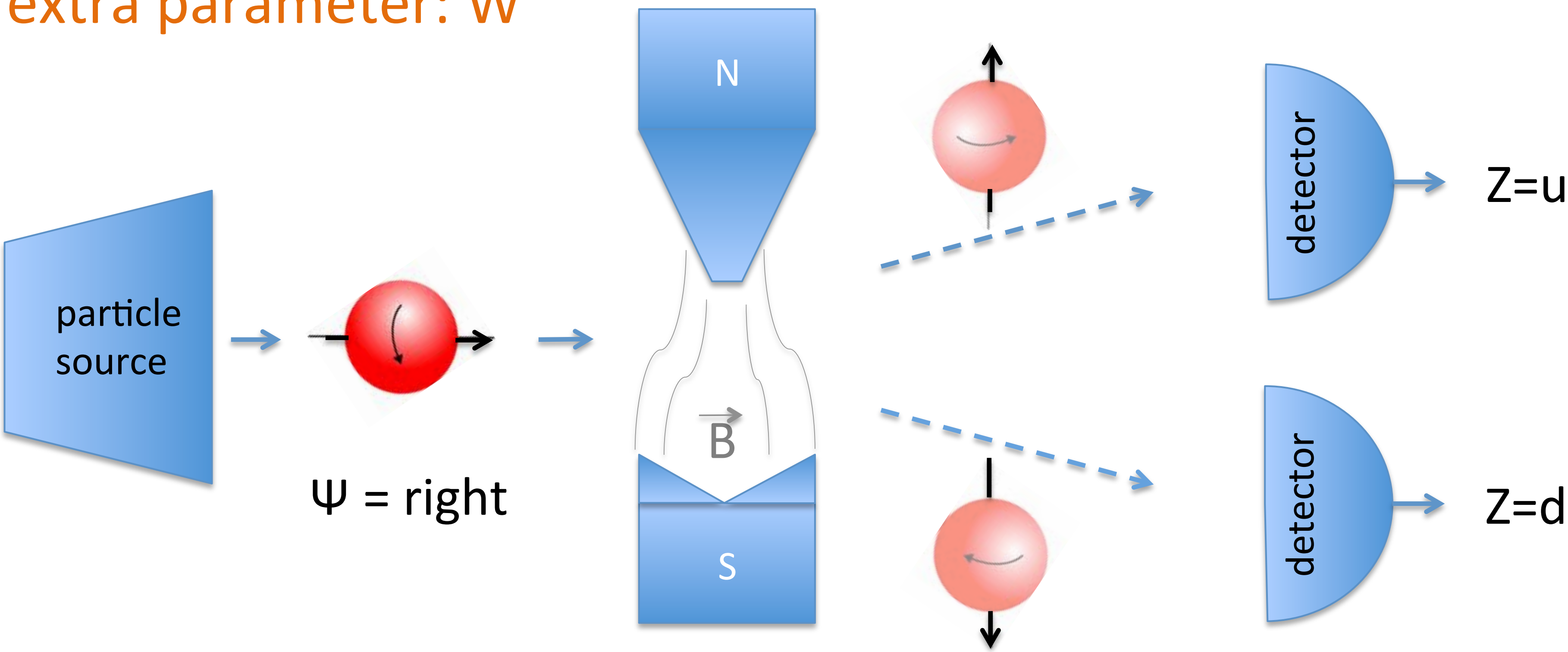
extra parameter: W



W takes either the value "u" or "d".

An extended theory

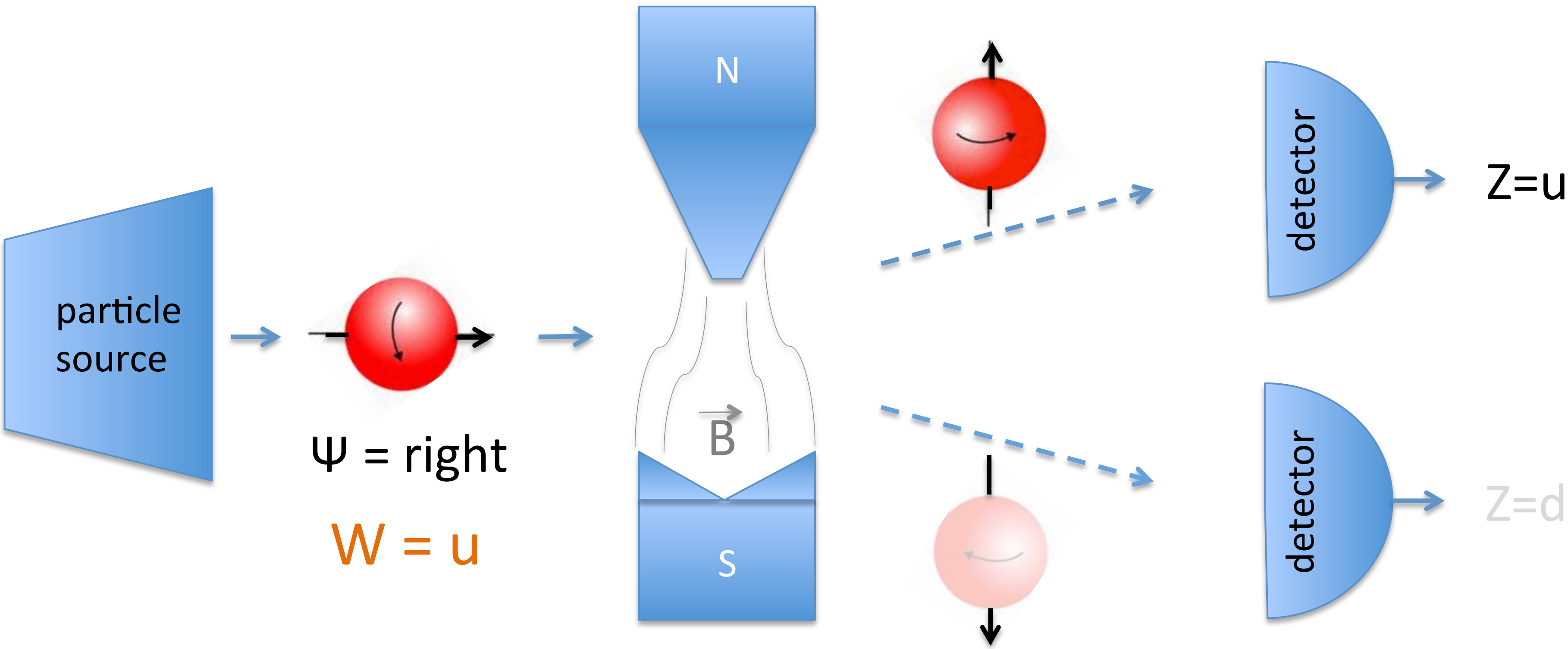
extra parameter: W



"Rule" of extended theory: Pr[Z=u | Ψ = right, W=u] = ¾

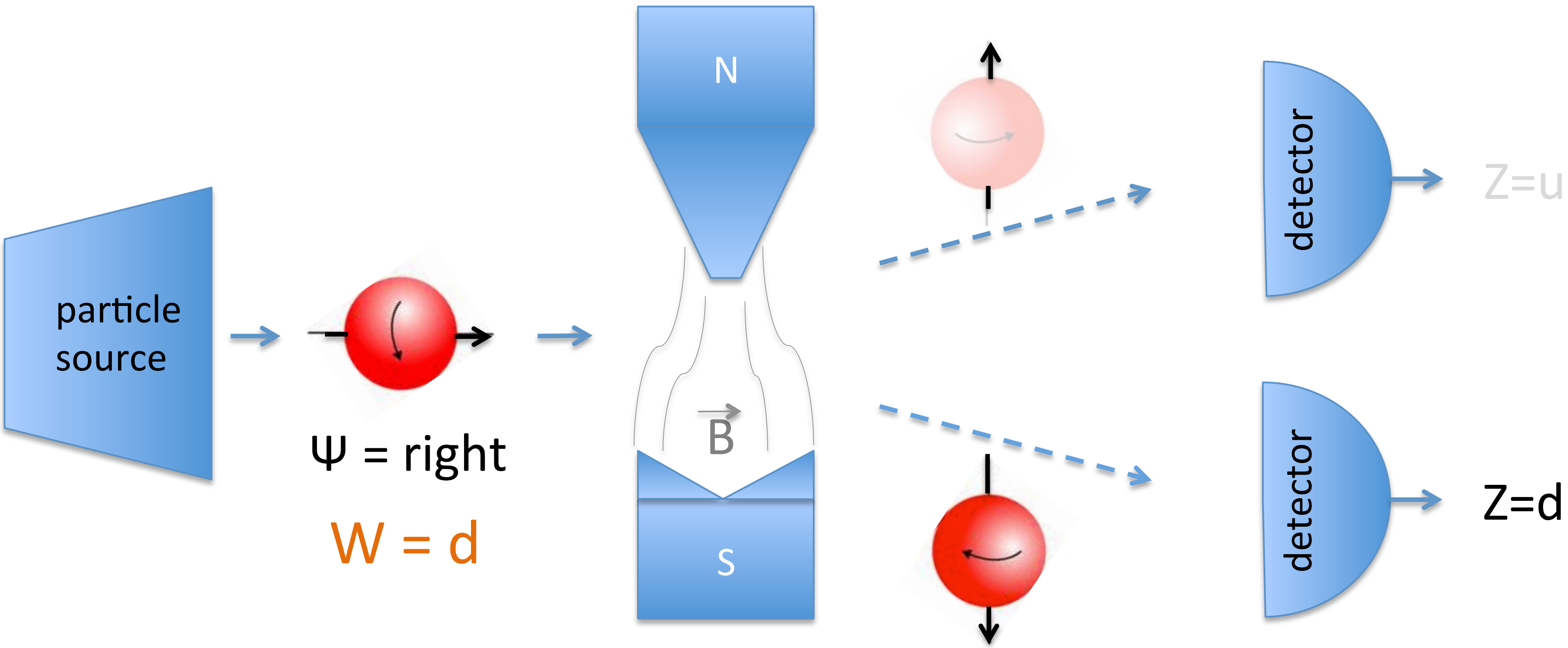
Pr[Z=u | Ψ = right, W=d] = ¼

An extended theory



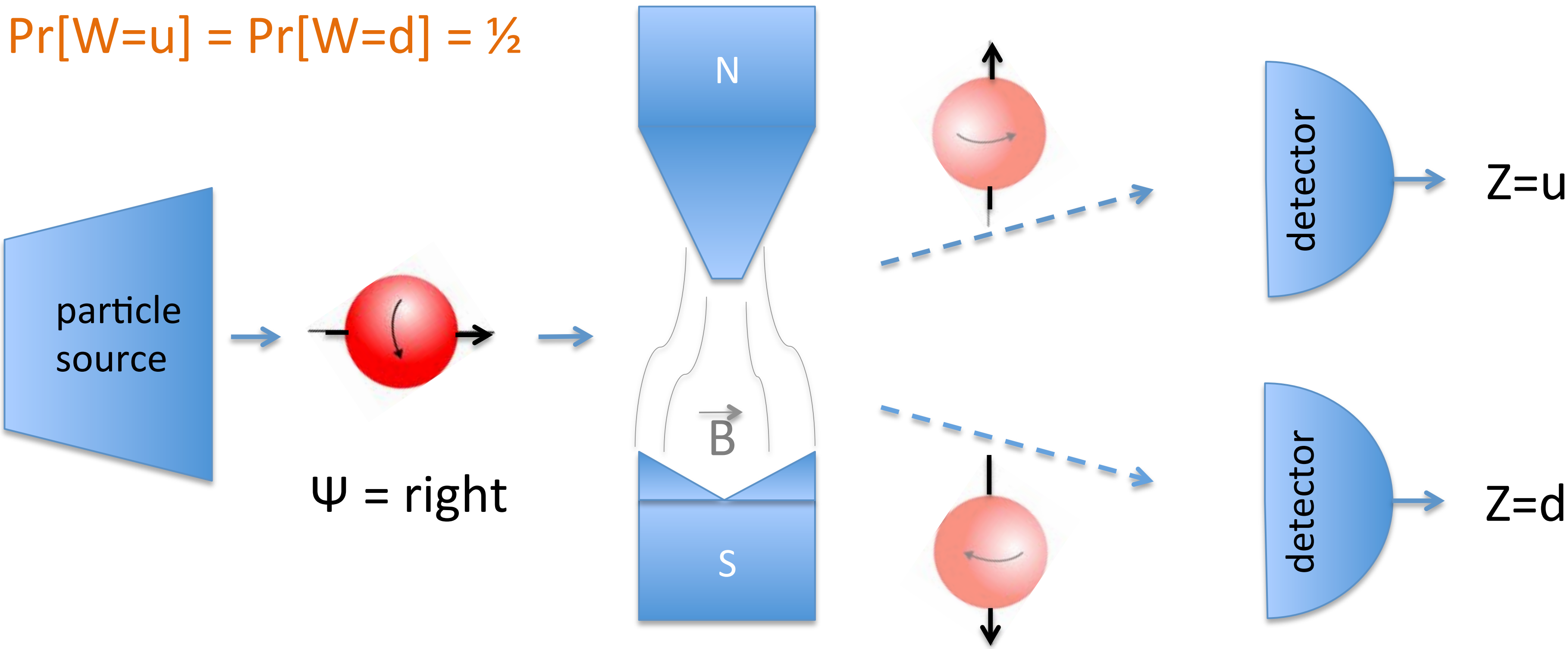
Improved prediction: Pr[Z=u | Ψ = right, W=u] = ¾

An extended theory



Improved prediction: Pr[Z=u | Ψ = right, W=d] = ¼

An extended theory



Prediction consistent with quantum theory when W is ignored:

Pr[Z=u | Ψ = right]

$$= \tfrac{1}{2}\underbrace{\Pr[Z{=}u \mid \Psi = \text{right}, W{=}u]}_{3/4} + \tfrac{1}{2}\underbrace{\Pr[Z{=}u \mid \Psi = \text{right}, W{=}d]}_{1/4} = \tfrac{1}{2}$$

Is quantum theory maximally informative?

The toy example shows that theories that are more informative than quantum theory are conceivable

(even if we require compatibility with standard quantum theory).

Is quantum theory maximally informative?

The toy example shows that theories that are more informative than quantum theory are conceivable

(even if we require compatibility with existing quantum theory).

There are many other examples of extensions of quantum theory (e.g., the “Leggett model”).

[A.J. Leggett, Foundations of Physics 33, 1469–1493 (2003)]

Is quantum theory maximally informative?

If the answer was "no", this may change our current understanding of physics.

How much information can we possibly have about the world around us?

How much information can we possibly have about the world around us?

Werner Heisenberg
1901 – 1976

There is some inevitable uncertainty in the outcomes of measurements carried out on quantum systems.

$$\Delta X \Delta P \geq \frac{\hbar}{2}$$

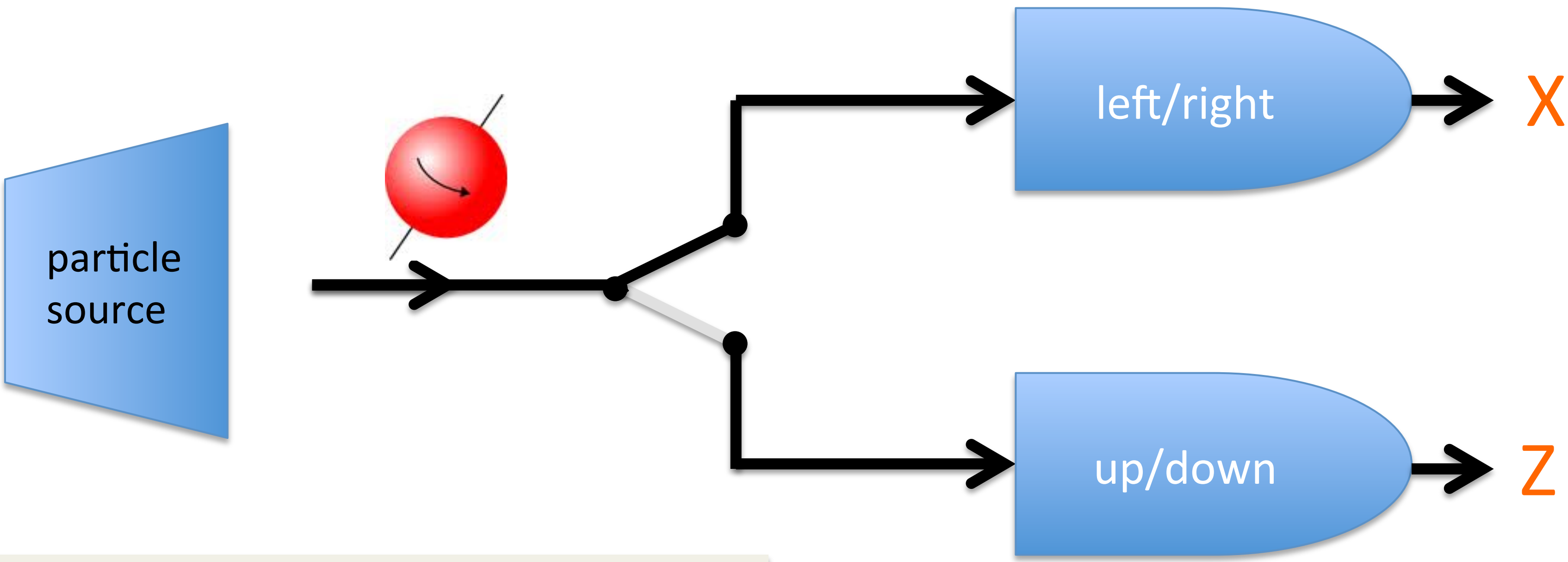
Heisenberg's Uncertainty Principle

läuterung der Relation $pq - qp = \frac{h}{2\pi i}$. Sei q_1 die Genauigkeit, mit der der Wert q bekannt ist (q_1 ist etwa der mittlere Fehler von q), also hier die Wellenlänge des Lichtes, p_1 die Genauigkeit, mit der der Wert p bestimmbar ist, also hier die unstetige Änderung von p beim Comptoneffekt, so stehen nach elementaren Formeln des Comptoneffekts p_1 und q_1 in der Beziehung

$$p_1 q_1 \sim h. \tag{1}$$

W. Heisenberg, Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik, *Zeitschrift für Physik* 43, 172–198 (1927)

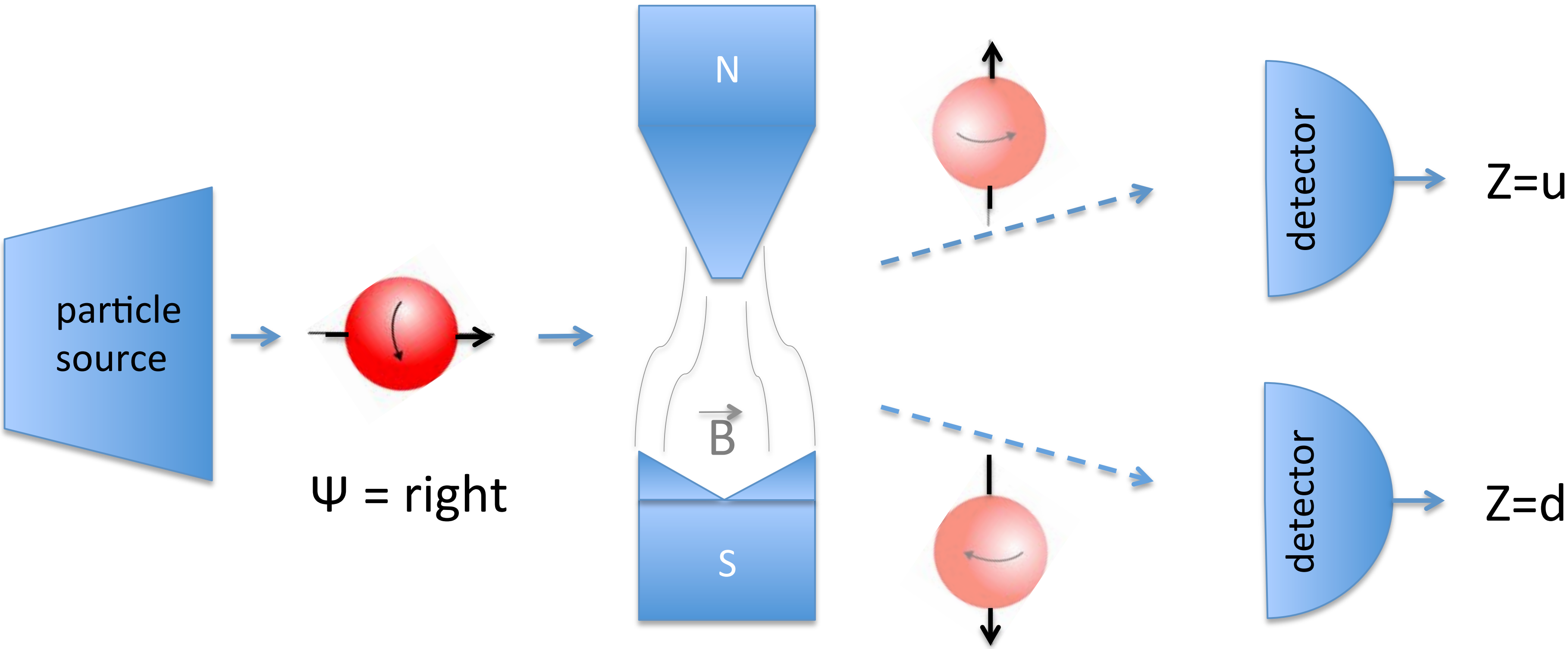
Heisenberg applied to Stern-Gerlach



$$H(X) + H(Z) \geq 1$$

Uncertainty measured in terms of entropy $H(X) = -\sum_x P_X(x) \log_2 P_X(x)$

Heisenberg applied to Stern-Gerlach



Heisenberg's uncertainty principle implies that If the particle is in state "right" we have maximal uncertainty about the outcome X.

How much information can we possibly have about the state of the world around us?

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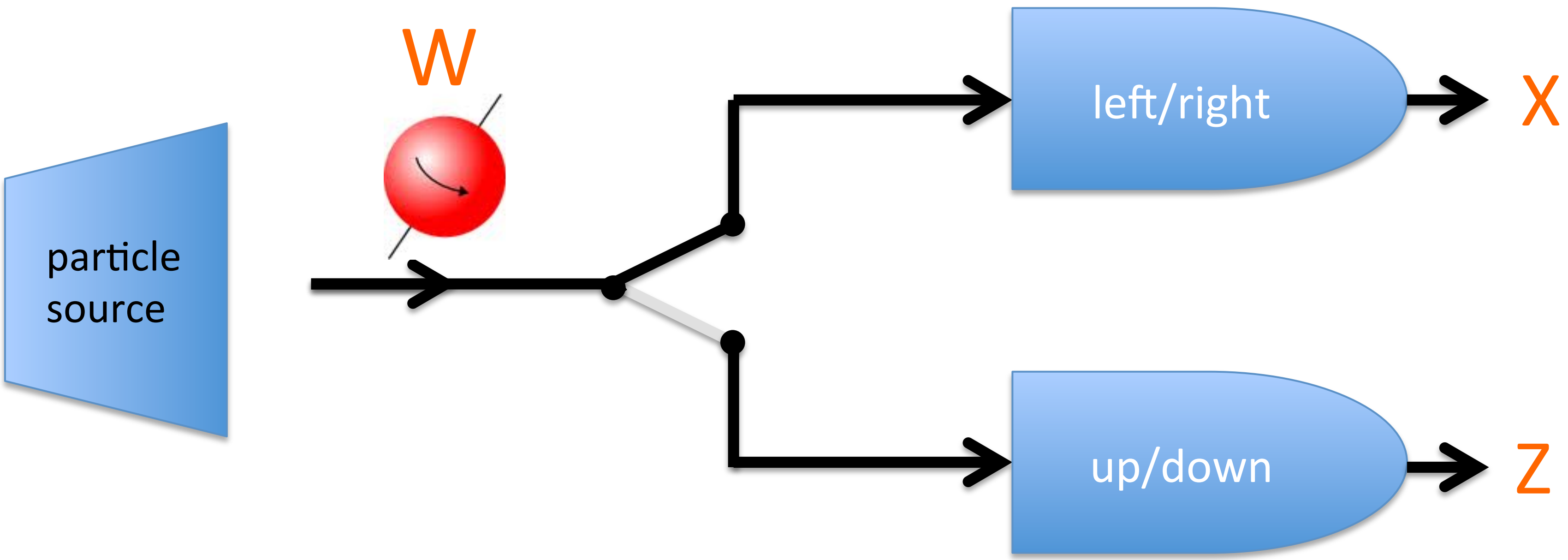
But: The derivation of this principle *assumes* that quantum theory is informationally complete.

Is the uncertainty intrinsic?



$$H(X|W) + H(Z|W) \overset{?}{\geq} 1$$

$H(X|W)$ uncertainty about X conditioned on W

This leads us back to the question whether quantum theory is complete

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

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ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question

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Einstein, Podolsky, and Rosen (EPR)

MAY 15, 19 … VOLUME 47

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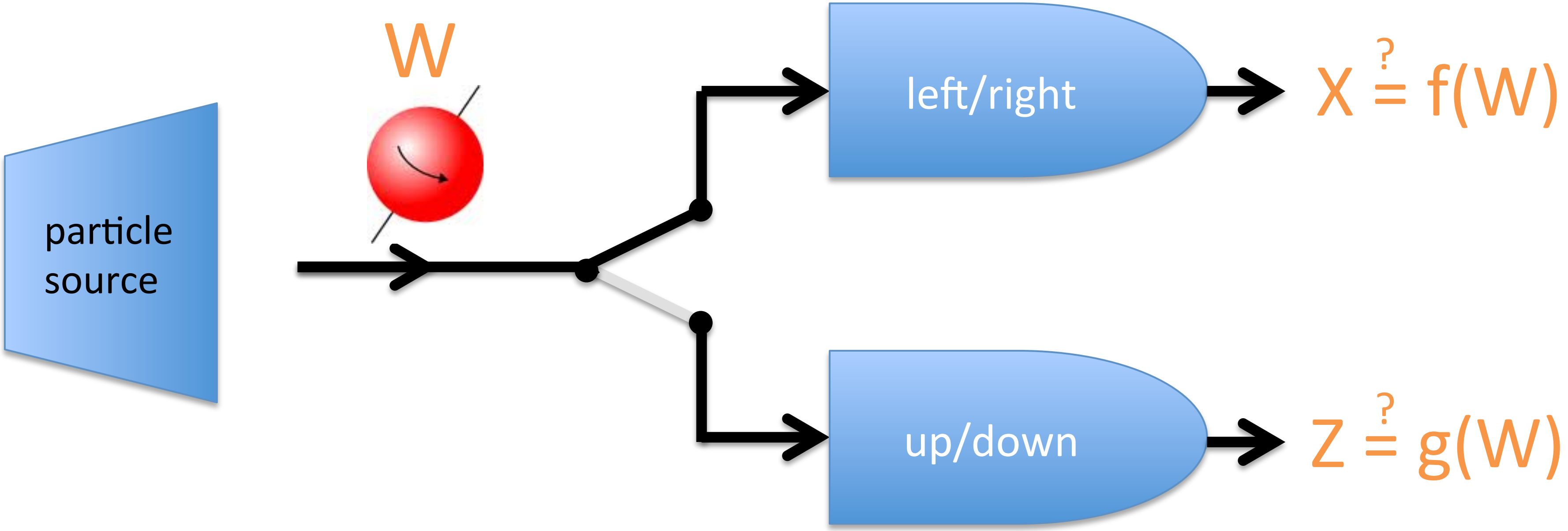
Simpler question: Could there be a deterministic theory?



$$H(X|W) + H(Z|W) \stackrel{?}{=} 0$$

Logic in a quantum world

Ernst Specker
1920 – 2011

DIE LOGIK NICHT GLEICHZEITIG ENTSCHEIDBARER AUSSAGEN

von Ernst SPECKER, Zürich

La logique est d'abord une science naturelle.
F. GONSETH.

Das der Arbeit vorangestellte Motto ist der Untertitel des Kapitels *La physique de l'objet quelconque* aus dem Werk *Les mathématiques et la réalité*; diese Physik erweist sich im wesentlichen als eine Form der klassischen Aussagenlogik, welche so einerseits eine typische Realisation erhält und sich anderseits auf fast selbstverständliche Art des Absolutheitsanspruches entkleidet findet, mit dem sie zeitweise behängt wurde. Die folgenden Ausführungen schliessen sich an diese Betrachtungsweise an und möchten in demselben empirischen Sinn verstanden sein.

Wir gehen aus von einem Bereich B von Aussagen und stellen uns die Aufgabe, die Struktur dieses Bereiches zu untersuchen. Eine solche strukturelle Beschreibung von B ist erst möglich, wenn

Non-existence of hidden variables

- E. Specker, Logic of Non-Simultaneously Decidable Propositions (1960)
- S. Kochen, E. Specker, The Problem of Hidden Variables in Quantum Mechanics (1967)
- J. Bell, On the Problem of Hidden Variables in Quantum Mechanics (1964/66)

Where did this lead us to?

- Kochen and Specker's as well as Bell's results imply that

$$H(X|W) + H(Z|W) > 0$$

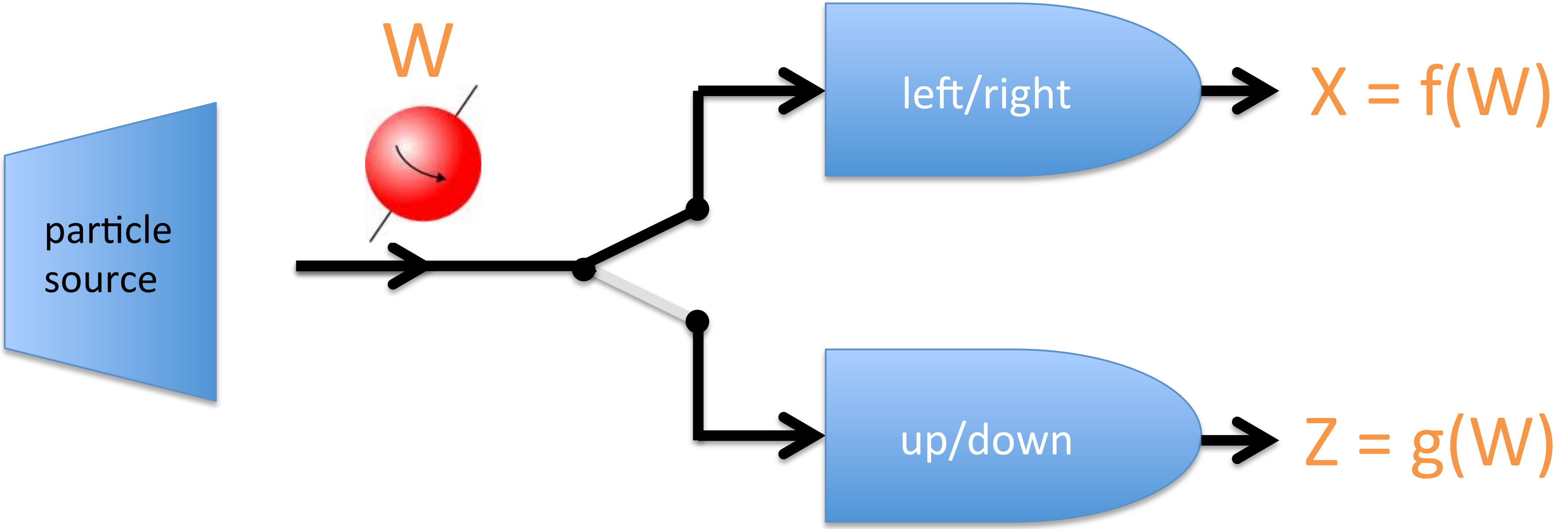
i.e., there is some intrinsic uncertainty.

- However, these results do not exclude the possibility that

$$H(X|W) + H(Z|W) < 1$$

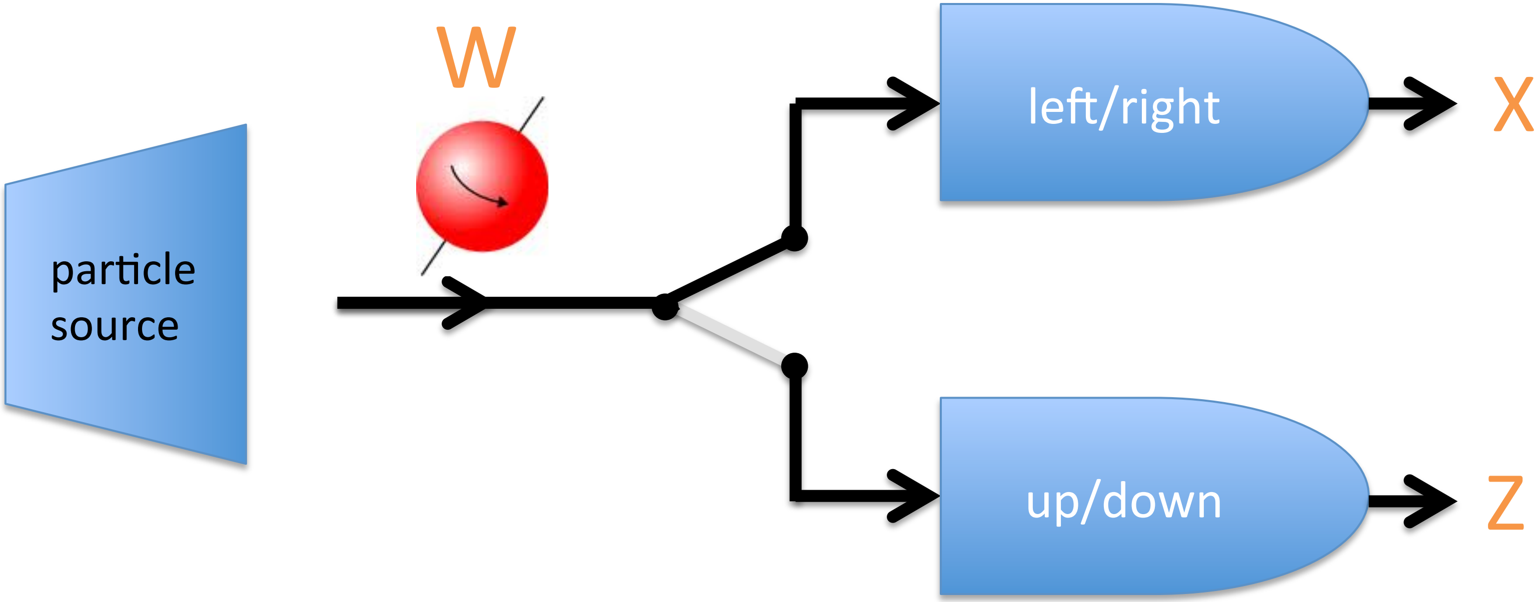
i.e., quantum mechanics may still be incomplete, (not maximally informative).

This is excluded by Bell's arguments …



$$H(X|W) + H(Z|W) = 0$$

… but quantum theory may still be incomplete



Example:

W may be such that Z = f(W) and X=g(W) hold with some probability, e.g., 80%.

A completeness theorem

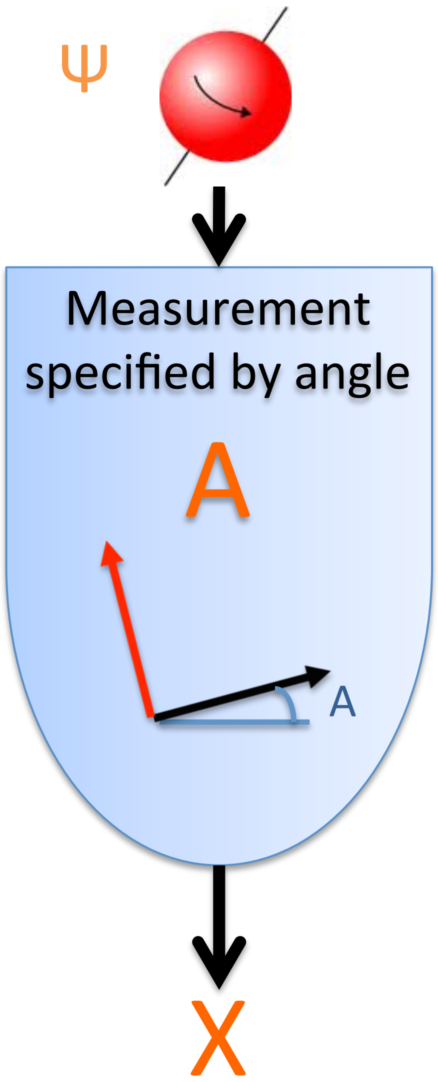
Theorem (informal version)
Consider an extended theory which allows us to make predictions based on additional parameters W and assume that

- when ignoring W, the theory reproduces the predictions of quantum theory
- measurement settings A can be chosen freely.

Then W does not provide any information about the outcomes of future measurements (beyond quantum theory).

RR and R. Colbeck, No extension of quantum theory can have improved predictive power, *Nature Communications* 2, 411, 2011

General setup



Claim: $P_X = P_{X|W}$

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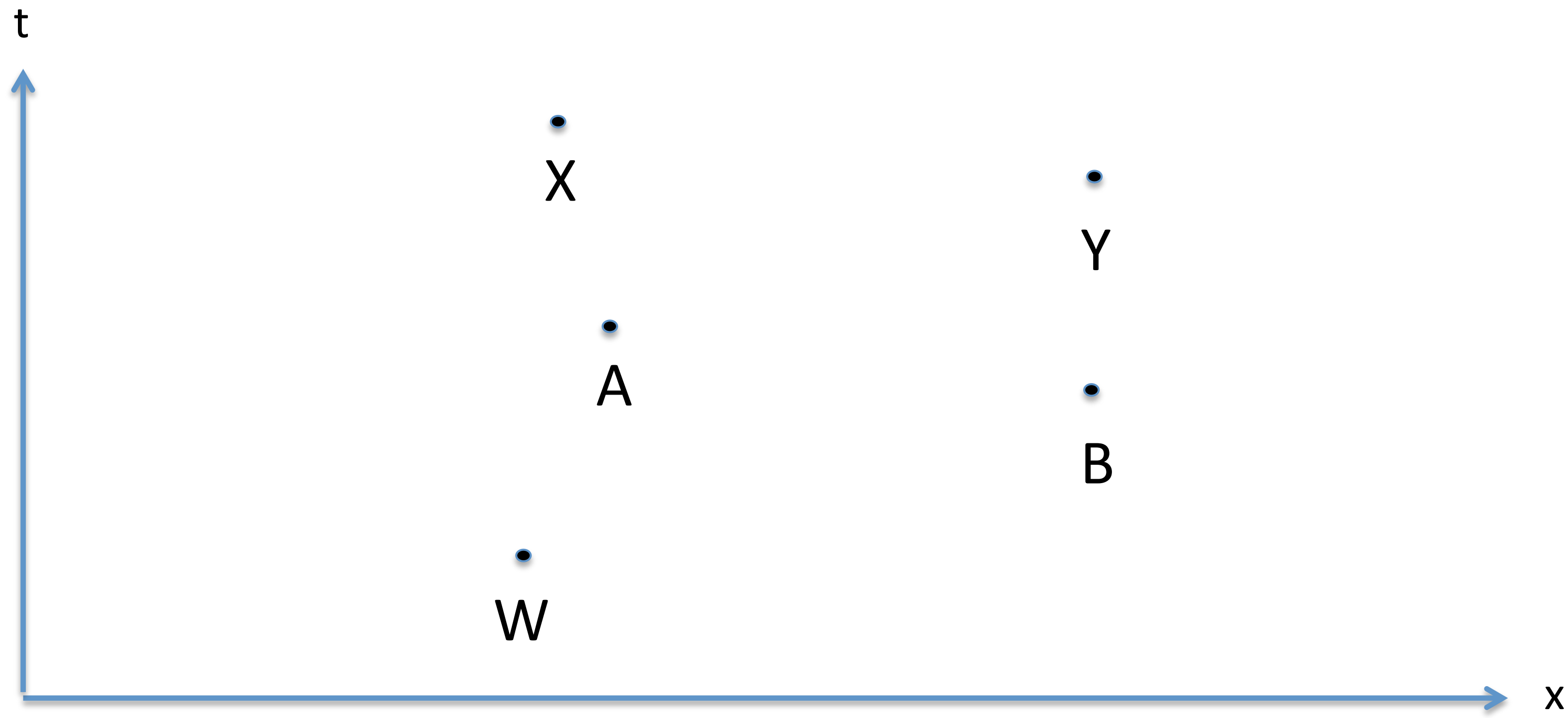
Formally, $P_{X|\Psi A} = P_{X|\Psi AW}$ where Ψ is the initial state.

What does it mean that we can choose measurement settings “freely”?

We are going to use an intuitive notion of free choice that is implicit in most of the literature

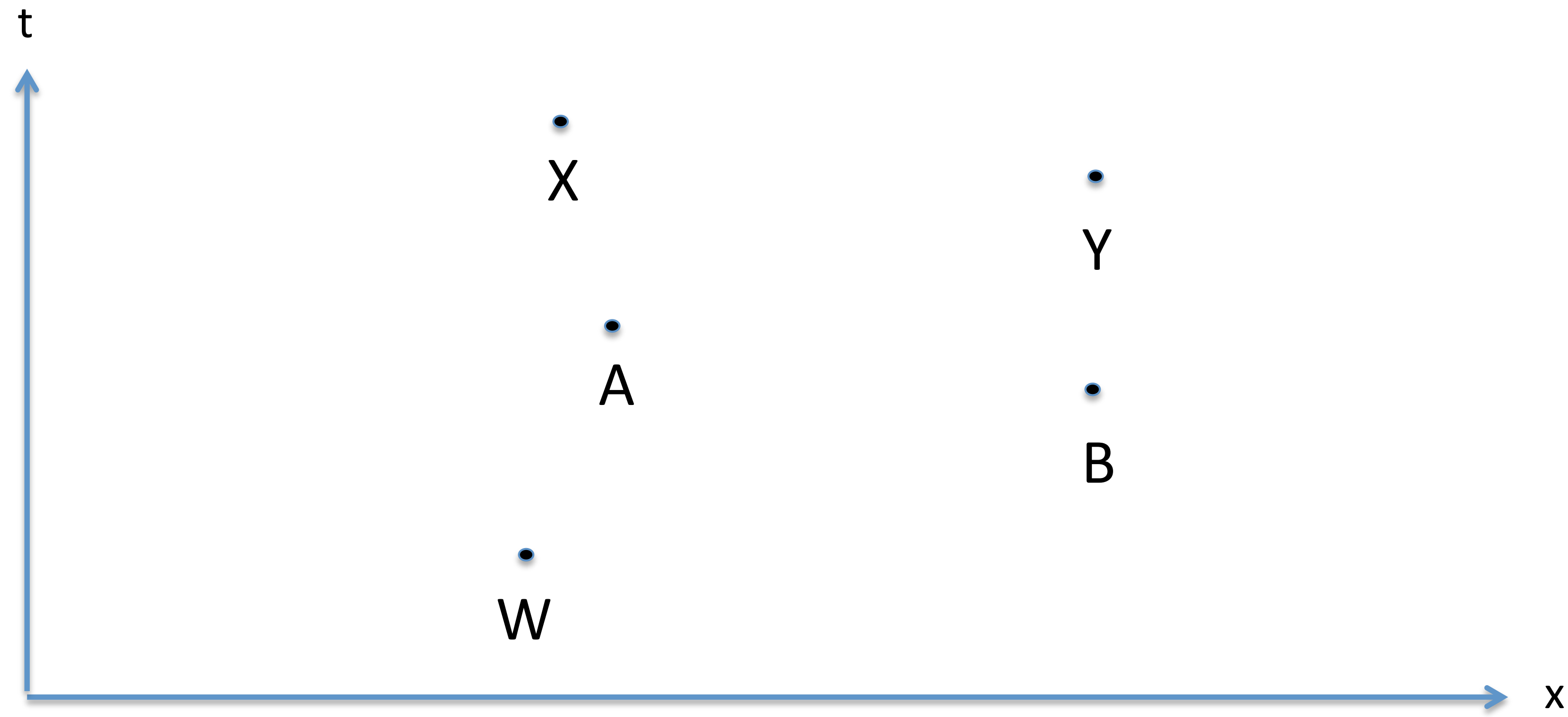
(and sometimes mentioned explicitly, e.g., in Bell’s work).

Space-time random variables



Idea: Associate to any observable value a coordinate (t, x_1, x_2, x_3) indicating its position in space time.

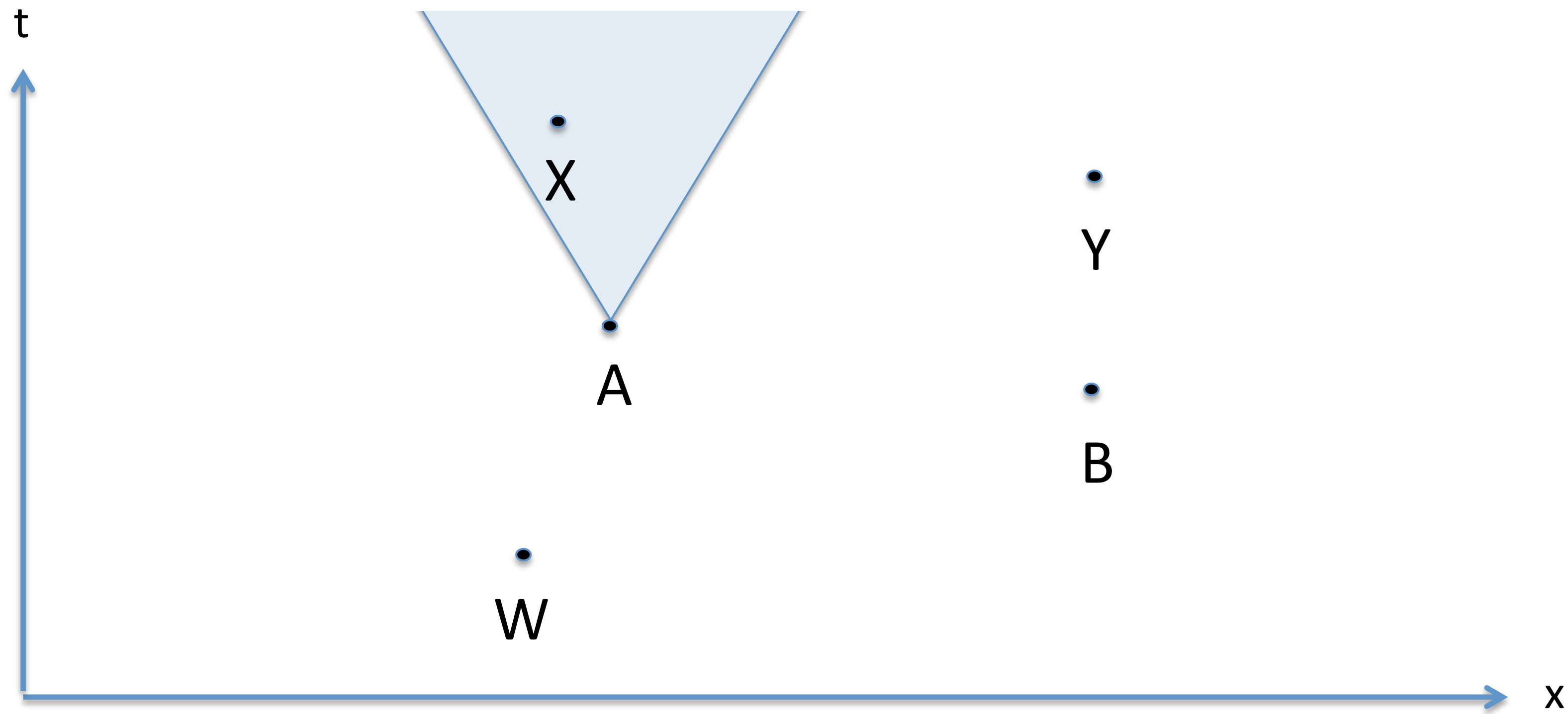
Space-time random variables



Definition

A space-time random variable (SV) is a random variable with associated coordinates (t, x_1, x_2, x_3).

Space-time random variables



Definition

We say that an SV X can be caused by an SV A if it lies in the future lightcone of A.

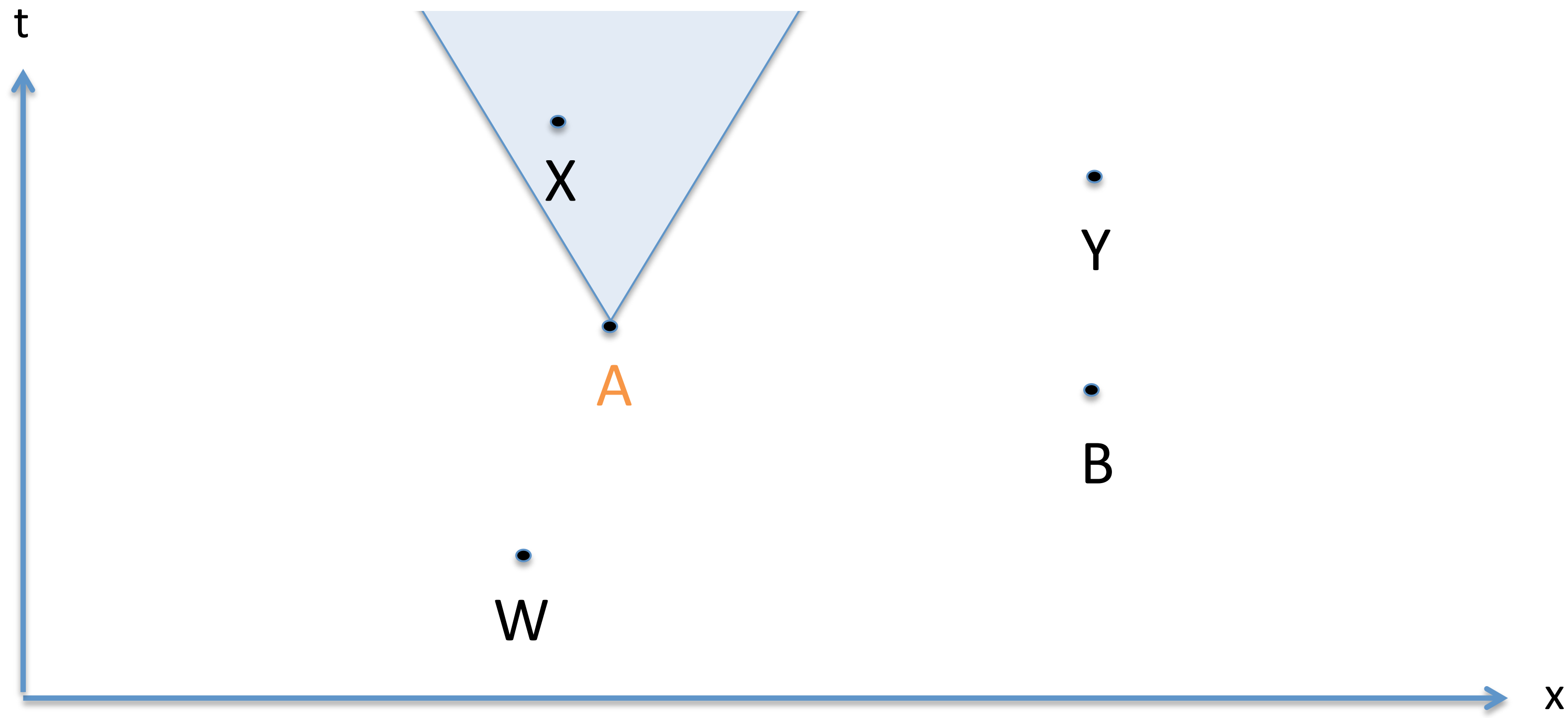
Free choice assumption

Definition

We say that a choice A is free (with respect to a set Γ of SVs) if A is statistically independent of the set of all values W ∈ Γ that cannot be caused by A.

Statistical independence means that $P_{WA} = P_W \times P_A$

Free choice assumption



Example

A is *free* with respect to all other SVs if it is statistically independent of B, Y, and W.

A completeness theorem

Theorem

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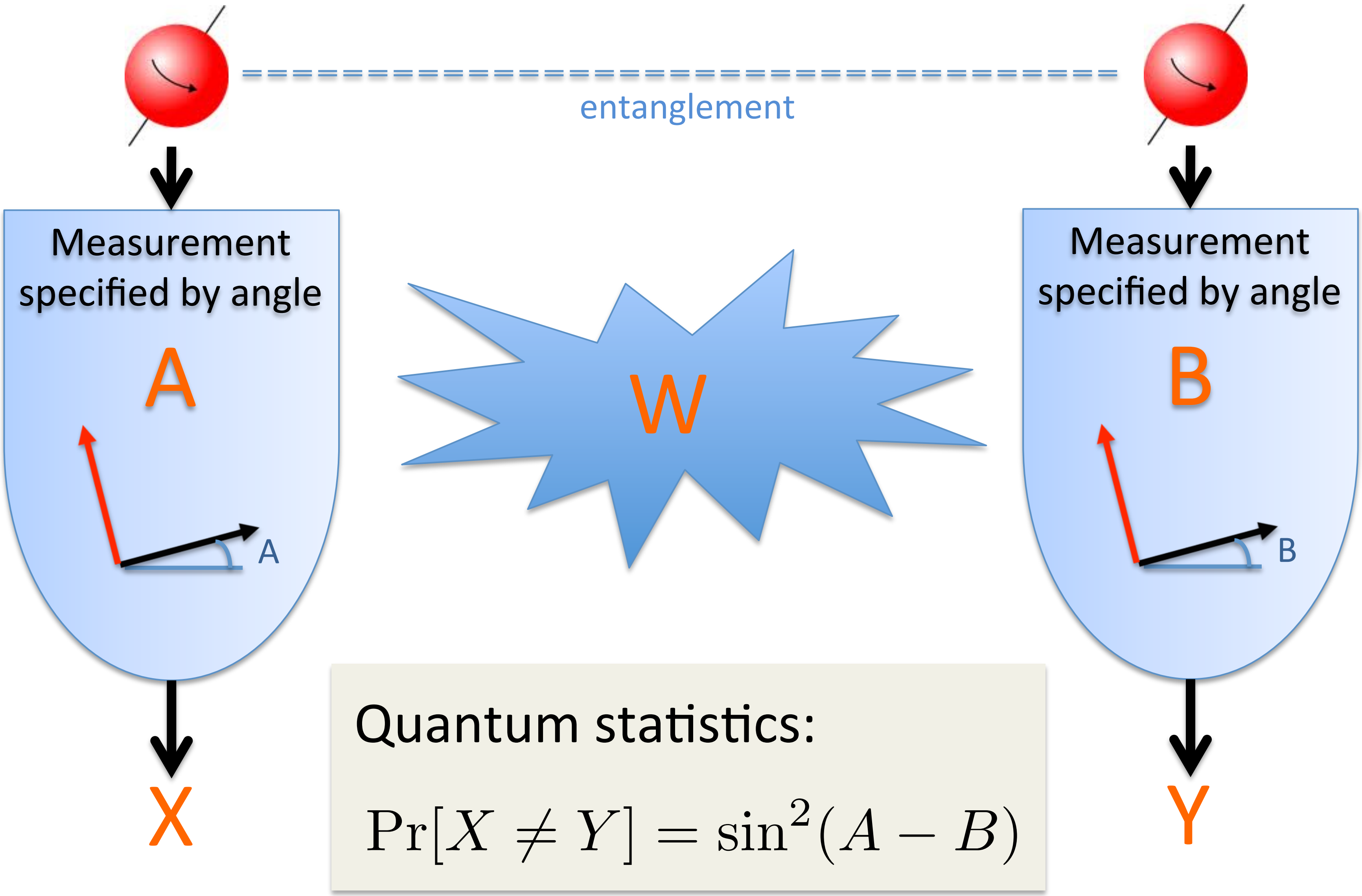
Proof idea

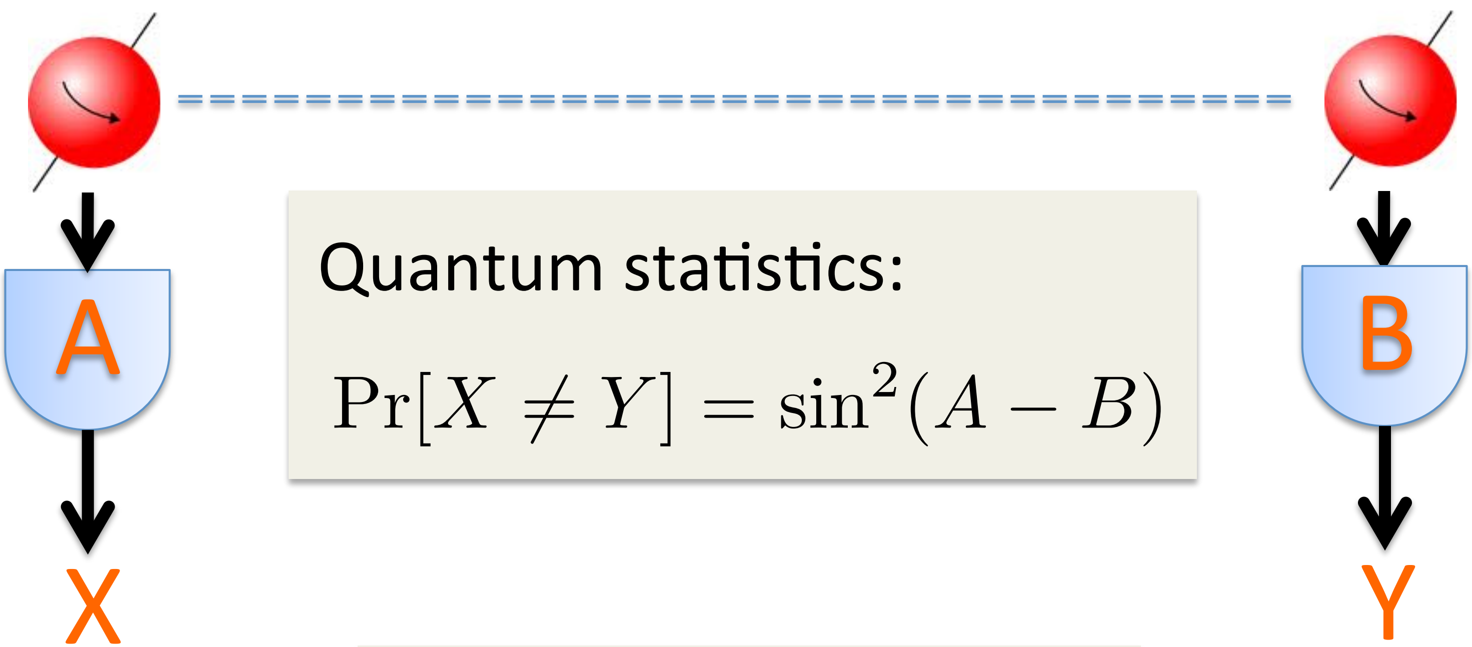

entanglement
Measurement specified by angle
A
A
Measurement specified by angle
B
B
W
X
Y
Quantum statistics:
$\Pr[X \neq Y] = \sin^2(A - B)$

Proof idea

based on ideas by Braunstein and Caves, as well as by Barrett, Hardy and Kent

$A \in \{0\delta, 2\delta, \ldots, (N-2)\delta\}$

$B \in \{1\delta, 3\delta, \ldots, (N-1)\delta\}$



Quantum statistics:

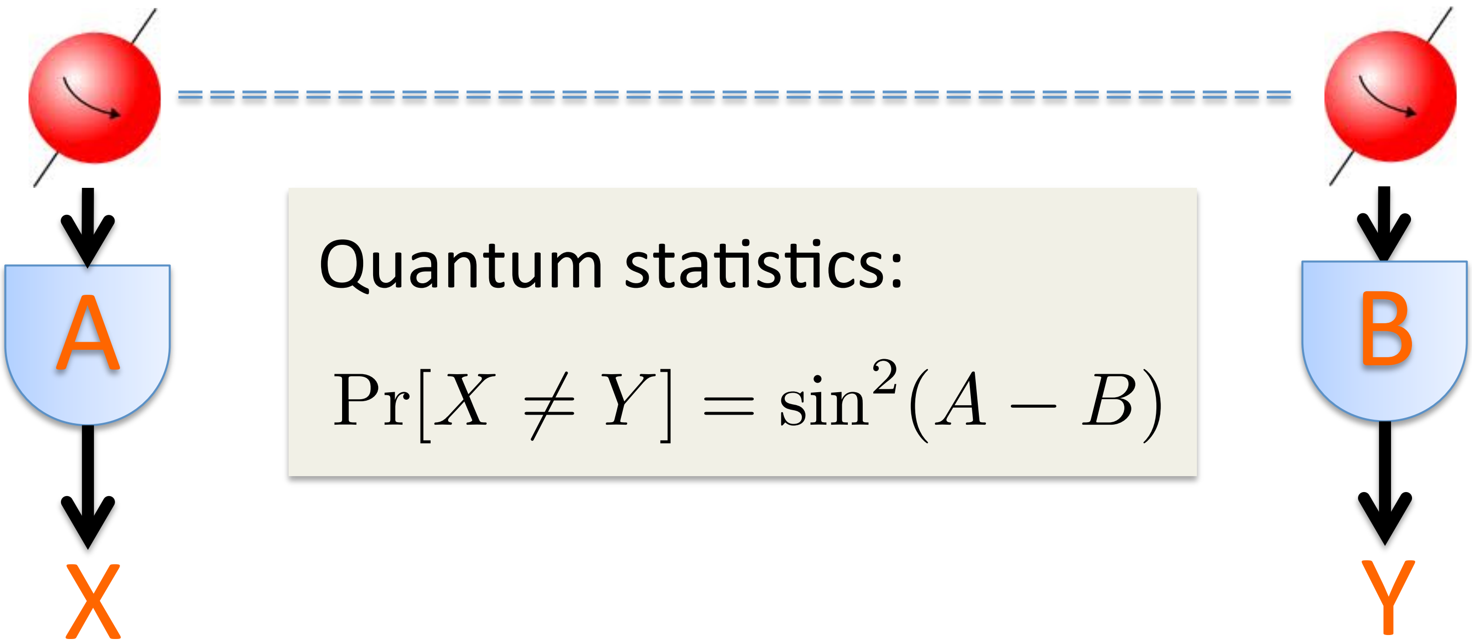
$$\Pr[X \neq Y] = \sin^2(A - B)$$

$\delta := \frac{\pi}{2N}$ for some large N

Proof idea

$A \in \{0\delta, 2\delta, \ldots, (N-2)\delta\}$

$B \in \{1\delta, 3\delta, \ldots, (N-1)\delta\}$



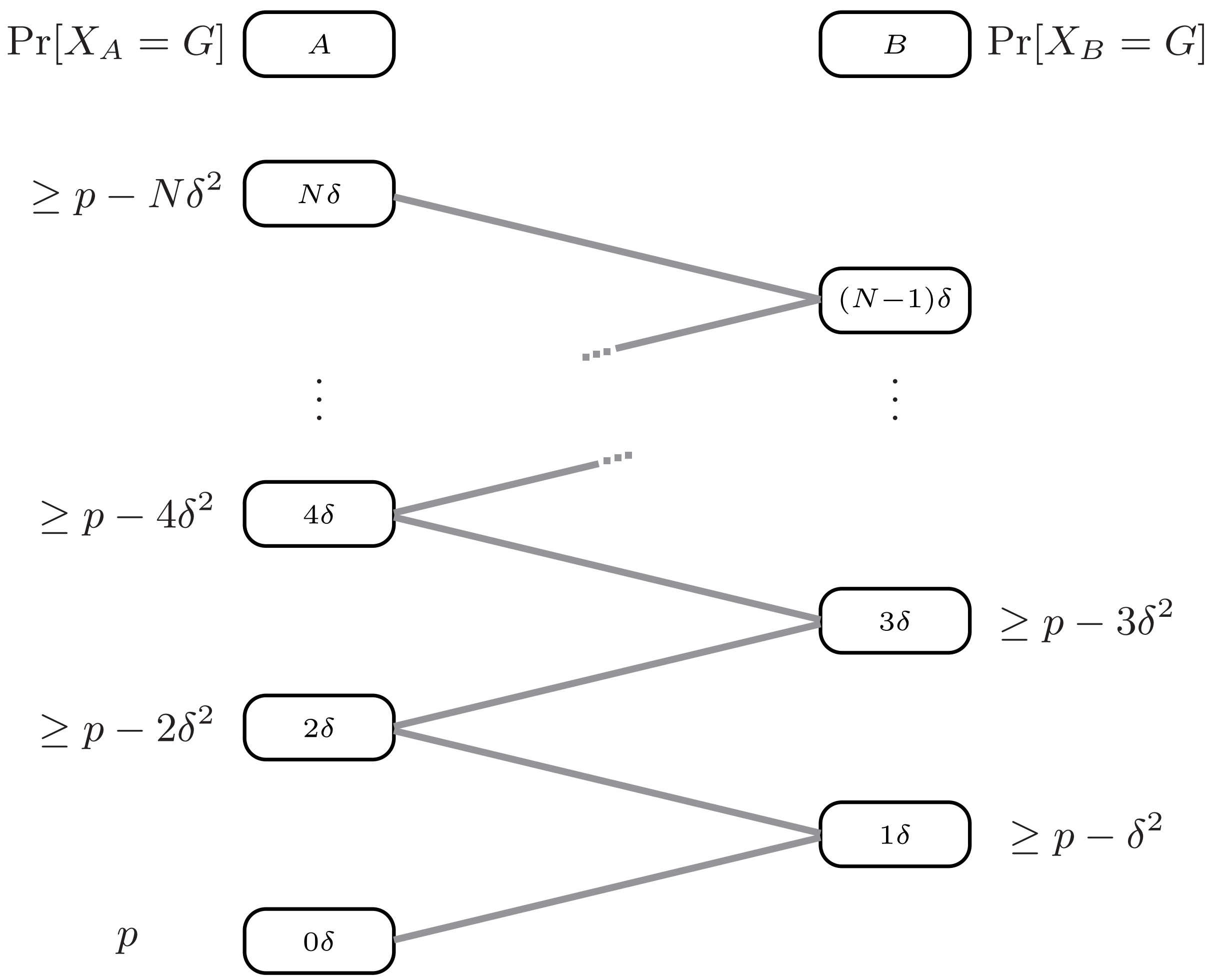
Quantum statistics:

$$\Pr[X \neq Y] = \sin^2(A - B)$$

Hence, for neighbouring angles:

$$\Pr[X \neq Y] = \sin^2 \delta \approx \delta^2$$

Proof idea



Let G be a "guess" for X_0.

Set $p := \Pr[G=X_0]$

It follows that

$\Pr[G=X_{N\delta}] \geq p - N\delta^2$

On the other hand

$\Pr[G=X_{N\delta}] = 1-p$.

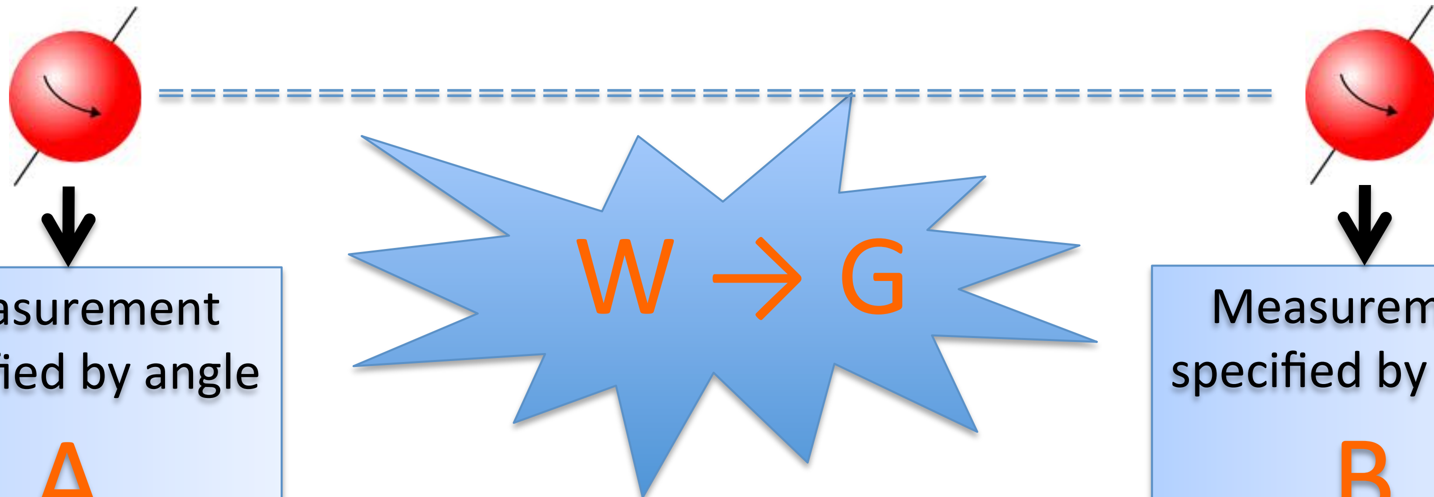
Combining the above, we find

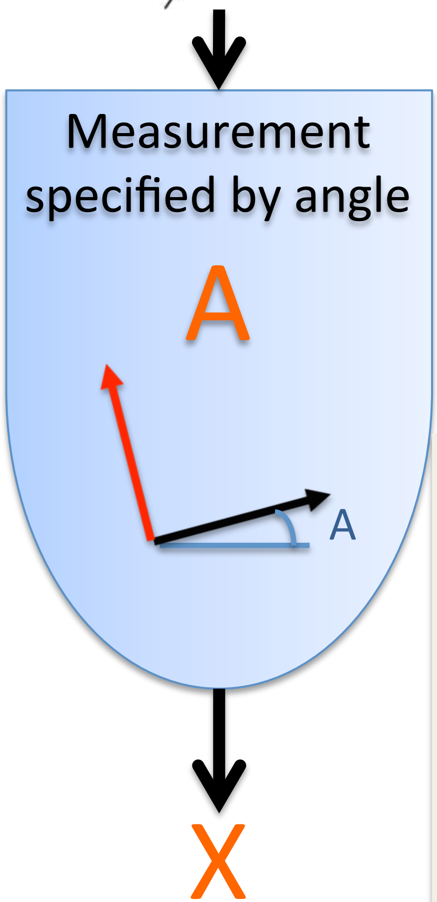
$1-p \geq p - N\delta^2$.

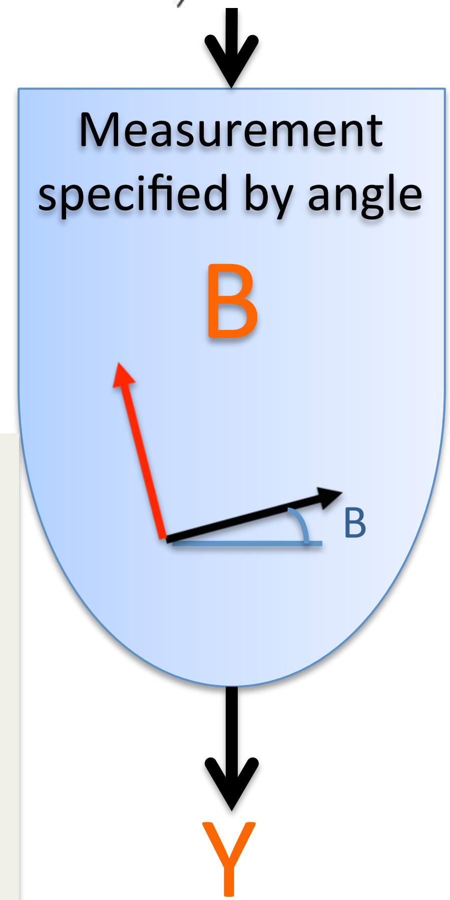
We thus conclude that

$p \leq ½ + ½ N\delta^2 \approx ½$.

Proof idea







We have thus shown that

$$\Pr[X_0 = G] \leq \frac{1}{2}$$

That is, X_0 is completely random.

A completeness theorem

Theorem

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Then W does not provide any information about the outcomes of future measurements (beyond quantum theory).

Note: We have now seen a proof sketch in the special case of measurements on entangled particles. However the statement can be extended to arbitrary measurements.

What does this tell us about the EPR question?

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Implications and questions

- The completeness theorem implies that

$$P_{X|\Psi} = P_{X|\Psi W}$$

- Therefore, the quantum mechanical wave function Ψ is maximally informative.

- But is the wave function also unique?

Further implications

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NATURE | NEWS

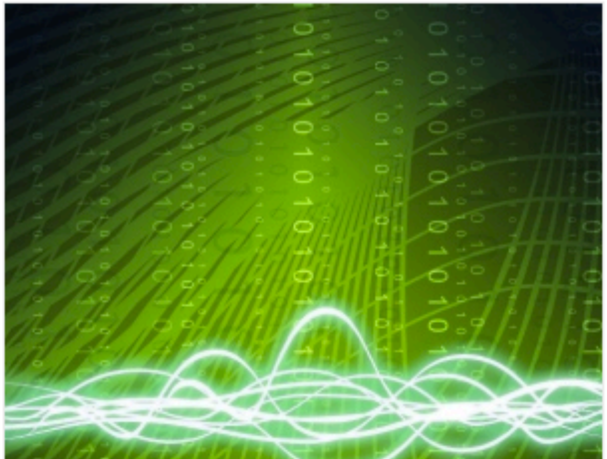
Quantum theorem shakes foundations

The wavefunction is a real physical object after all, say researchers.

Eugenie Samuel Reich

17 November 2011

At the heart of the weirdness for which the field of quantum mechanics is famous is the wavefunction, a powerful but mysterious entity that is used to determine the probabilities that quantum particles will have certain properties. Now, a preprint posted online on 14 November[1] reopens the question of what the wavefunction represents — with an answer that could rock quantum theory to its core. Whereas many physicists have generally interpreted the wavefunction as a statistical tool that reflects our ignorance of the particles being measured, the authors of the latest paper argue that, instead, it is



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Implications

"The wave function is a real physical object after all, say researchers."

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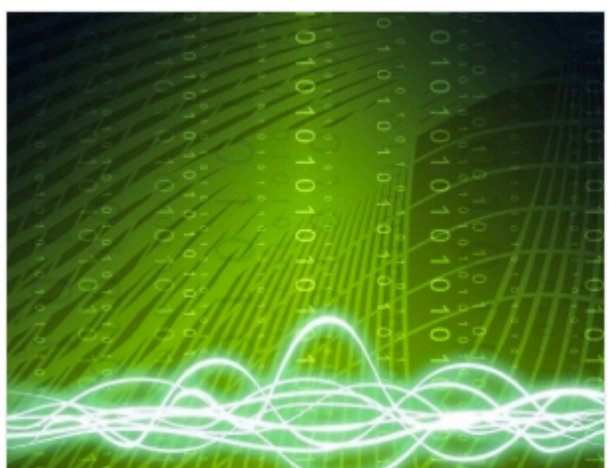

The wave function is unique

Corollary

Consider an extended theory which allows us to make predictions based on additional parameters W and assume that

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If W is maximally informative then there exists a function f such that $\Psi = f(W)$.

The wave function is unique

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If W is maximally informative then there exists a function f such that $\Psi = f(W)$.

Hence, any maximally informative theory compatible with quantum theory is equivalent to quantum theory.

Conclusions

- It is impossible to extend quantum theory so that it allows us to make more certain predictions (unless we give up “free will”).
- The wave function of a physical system is uniquely determined by any maximally informative theory.

Many thanks for your attention