

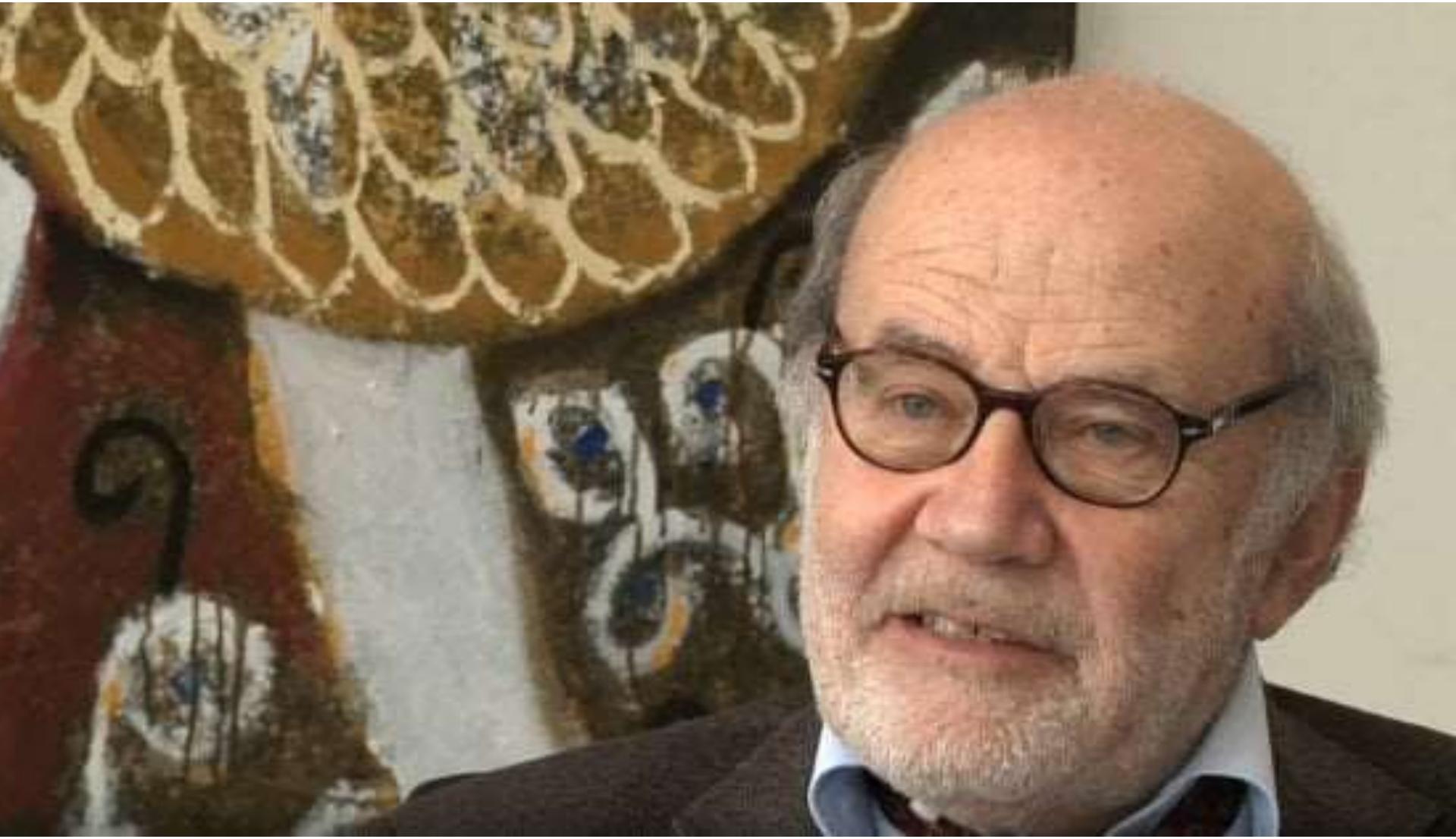


*Testing Quantum Mechanics Underground:  
a hunt of the impossible radiation*

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Catalina Curceanu, LNF-INFN  
University of Pavia  
21 March 2019

This talk is dedicated to  
**GianCarlo Ghirardi**







Picture: Danilo Pivato © copyright - All rights reserved







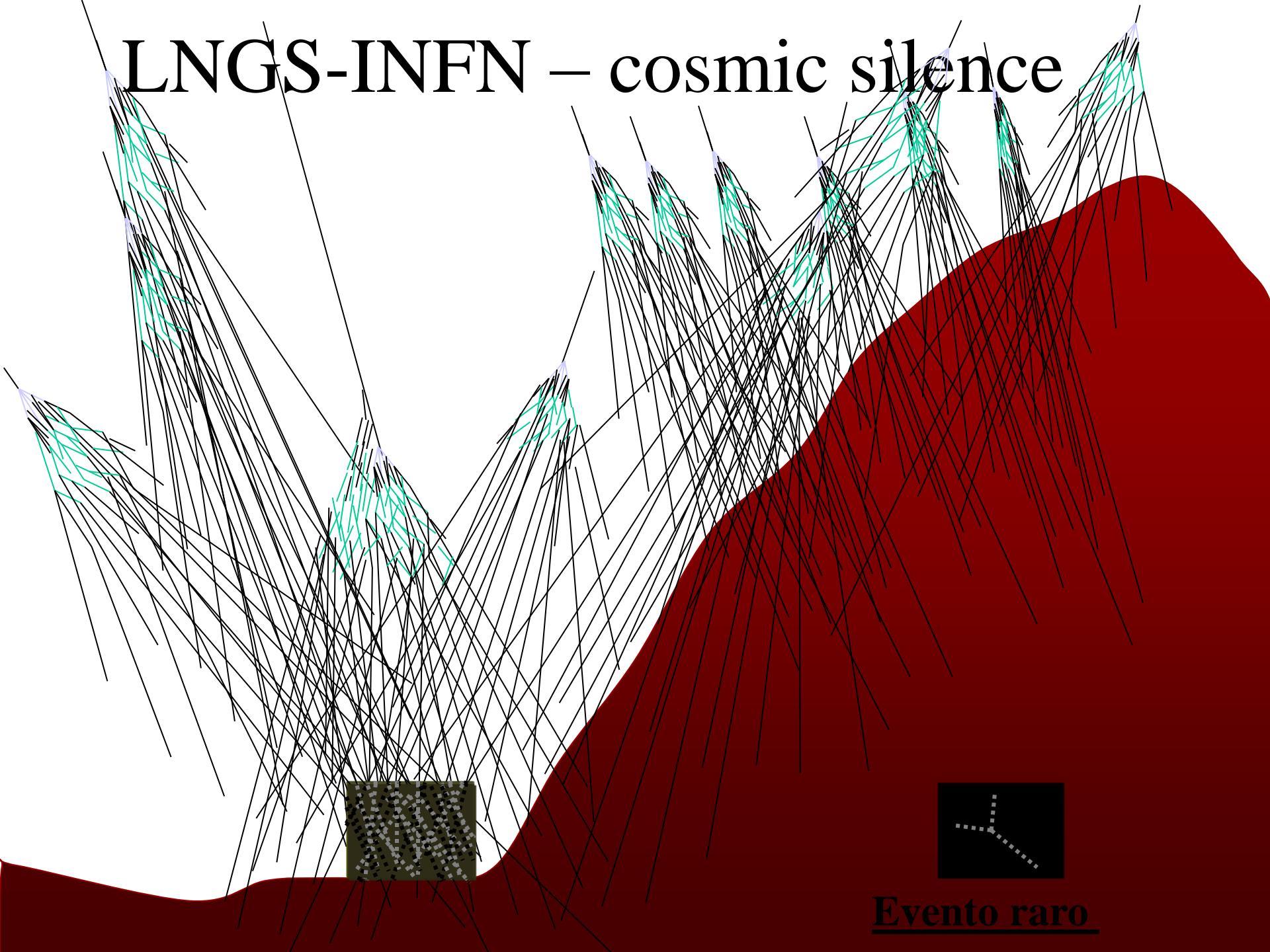








# LNGS-INFN – cosmic silence



Evento raro





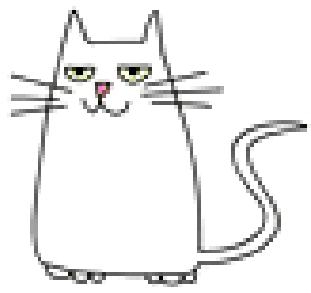
**Test Quantum Mechanics  
In the Cosmic Silence**

# Quantum Mechanics







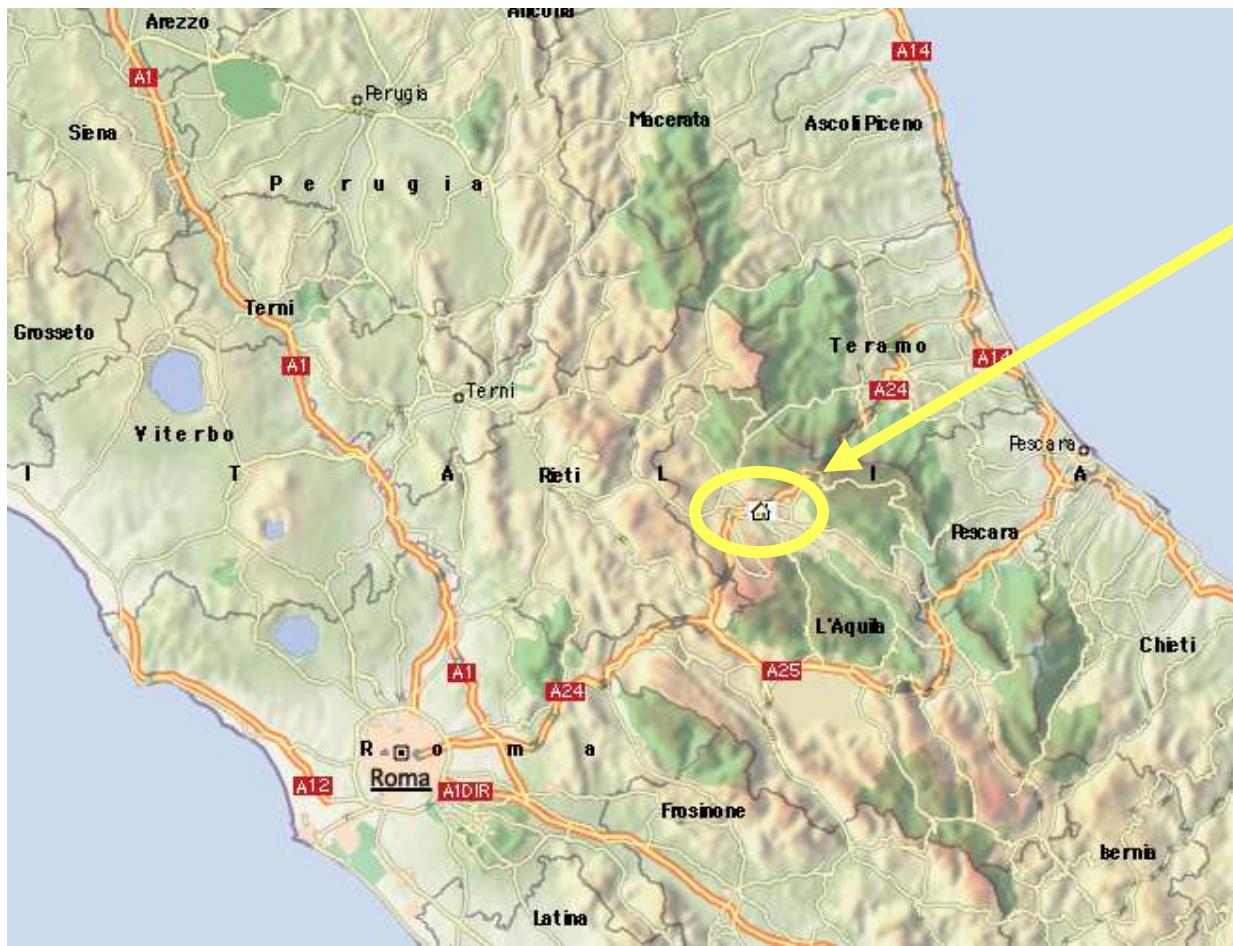


## Possible solutions:

- De Broglie - Bohm
- Many-World Interpretations
- Collapse of the w.f.
- .....

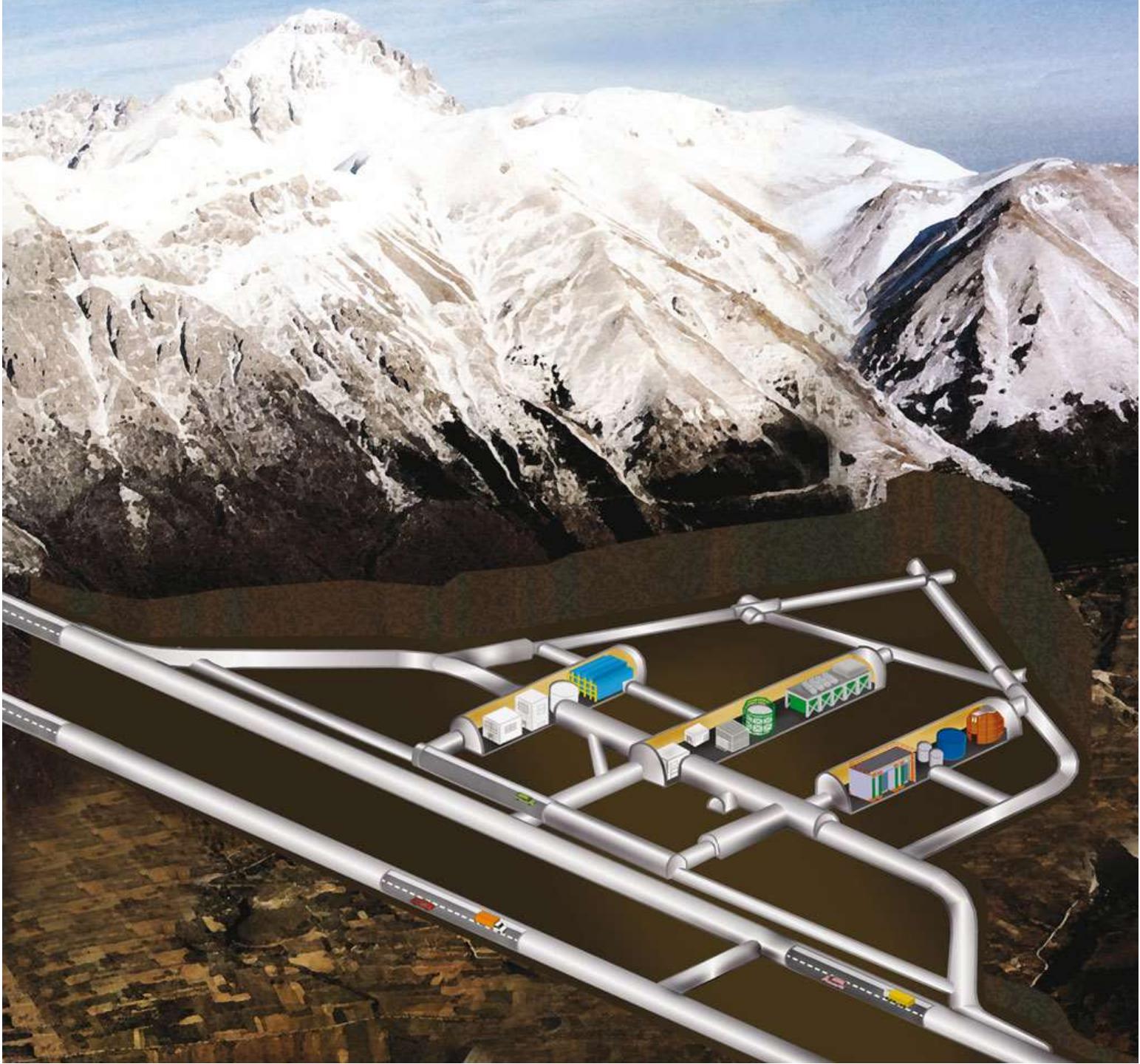
# Laboratori Nazionali del Gran Sasso, Istituto Nazionale di Fisica Nucleare

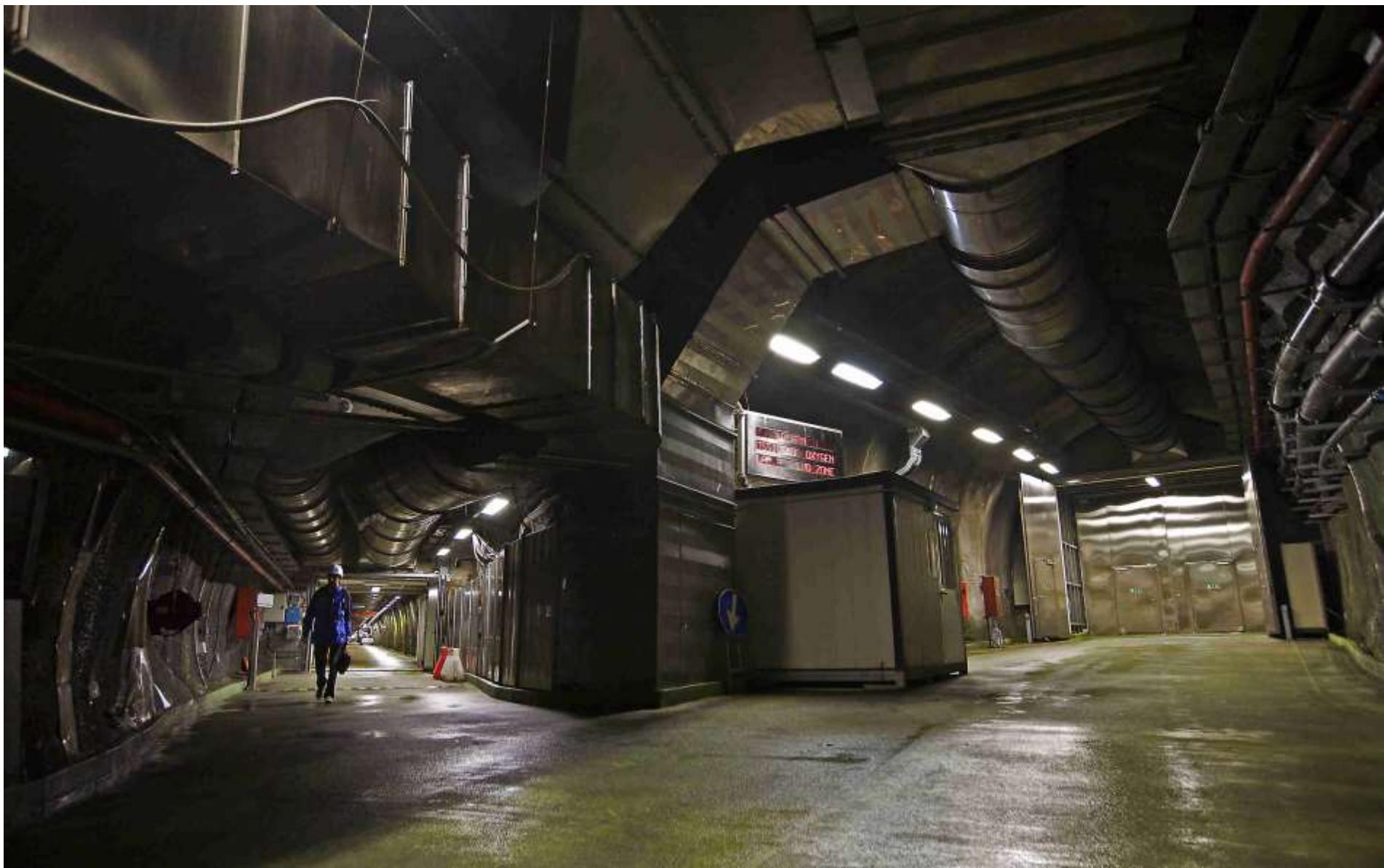
LNGS

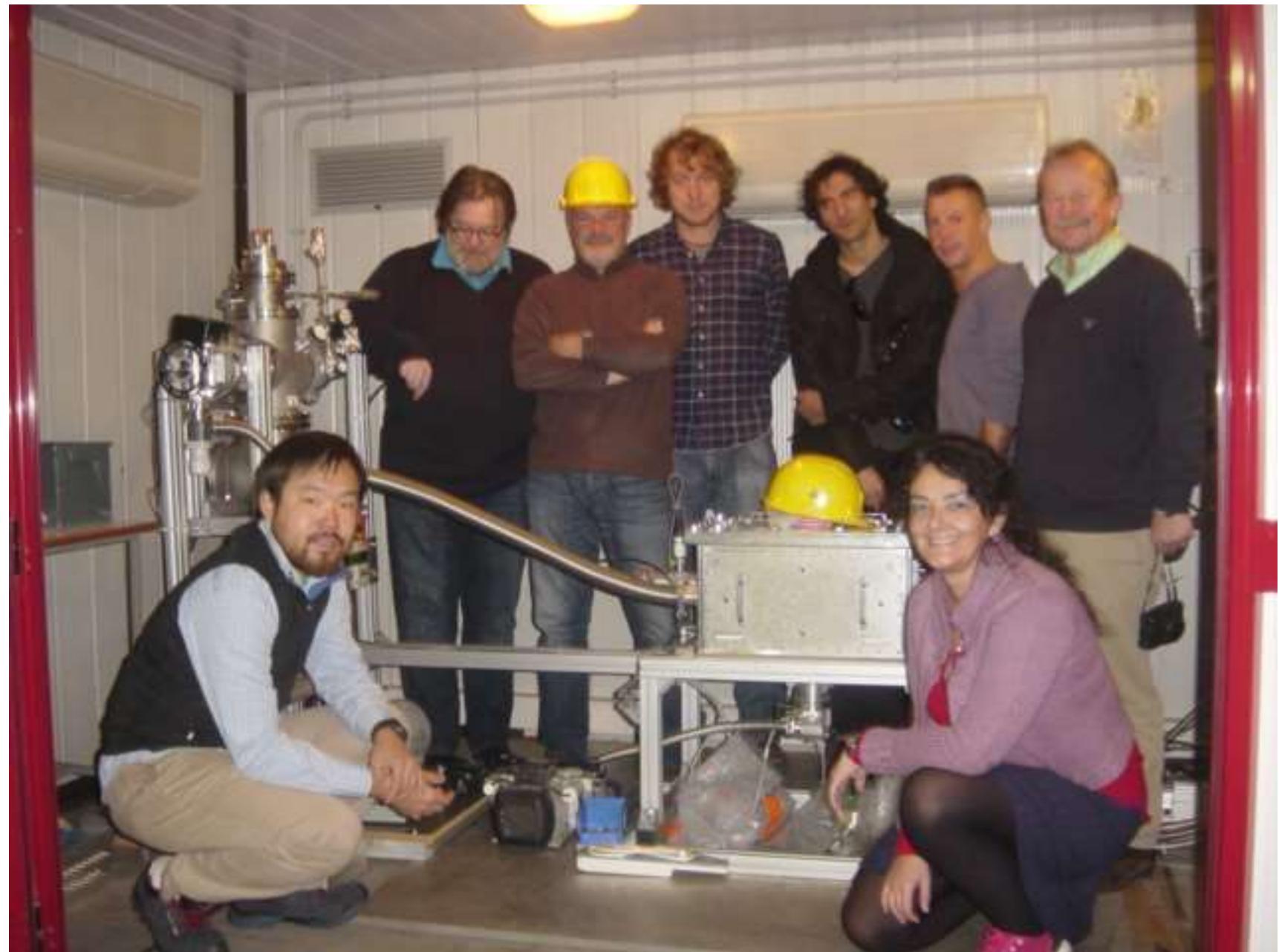












# What are collapse models

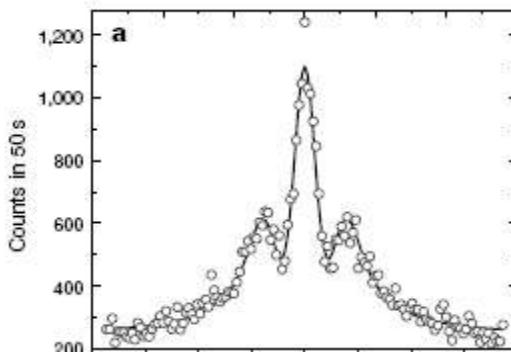
## 1. Collapse models = solution of the measurement problem

Paradox-free description of the quantum world

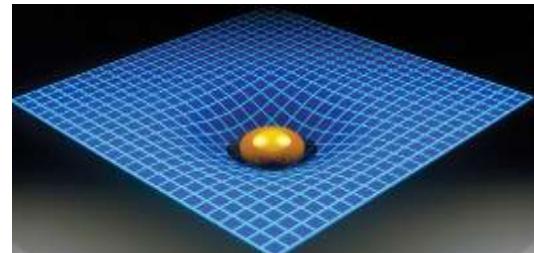


## 2. Collapse models = rival theory of Quantum Mechanics

They give quantitative meaning to experiments testing quantum linearity



## 3. Collapse models as phenomenological models of an underlying pre-quantum theory



Can gravity causes the collapse?

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}) \Psi(\mathbf{r}, t)$$

sinus  
at 45°

# CSL model

$$d|\psi_t\rangle = \left[ -\frac{i}{\hbar} H dt + \sqrt{\lambda} \int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle_t) dW_t(\mathbf{x}) - \frac{\lambda}{2} \int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle_t)^2 dt \right] |\psi_t\rangle$$

System's Hamiltonian

NEW COLLAPSE TERMS



New Physics

$$N(\mathbf{x}) = a^\dagger(\mathbf{x})a(\mathbf{x}) \quad \text{particle density operator}$$

**choice of the preferred basis**

$$\langle N(\mathbf{x}) \rangle_t = \langle \psi_t | N(\mathbf{x}) | \psi_t \rangle$$

**nonlinearity**

$$W_t(\mathbf{x}) = \text{noise} \quad \mathbb{E}[W_t(\mathbf{x})] = 0, \quad \mathbb{E}[W_t(\mathbf{x})W_s(\mathbf{y})] = \delta(t-s)e^{-(\alpha/4)(\mathbf{x}-\mathbf{y})^2}$$

**stochasticity**

$$\lambda = \text{collapse strength} \quad r_C = 1/\sqrt{\alpha} = \text{correlation length}$$

**two parameters**

**the only possible modification of the Schrödinger equation, compatible with the non-faster-than-light signaling condition!**

# Which values for $\lambda$ and $r_c$ ?

## Microscopic world (few particles)



$$\lambda \sim 10^{-8 \pm 2} \text{ s}^{-1}$$

QUANTUM - CLASSICAL  
TRANSITION  
(Adler - 2007)

## Mesoscopic world Latent image formation + perception in the eye ( $\sim 10^4$ - $10^5$ particles)



S.L. Adler, JPA 40, 2935 (2007)

A. Bassi, D.A. Deckert & L. Ferrialdi, EPL 92, 50006 (2010)

$$\lambda \sim 10^{-17} \text{ s}^{-1}$$

QUANTUM - CLASSICAL  
TRANSITION  
(GRW - 1986)

## Macroscopic world ( $> 10^{13}$ particles)

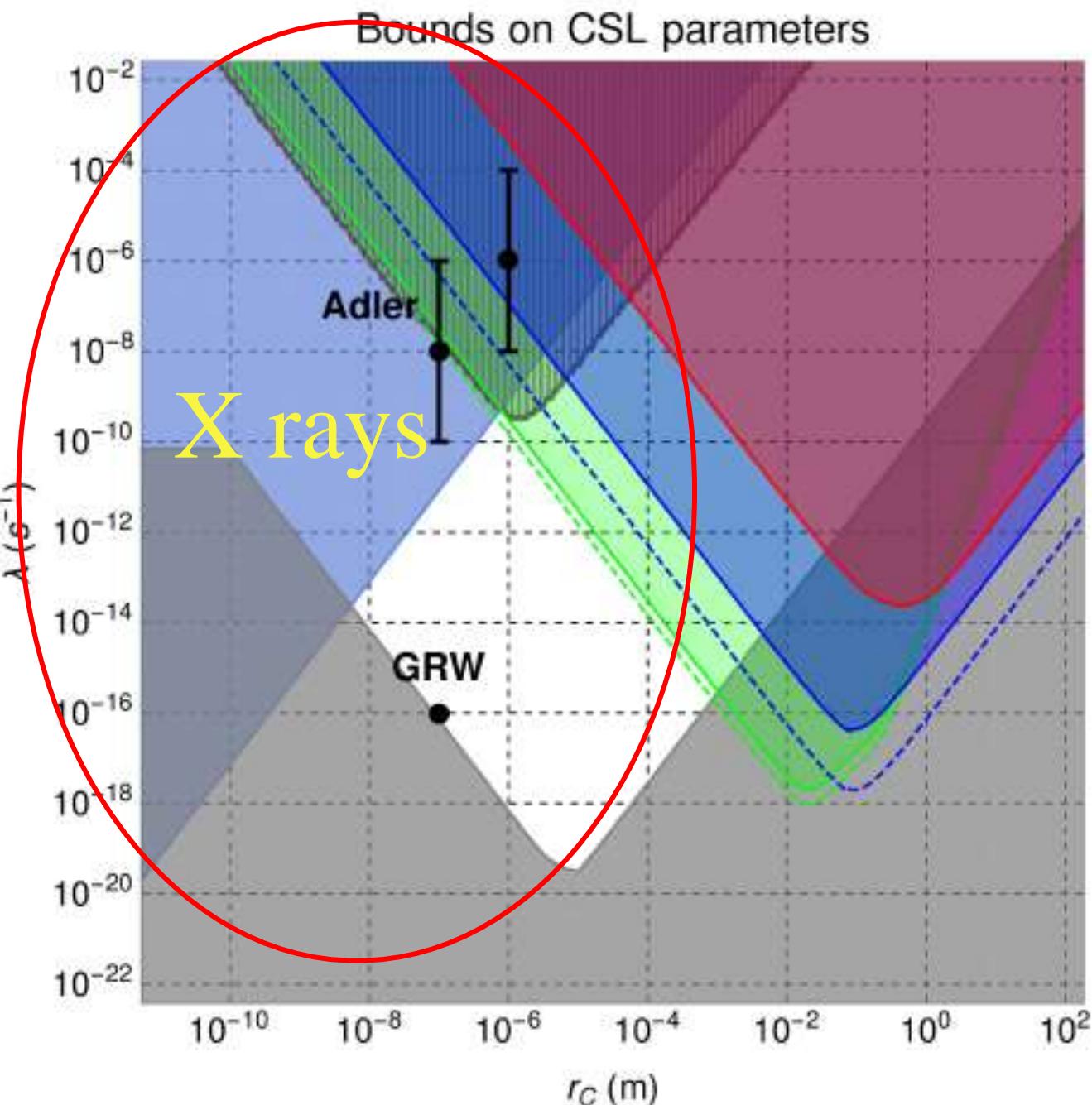


G.C. Ghirardi, A. Rimini and T. Weber, PRD 34, 470 (1986)

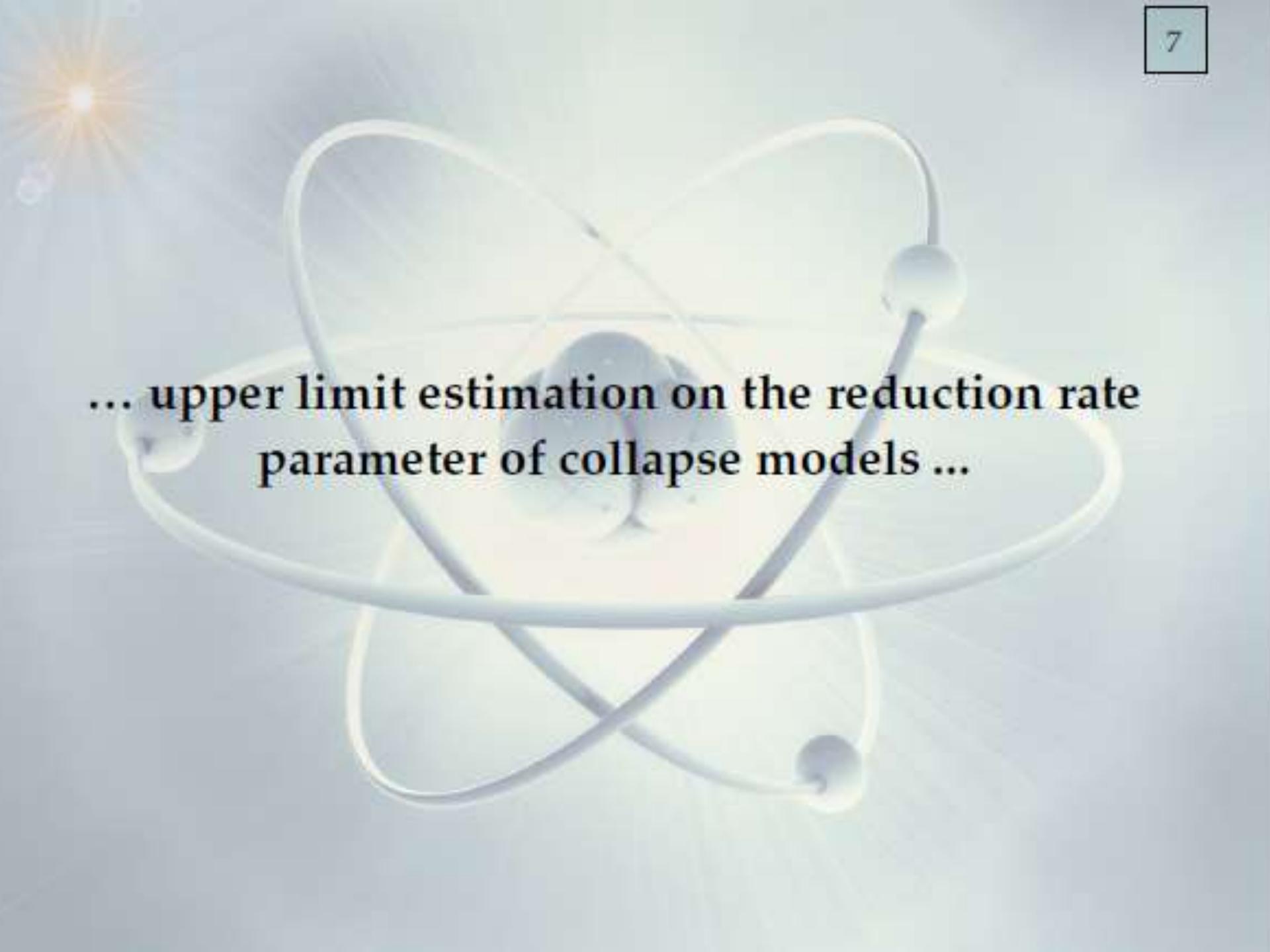
$$r_C = 1/\sqrt{\alpha} \sim 10^{-5} \text{ cm}$$

**PREDICTIONS** of collapse models are different from standard quantum mechanical predictions ... they can be tested experimentally! ...

Increasing size of the system



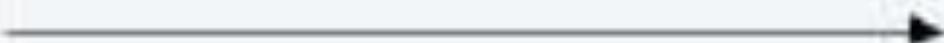
[22] C. Curceanu, B. C. Hiesmayr, and K. Piscicchia,  
J. Adv. Phys. **4**, 263 (2015).

A stylized atomic model is centered in the background. It features three elliptical orbits represented by thin, glowing white lines. Three small, dark blue spheres representing electrons are positioned at the intersections of the orbits. The background is a light gray.

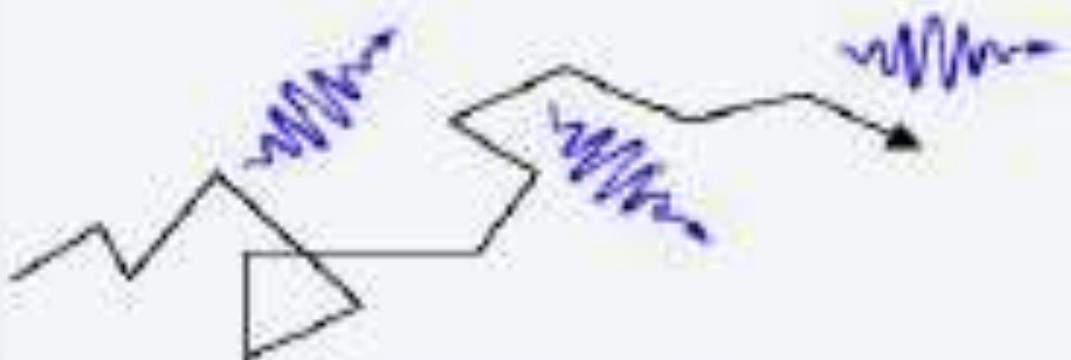
... upper limit estimation on the reduction rate  
parameter of collapse models ...

# FREE PARTICLE

1. Quantum mechanics



2. Collapse models



# ... spontaneous photon emission

Besides collapsing the state vector to the position basis in non relativistic QM  
 the interaction with the stochastic field increases the expectation value of particle's energy



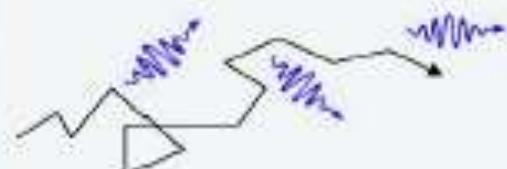
implies for a charged particle energy radiation (not present in standard QM) !!!

- 1) Plausibility test of collapse models (ex. Karolyhazy model, collapse is induced by fluctuations in space-time → unreasonable amount of radiation in the X-ray range).
- 2) The comparison between theoretical prediction and experimental results will provide constraints on the parameters of the CSL model

## FREE PARTICLE

1. Quantum mechanics

2. Collapse models



$$\frac{d\Gamma_k}{dk} = \frac{e^2 \lambda \hbar}{2\pi^2 \epsilon_0 m^2 c^3 k}$$

Q. Fu, Phys. Rev. A 56, 1806 (1997)

S.L. Adler, A. Bassi & S. Donadi,  
 ArXiv 1011.3941

# Expected X-ray rate from Ge low activity experiments

Q. Fu, Phys. Rev. A 56, 1806 (1997) → only upper limit on  $\lambda$  based on comparison with the radiation appearing in an isolated slab of Ge (raw data not background subtracted)

H. S. Miley, et al., Phys. Rev. Lett. 65, 3092 (1990)

Energy (keV)	Expt. upper bound (counts/keV/kg/day)	Theory (counts/keV/kg/day)
11	0.049	0.071
101	0.031	0.0073
201	0.030	0.0037
301	0.024	0.0028
401	0.017	0.0019
501	0.014	0.0015

TABLE I. Experimental upper bounds and theoretical predictions of the spontaneous radiation by free electrons in Ge for a range of photon energy values.

Comparison with the lower energy bin, due to the non-relativistic constraint of the CSL model

$$\frac{d\Gamma_k}{dk} \Big|_{th} = (2.74 \cdot 10^{-31}) \cdot 4 \cdot (8.29 \cdot 10^{24}) \cdot (8.6 \cdot 10^4) \cdot \frac{1}{k} < \frac{d\Gamma_k}{dk} \Big|_{ex}$$

(Atoms / Kg) in Ge      1 day

4 valence electrons are considered  
BE ~ 10 eV « energy of emitted  $\gamma$  ~ 11 keV  
quasi-free electrons

$$\frac{e^2 \lambda}{4\pi^2 a^2 m^2}$$

Result →  $\lambda < 0.55 \times 10^{-16} \text{ s}^{-1}$  the GRW theory predicts 45% more radiation than the observed upper bound.

Result possibly biased by the punctual evaluation of the rate at one single energy bin.

# Expected X-ray rate from Ge low activity experiments

Q. Fu, Phys. Rev. A 56, 1806 (1997) → only upper limit on  $\lambda$  based on comparison with the radiation appearing in an isolated slab of Ge (raw data not background subtracted)  
H. S. Miley, et al., Phys. Rev. Lett. 65, 3092 (1990)

$$\text{Result} \rightarrow \lambda < 0.55 \times 10^{-16} \text{ s}^{-1}$$

According to S. L. Adler and F. M. Ramazanoglu, J. Phys. A40; 13395 (2007)

such value is to be divided by a factor  $4\pi$

No mass-proportional

$$\lambda < 4.38 \times 10^{-18} \text{ s}^{-1}$$

for a mass proportional coupling ...

$$\lambda \rightarrow \lambda \left( \frac{m_e}{m_N} \right)^2$$

mass-proportional

$$\lambda < 1.54 \times 10^{-11} \text{ s}^{-1}$$

# New analysis: using published data of the IGEX experiment (K. Piscicchia)

The IGEX experiment is a low-activity Ge based experiment dedicated to the  $\beta\beta 0\nu$  decay research. (C. E. Aalseth et al., IGEX collaboration Phys. Rev. C 59, 2108 (1999))

In (A. Morales et al., IGEX collaboration Phys. Lett. B 532, 8-14 (2002)) the published data acquired for an exposure of 80 kg day in the energy

Low-energy data from the IGEX RG-II detector (Mt = 80 kg day)

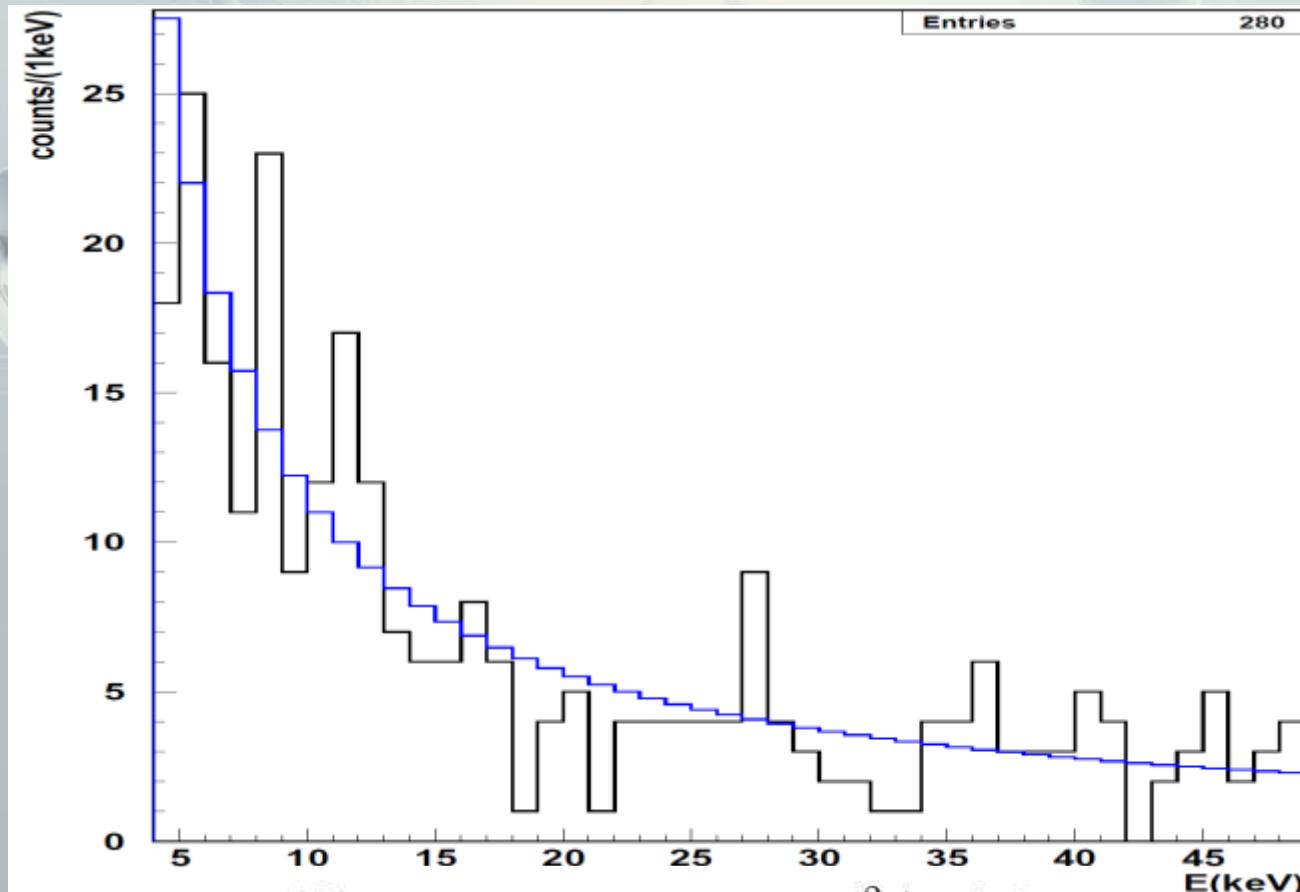
$E$ (keV)	Counts	$E$ (keV)	Counts	$E$ (keV)	Counts
4.5	18	19.5	4	34.5	4
5.5	25	20.5	5	35.5	4
6.5	16	21.5	1	36.5	6
7.5	11	22.5	4	37.5	3
8.5	23	23.5	4	38.5	3
9.5	9	24.5	4	39.5	3
10.5	12	25.5	4	40.5	5
11.5	17	26.5	4	41.5	4
12.5	12	27.5	9	42.5	0
13.5	7	28.5	4	43.5	2
14.5	6	29.5	3	44.5	3
15.5	6	30.5	2	45.5	5
16.5	8	31.5	2	46.5	2
17.5	6	32.5	1	47.5	3
18.5	1	33.5	1	48.5	4

## New analysis: results and discussion

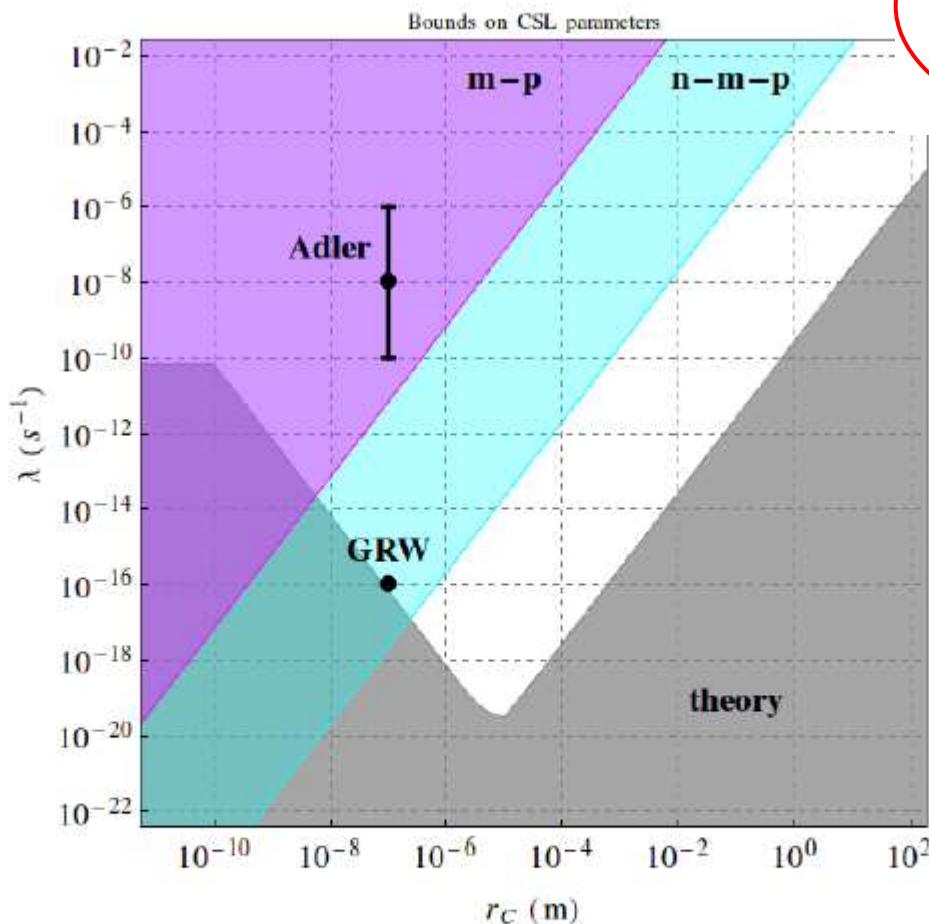
The X-ray spectrum was fitted assuming the predicted energy dependence:

$$\frac{d\Gamma_k}{dk} = \frac{\alpha(\lambda)}{k}$$

With  $\alpha(\lambda)$  free parameter, bin contents are treated with Poisson statistics.



Fit result  $\alpha(\lambda) = 110 \pm 7$  ,  $\chi^2/n.d.f = 1.1$



$\lambda \leq 6.8 \cdot 10^{-12} \text{ s}^{-1}$  mass prop.,

$\lambda \leq 2.0 \cdot 10^{-18} \text{ s}^{-1}$  non-mass prop.

Figure 2. Mapping of the  $\lambda - r_C$  Continuous Spontaneous Localization (CSL) parameters: the originally proposed theoretical values (GRW, Adler) are shown as black points; the region excluded by theory (theory) is represented in gray. The excluded region according to our analysis is shown in cyan for the non-mass proportional case (n-m-p) and in magenta for the mass proportional case (m-p).

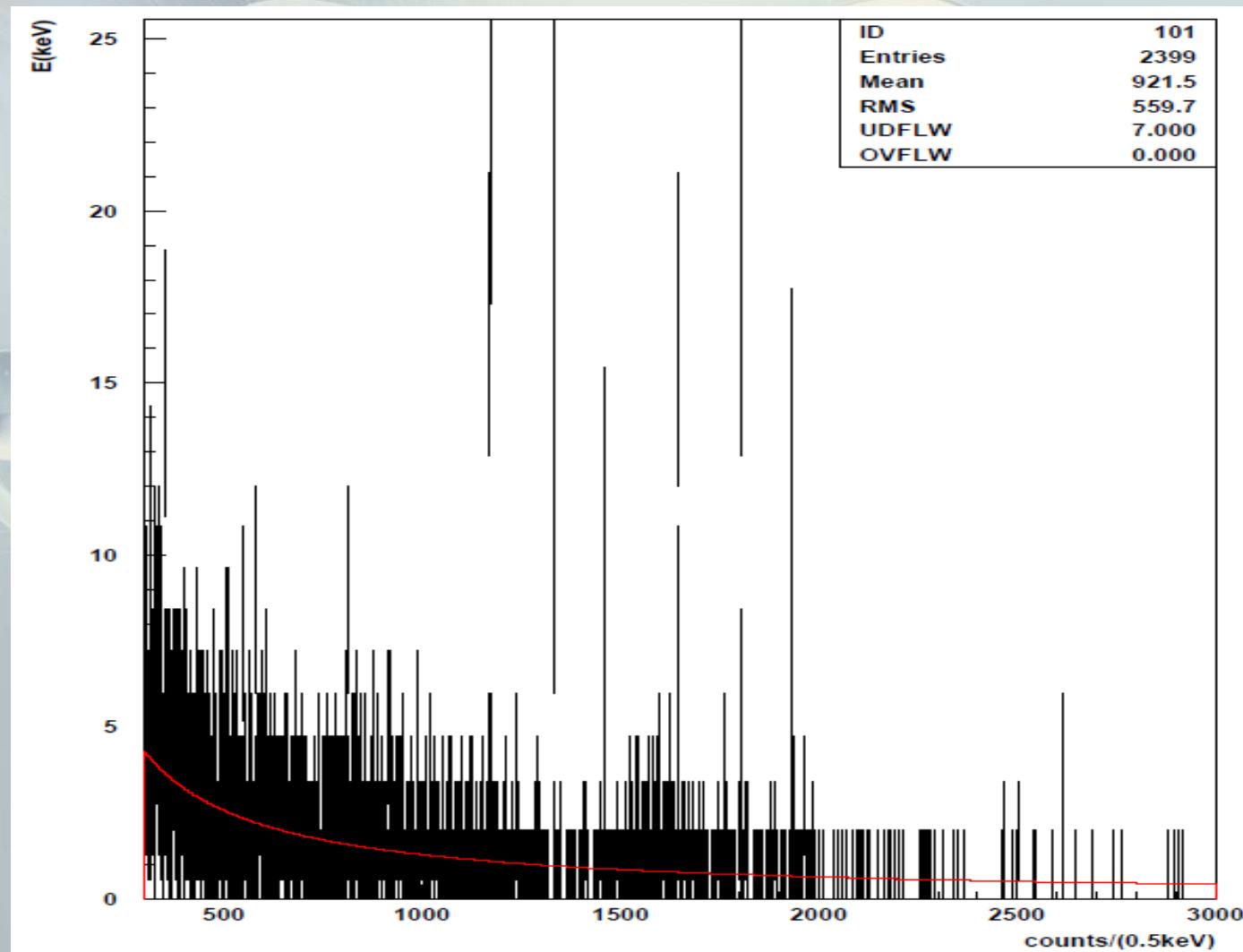
# Spontaneous emission including nuclear protons – data taking at LNGS (ultrapure Ge)!



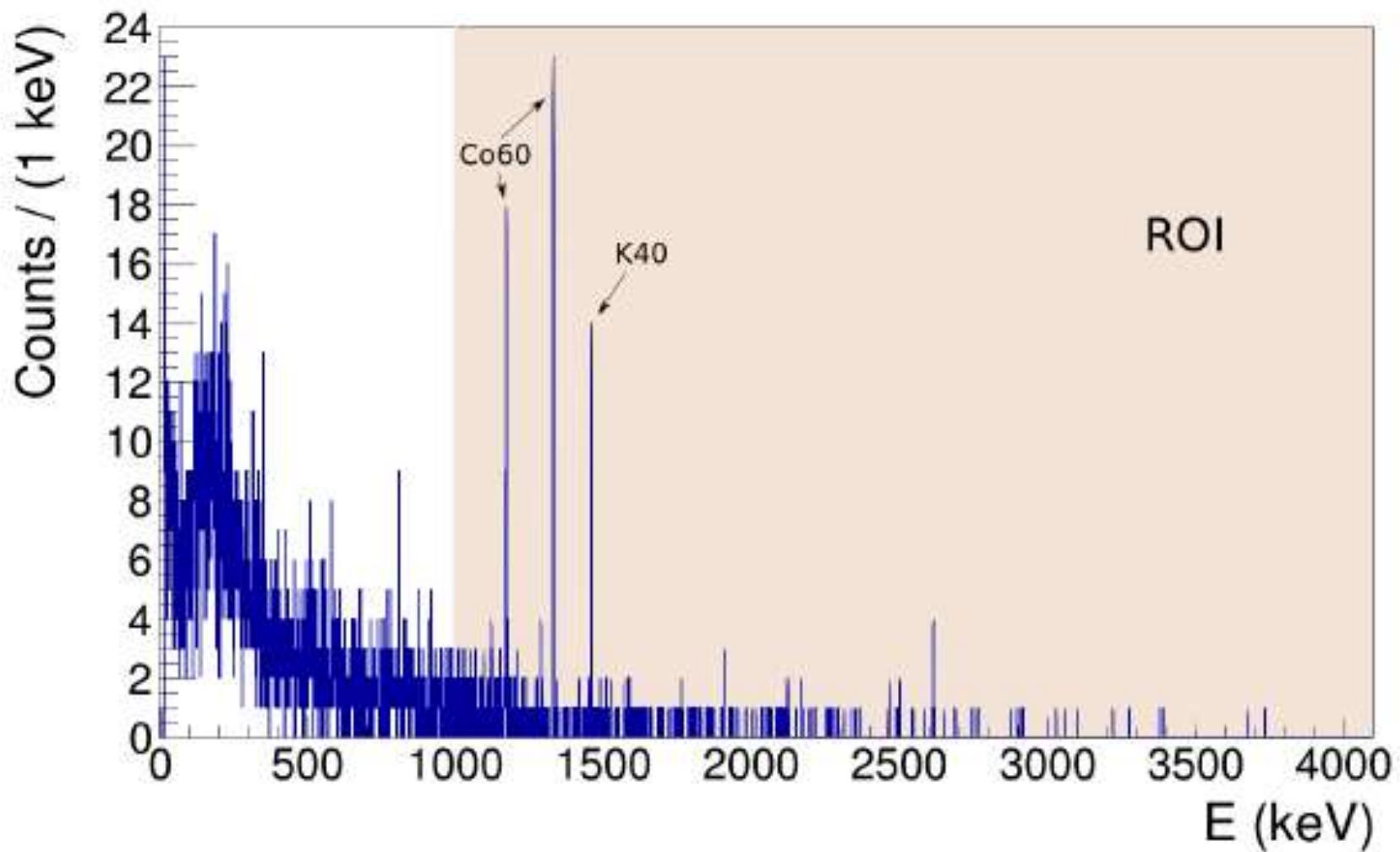
# Spontaneous emission – data taking at LNGS (ultrapure Ge)! - analysis including PROTON emission



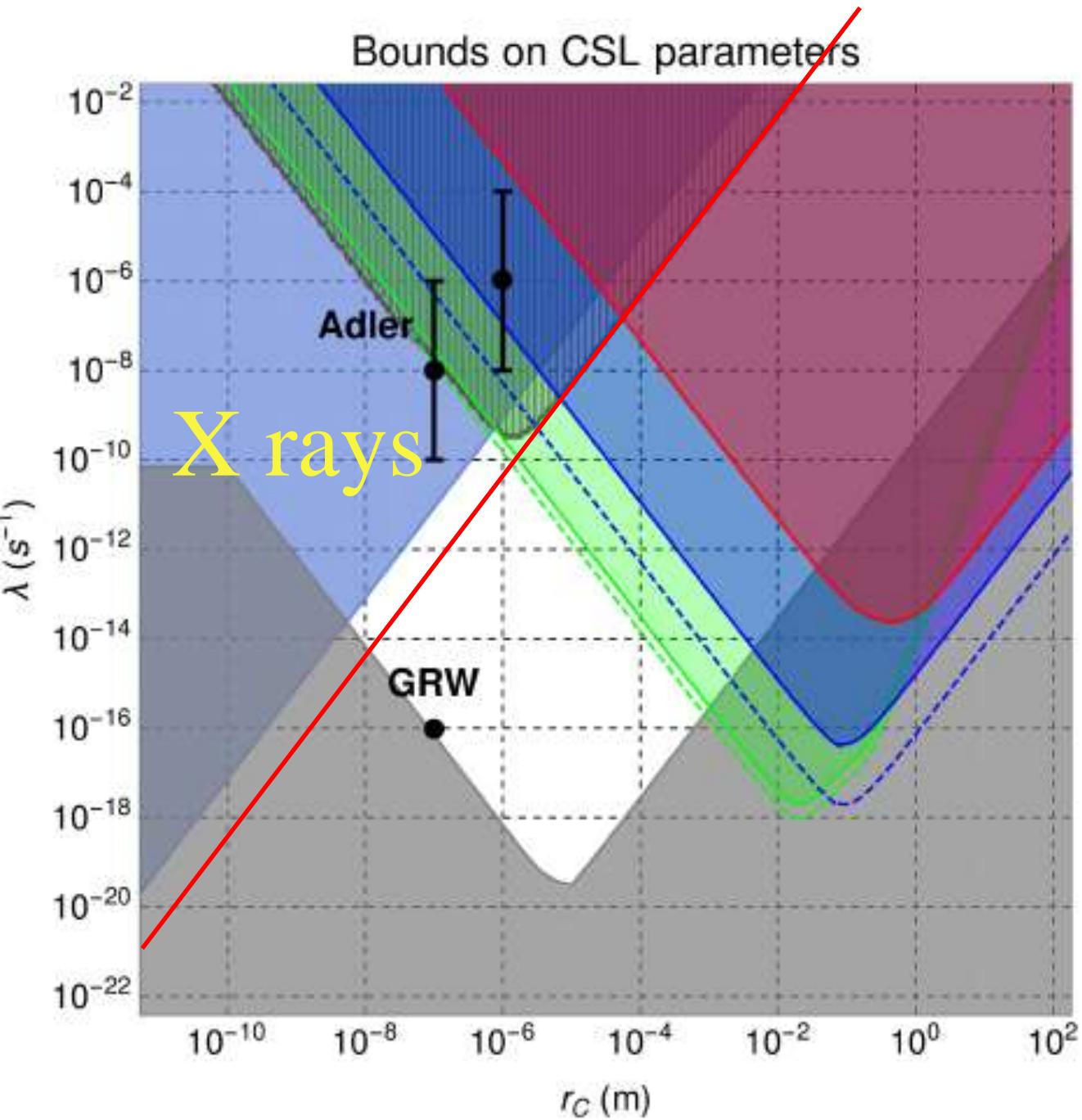
# Spontaneous emission – ultrapure Ge



$$\lambda < 5.2 \cdot 10^{-13} \text{ s}^{-1}$$



## Bounds on CSL parameters



[22] C. Curceanu, B. C. Hiesmayr, and K. Piscicchia,  
J. Adv. Phys. **4**, 263 (2015).

# Test of gravity-induced wave function collapse

Paper in preparation

Penrose proposed that a spatial quantum superposition collapses as a backreaction from spacetime, which is curved in different ways by each branch of the superposition; in this sense, people speak of gravity-induced wave function collapse. He also provided a heuristic formula to compute the decay time of the superposition, which is equivalent to that suggested earlier by Diósi, hence the name Diósi-Penrose model. The collapse is random, and this randomness

The photon emission rate for a crystal turns out to be:

$$\frac{d\Gamma_t}{d\omega} = \frac{2}{3} \frac{Ge^2 N_a^2}{\pi^{3/2} \varepsilon_0 c^3 R_0^3 \omega},$$

$N_a$  the total number of atoms;  $R_0$  which according to Penrose corresponds to the width of the nucleus wave function in the stationary state of the crystal ( $\sim 10^{-14}$  m (32)), is left as a free

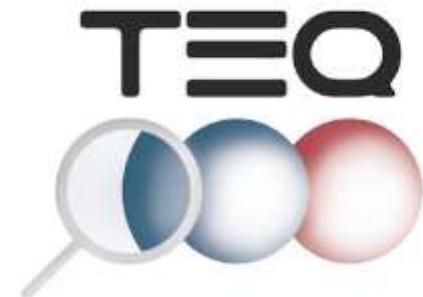
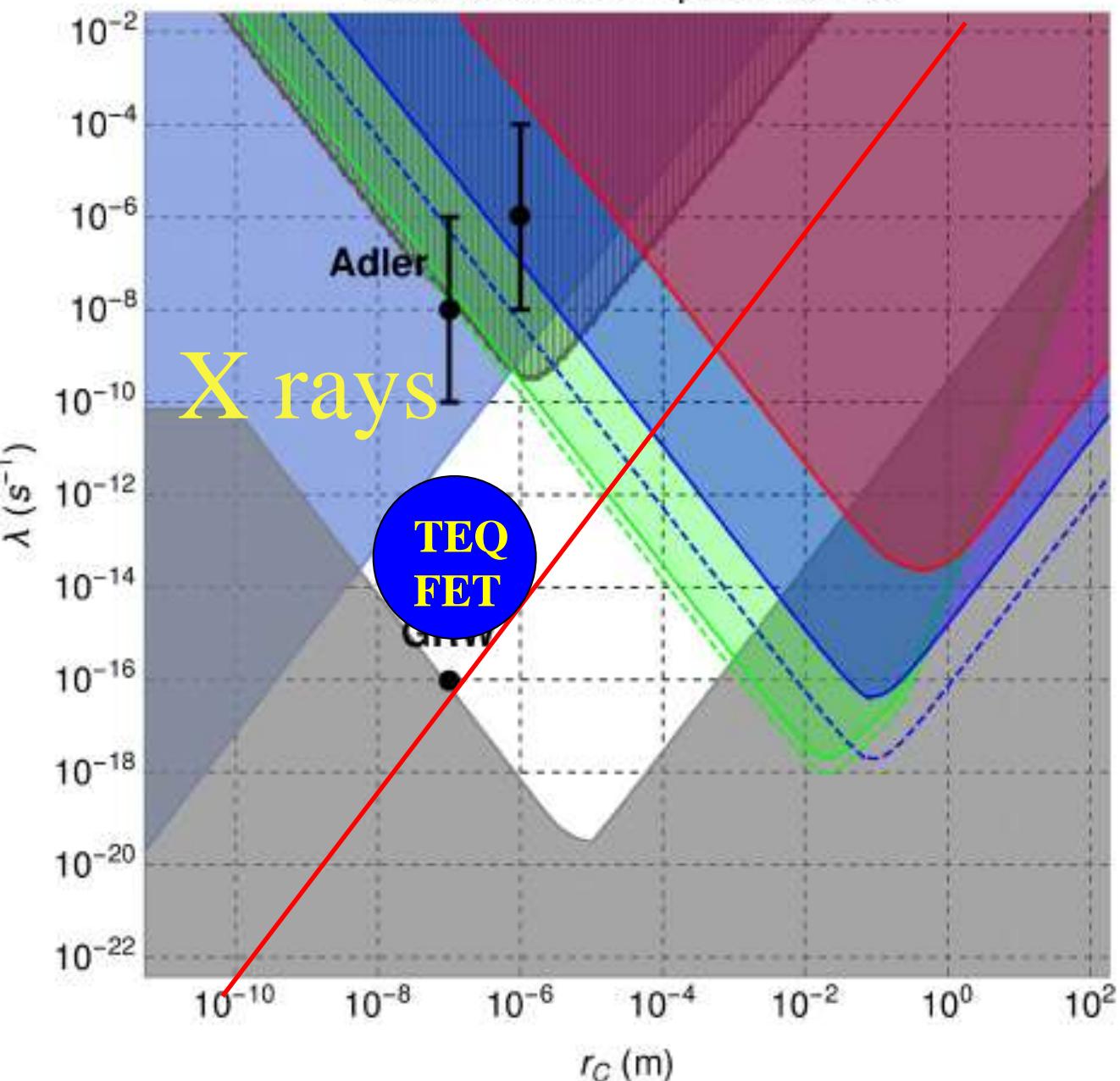
$$R_0 > 0.54 \times 10^{-10} \text{ m}$$

Therefore, we conclude that the model of a gravity-induced collapse of the wave function, as proposed by Penrose is ruled out.

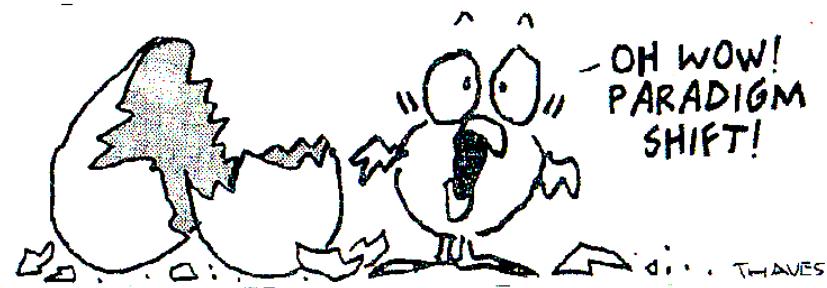
## *Future plans:*

- we have other data taking with pure -lead target (roman lead) -> data analyses
- Work on the “color of the noise”
- Other measurements at LNGS and at Stawell (Australia)

## Bounds on CSL parameters



[22] C. Curceanu, B. C. Hiesmayr, and K. Piscicchia,  
J. Adv. Phys. **4**, 263 (2015).



## Questions:

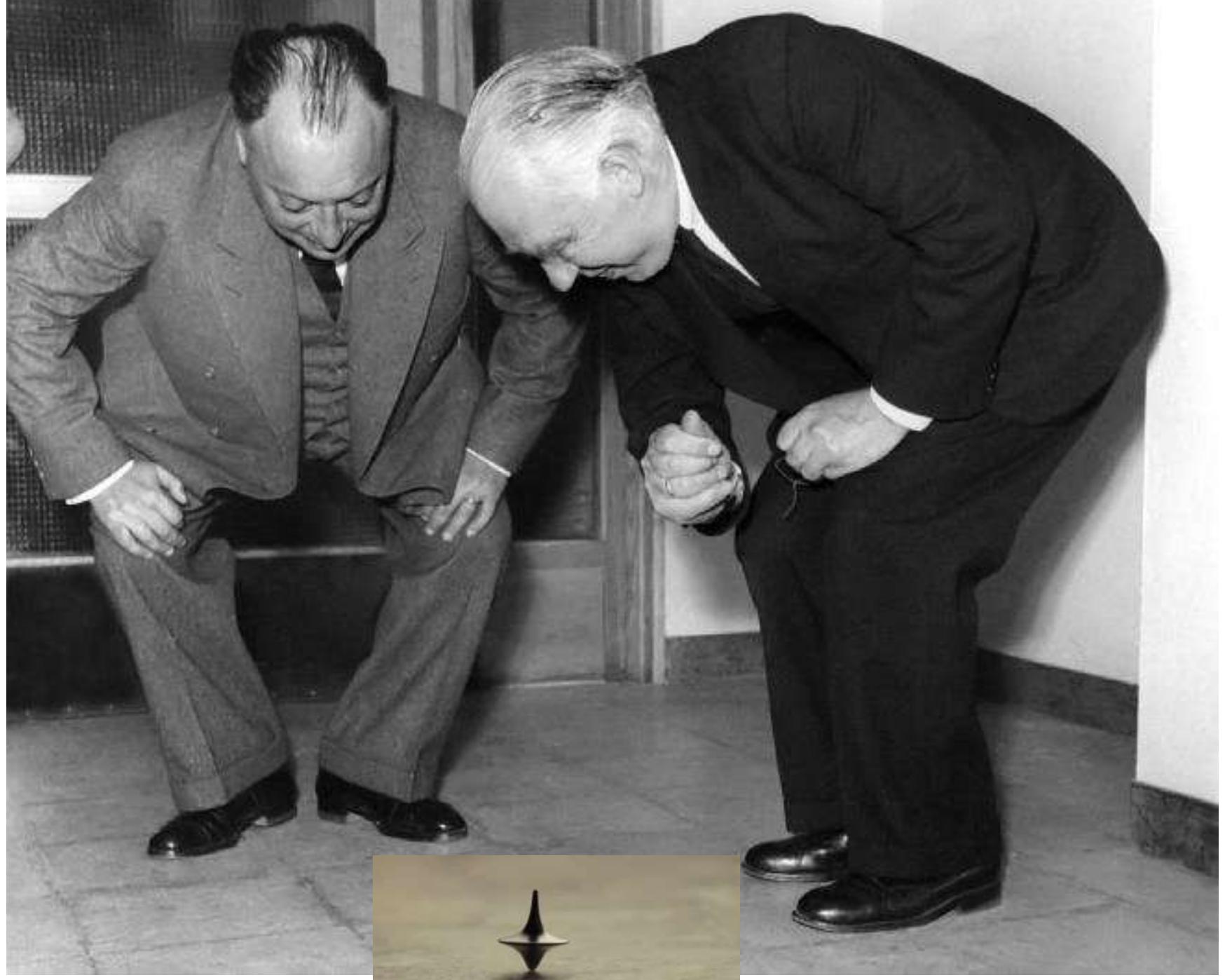
- *What induces the collapse:  
Could be related with gravity?*
- *Has it anything to do with dark  
Sector (matter, energy)?*
- *Is there any theory beyond QM?*

**We also search for the *impossible atoms***

***An experiment to test the Pauli Exclusion***

***Principle (PEP) for electrons in a clean  
environment (LNGS) using atomic physics***

***methods – the VIP experiment***





Required for  
bosons.

$$\psi = \psi_1(a)\psi_2(b) \pm \psi_1(b)\psi_2(a)$$

Probability amplitude that  
both states "a" and "b" are  
occupied by electrons 1 and  
2 in either order.

Required for  
fermions.



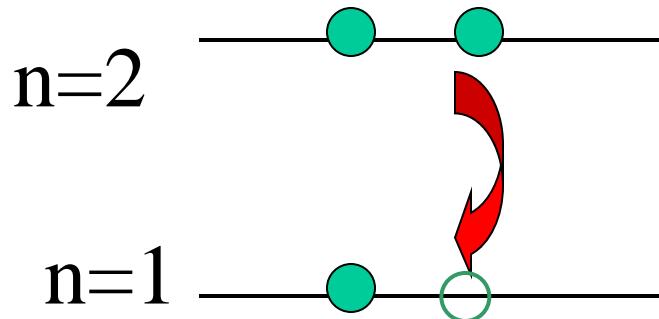
## Theories of Violation of Statistics

O.W. Greenberg: AIP Conf.Proc.545:113-127,2004

*“Possible external motivations for violation of statistics include: (a) violation of CPT, (b) violation of locality, (c) violation of Lorentz invariance, (d) extra space dimensions, (e) discrete space and/or time and (f) noncommutative spacetime. Of these (a) seems unlikely because the quon theory which obeys CPT allows violations, (b) seems likely because if locality is satisfied we can prove the spin-statistics connection and there will be no violations, (c), (d), (e) and (f) seem possible.....”*

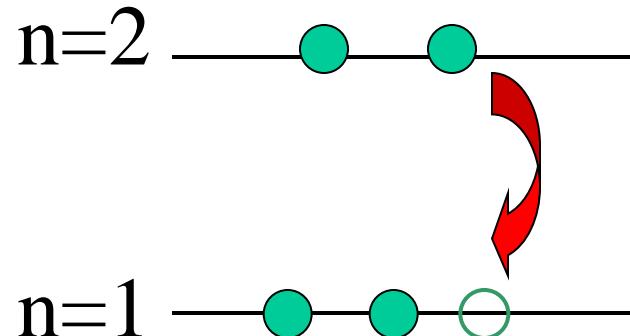
***Hopefully either violation will be found experimentally or our theoretical efforts will lead to understanding of why only bose and fermi statistics occur in Nature.”***

# Experimental method: Search for anomalous X-ray transitions when bringing “new” electrons



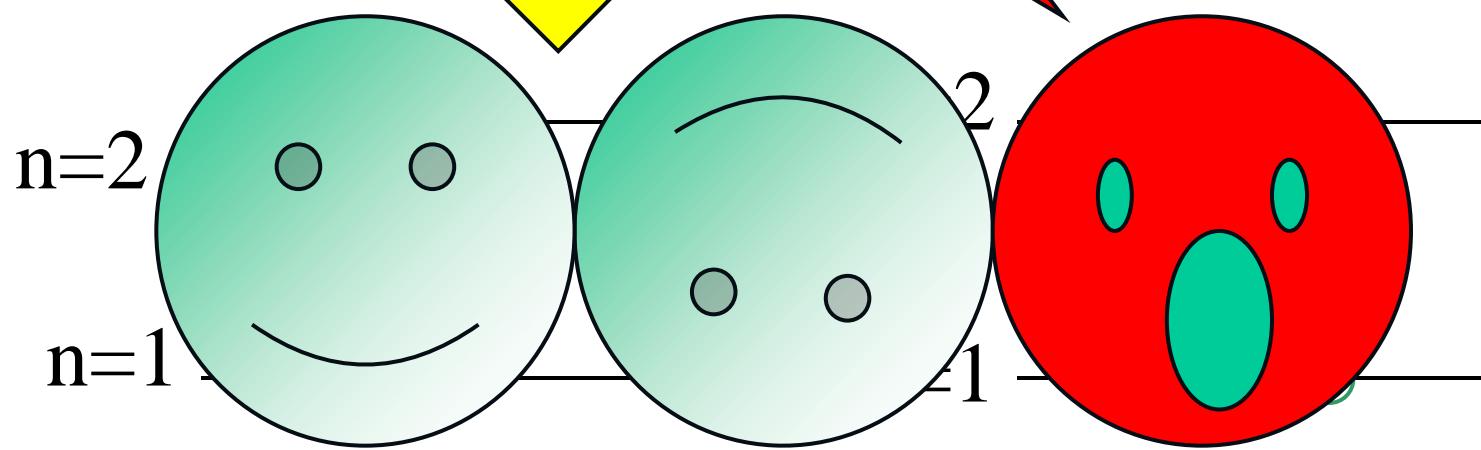
Normal  $2p \rightarrow 1s$   
transition

Energy 8.04 keV



$2p \rightarrow 1s$  transition  
violating  
Pauli principle  
Energy 7.7 keV

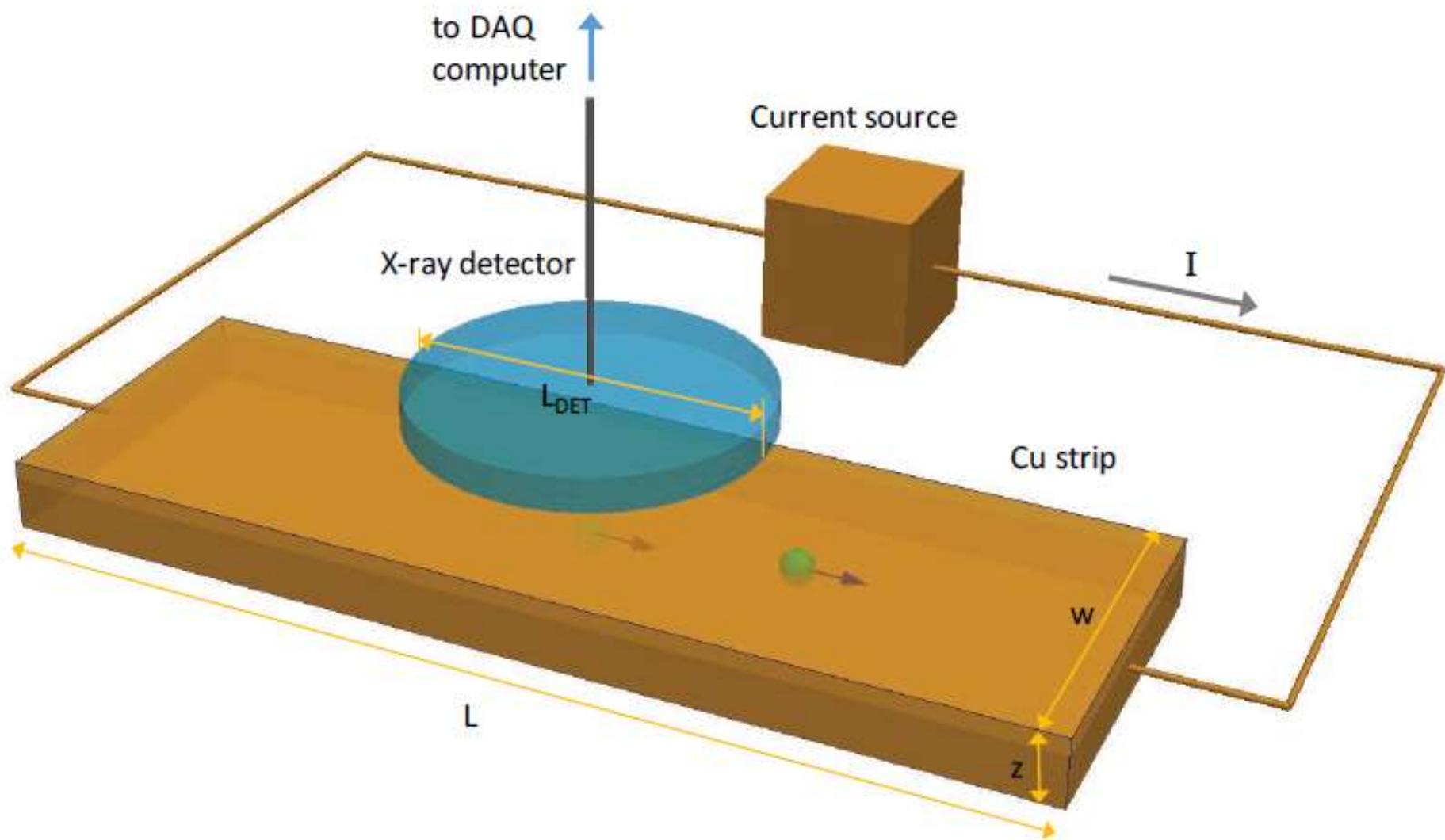
# Experimental method: Search for anomalous X-ray transitions when bringing “n” electrons

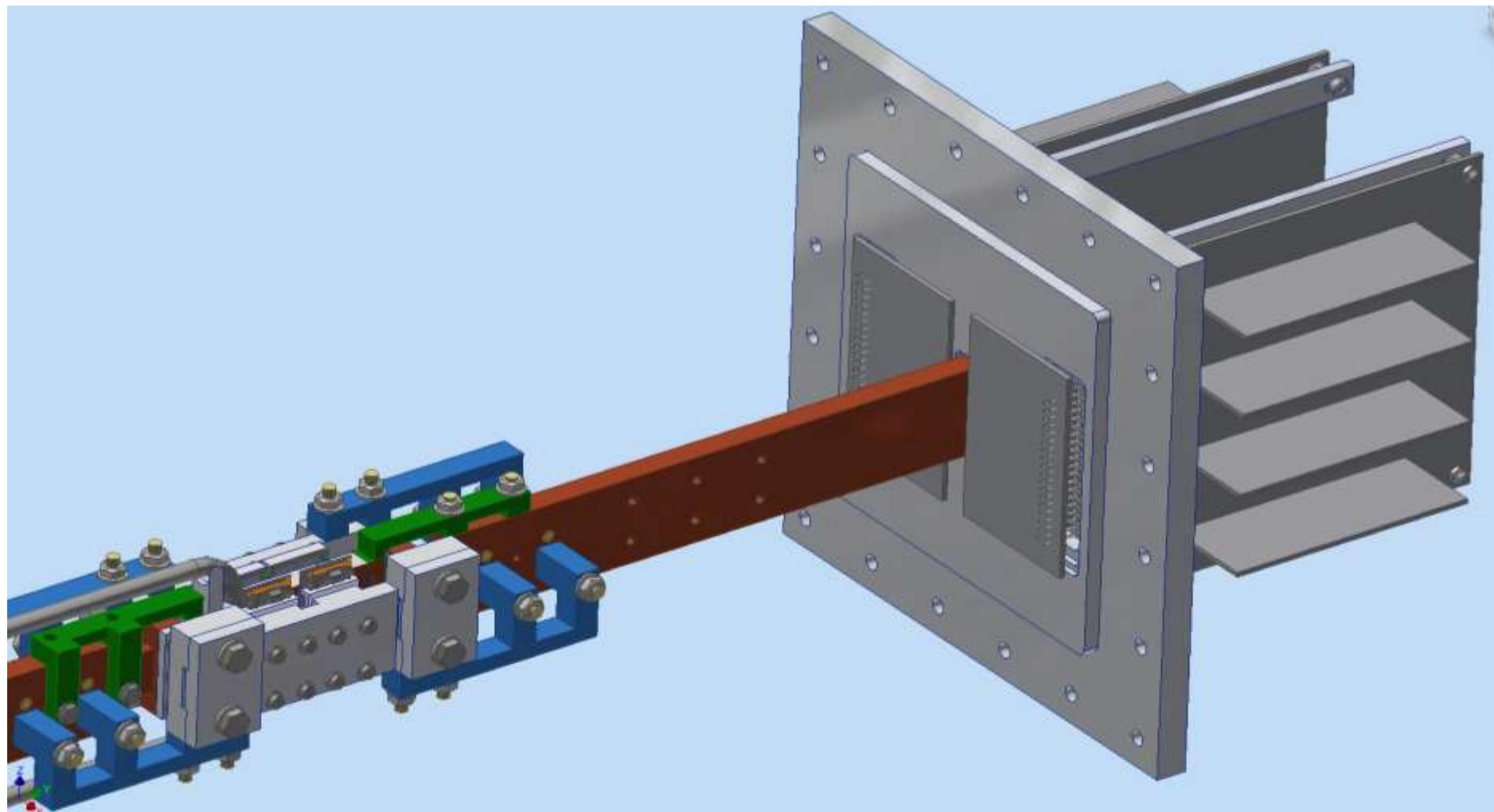


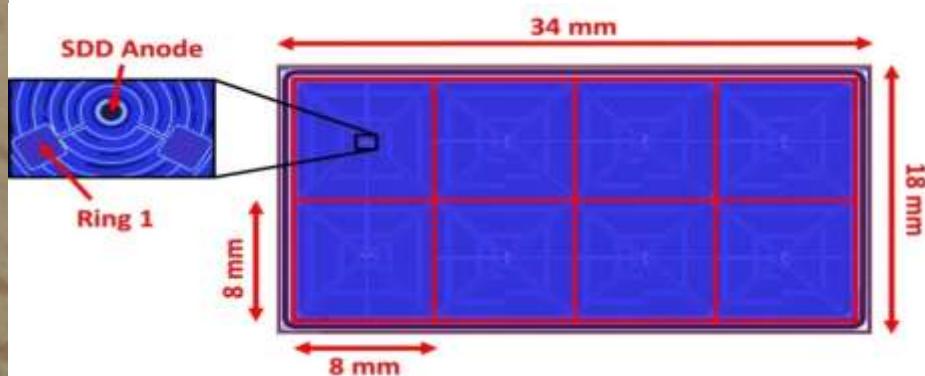
Normal  $2p \rightarrow 1s$   
transition

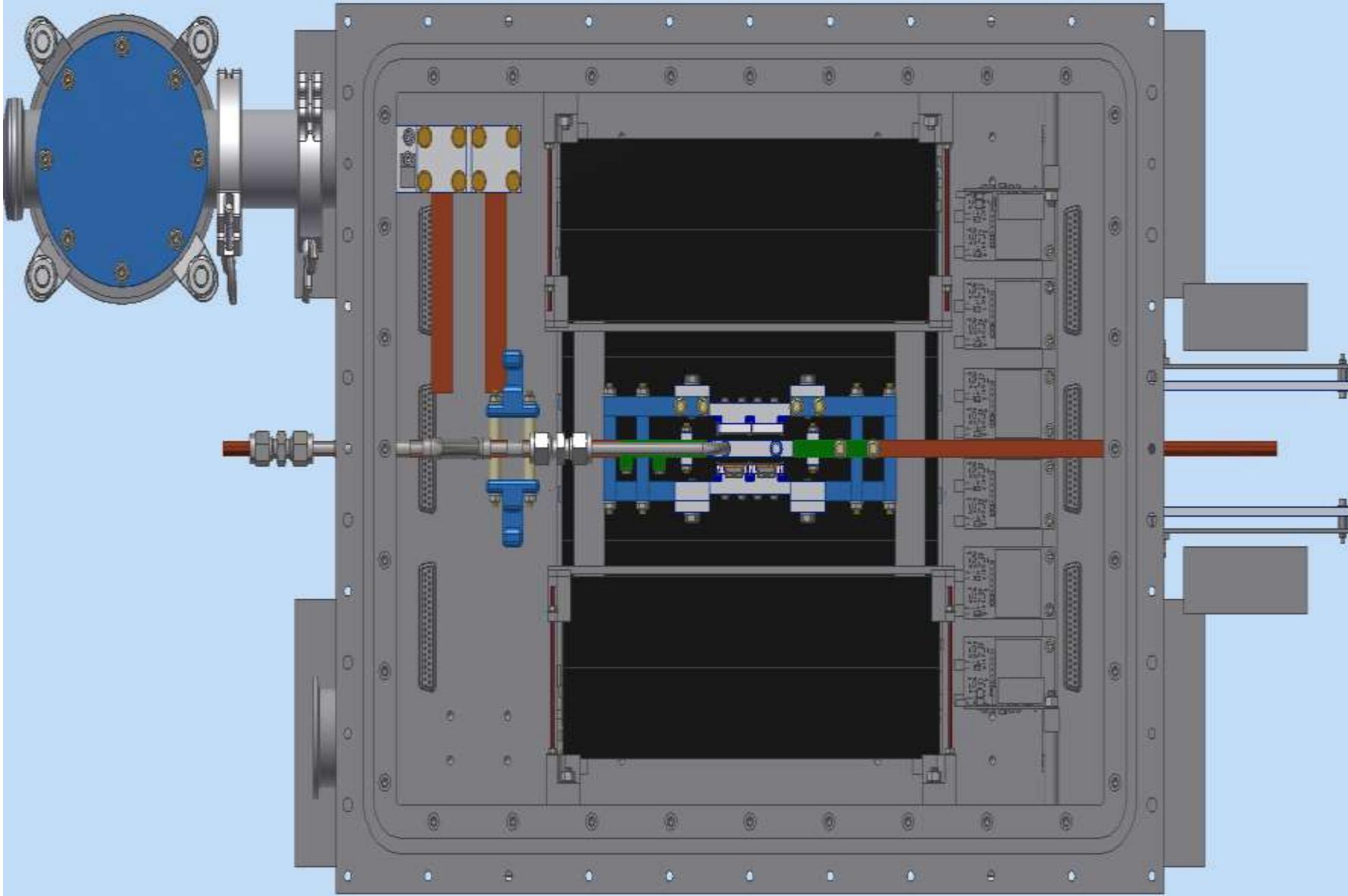
Energy 8.04 keV

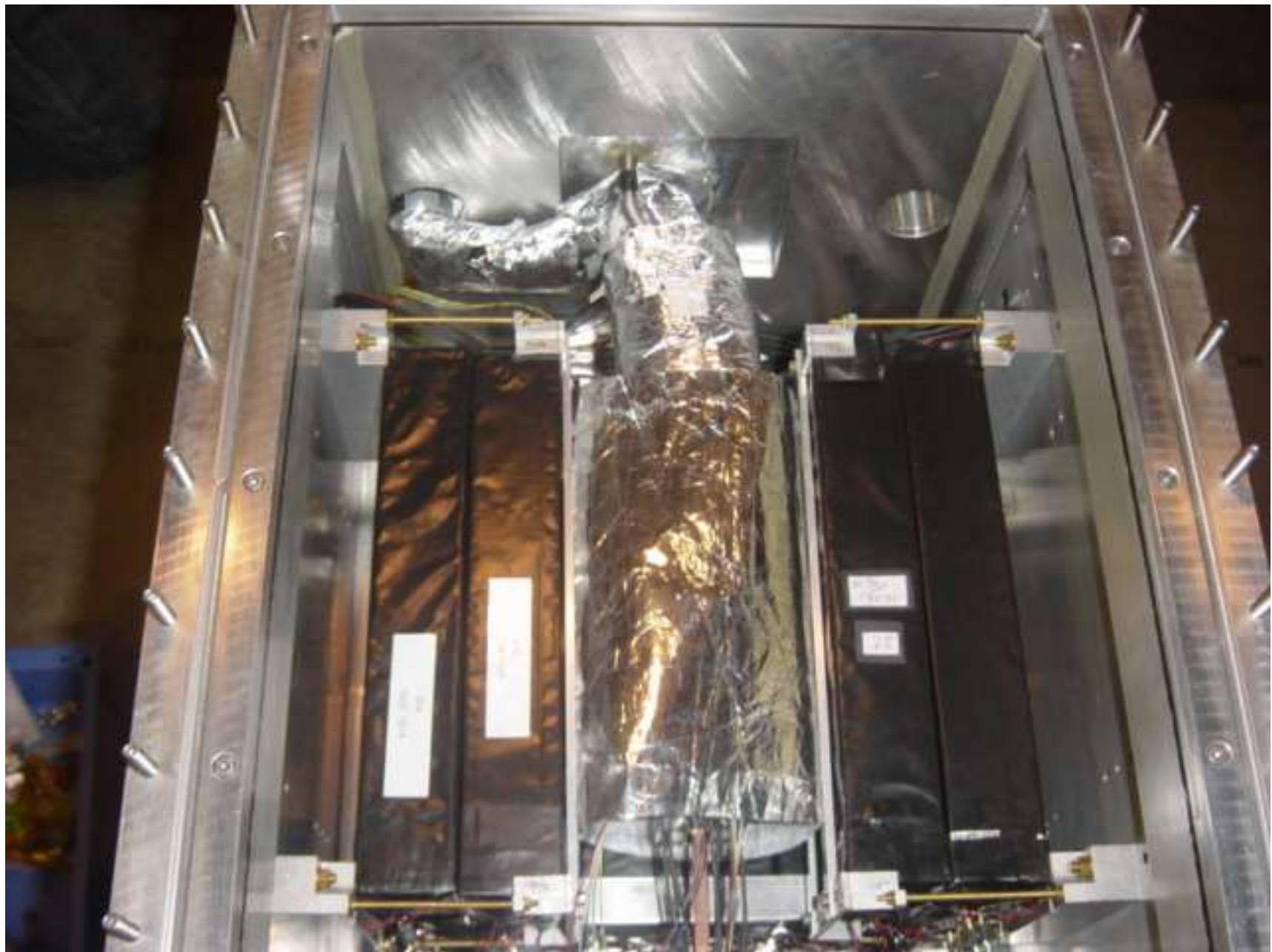
$2p \rightarrow 1s$  transition  
violating  
Pauli principle  
Energy 7.7 keV



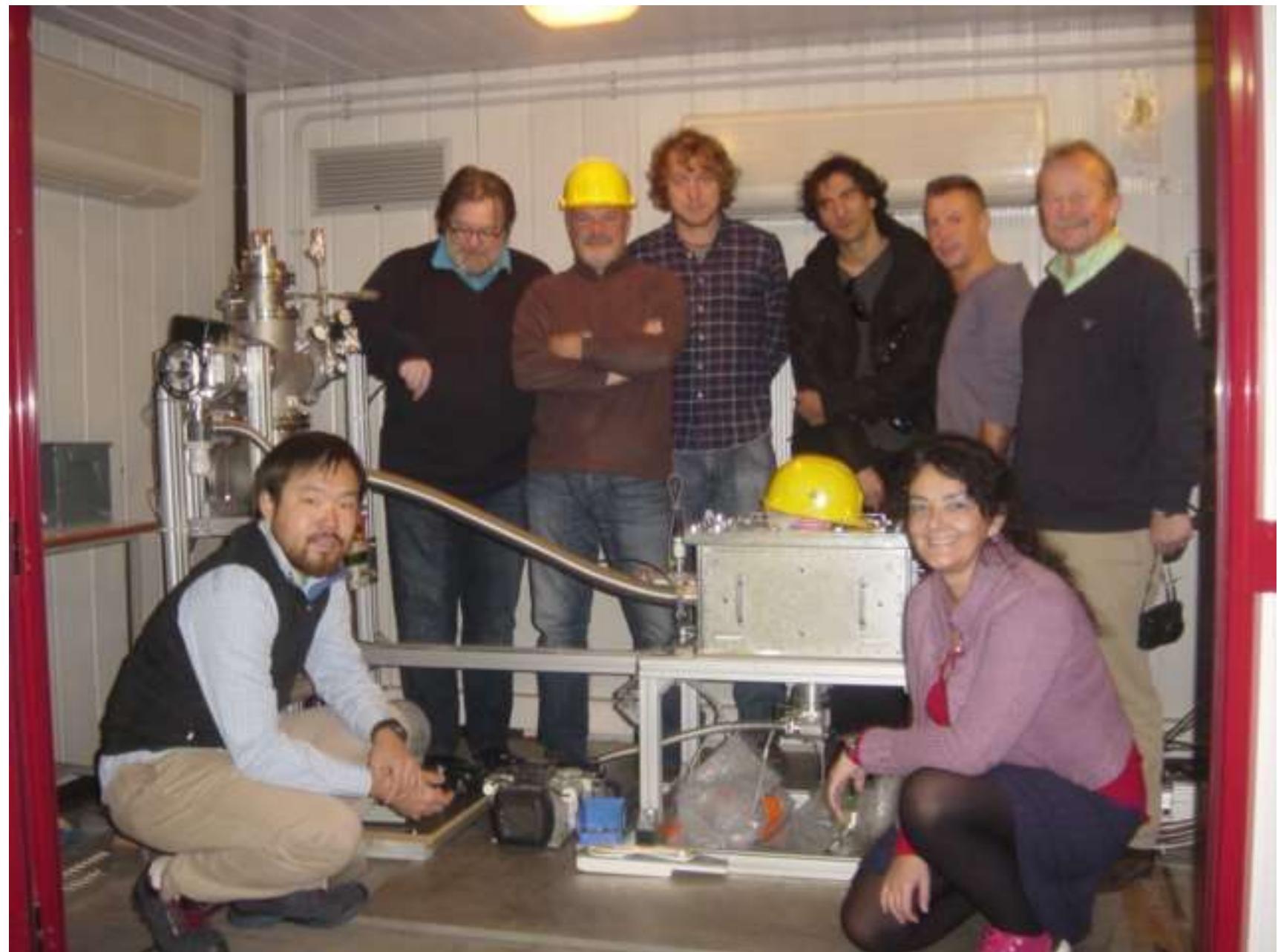












# Experimental search for the violation of Pauli exclusion principle

VIP-2 Collaboration

H. Shi<sup>1,2,12,a</sup>, E. Milotti<sup>3,b</sup>, S. Bartalucci<sup>1</sup>, M. Bazzi<sup>1</sup>, S. Bertolucci<sup>4</sup>, A. M. Bragadireanu<sup>1,5</sup>, M. Cargnelli<sup>1,2</sup>, A. Clozza<sup>1</sup>, L. De Paolis<sup>1</sup>, S. Di Matteo<sup>6</sup>, J.-P. Egger<sup>7</sup>, H. Elnaggar<sup>8</sup>, C. Guaraldo<sup>1</sup>, M. Iliescu<sup>1</sup>, M. Laubenstein<sup>9</sup>, J. Marton<sup>1,2</sup>, M. Miliucci<sup>1</sup>, A. Pichler<sup>1,2</sup>, D. Pietreanu<sup>1,5</sup>, K. Piscicchia<sup>1,10</sup>, A. Scordo<sup>1</sup>, D. L. Sirghi<sup>1,5</sup>, F. Sirghi<sup>1,5</sup>, L. Sperandio<sup>1</sup>, O. Vazquez Doce<sup>1,11</sup>, E. Widmann<sup>2</sup>, J. Zmeskal<sup>1,2</sup>, C. Curceanu<sup>1,5,10</sup>

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<sup>4</sup> Dipartimento di Fisica e Astronomia, Università di Bologna, Viale Berti Pichat 6/2, Bologna, Italy

<sup>5</sup> IFIN-HH, Institutul National Pentru Fizica si Inginerie Nucleara Horia Hulubbei, Reactorului 30, Magurele, Romania

<sup>6</sup> UMR 6251, IPR (Institut de Physique de Rennes), CNRS, Université de Rennes, 35000 Rennes, France

<sup>7</sup> Institut de Physique, Université de Neuchâtel, 1 Rue A.-L. Breguet, 2000 Neuchâtel, Switzerland

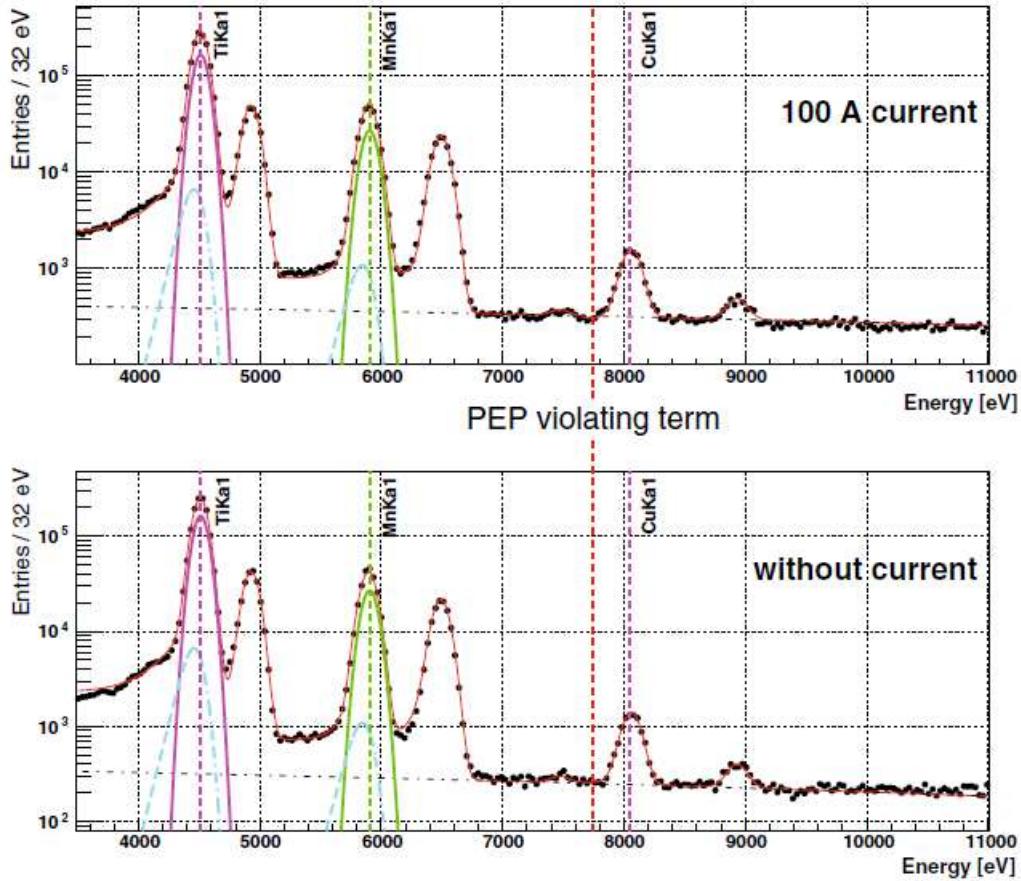
<sup>8</sup> Debye Institute for Nanomaterial Science, Utrecht University, P.O. Box 80.000, 3508 TA Utrecht, The Netherlands

<sup>9</sup> INFN, Laboratori Nazionali del Gran Sasso, S.S. 17/bis, 67010 Assergi, AQ, Italy

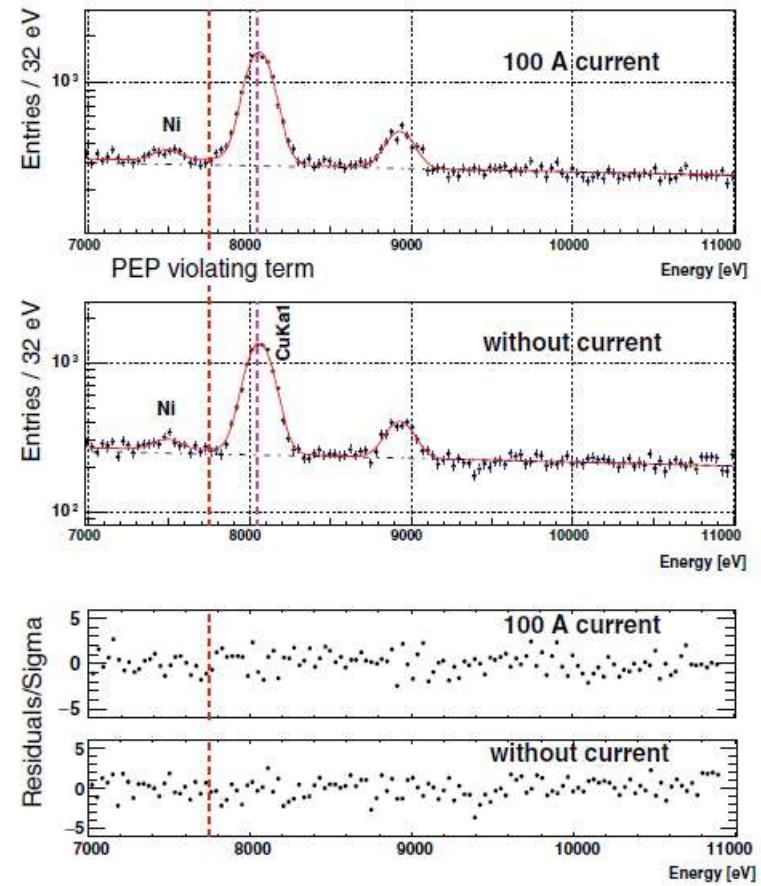
<sup>10</sup> Museo Storico della Fisica e Centro Studi e Ricerche Enrico Fermi, Piazza del Viminale 1, 00183 Rome, Italy

<sup>11</sup> Excellence Cluster Universe, Technische Universität München, Boltzmannstraße 2, 85748 Garching, Germany

<sup>12</sup> Present address: Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, Nikolsdorfer Gasse 18, 1050 Wien, Austria



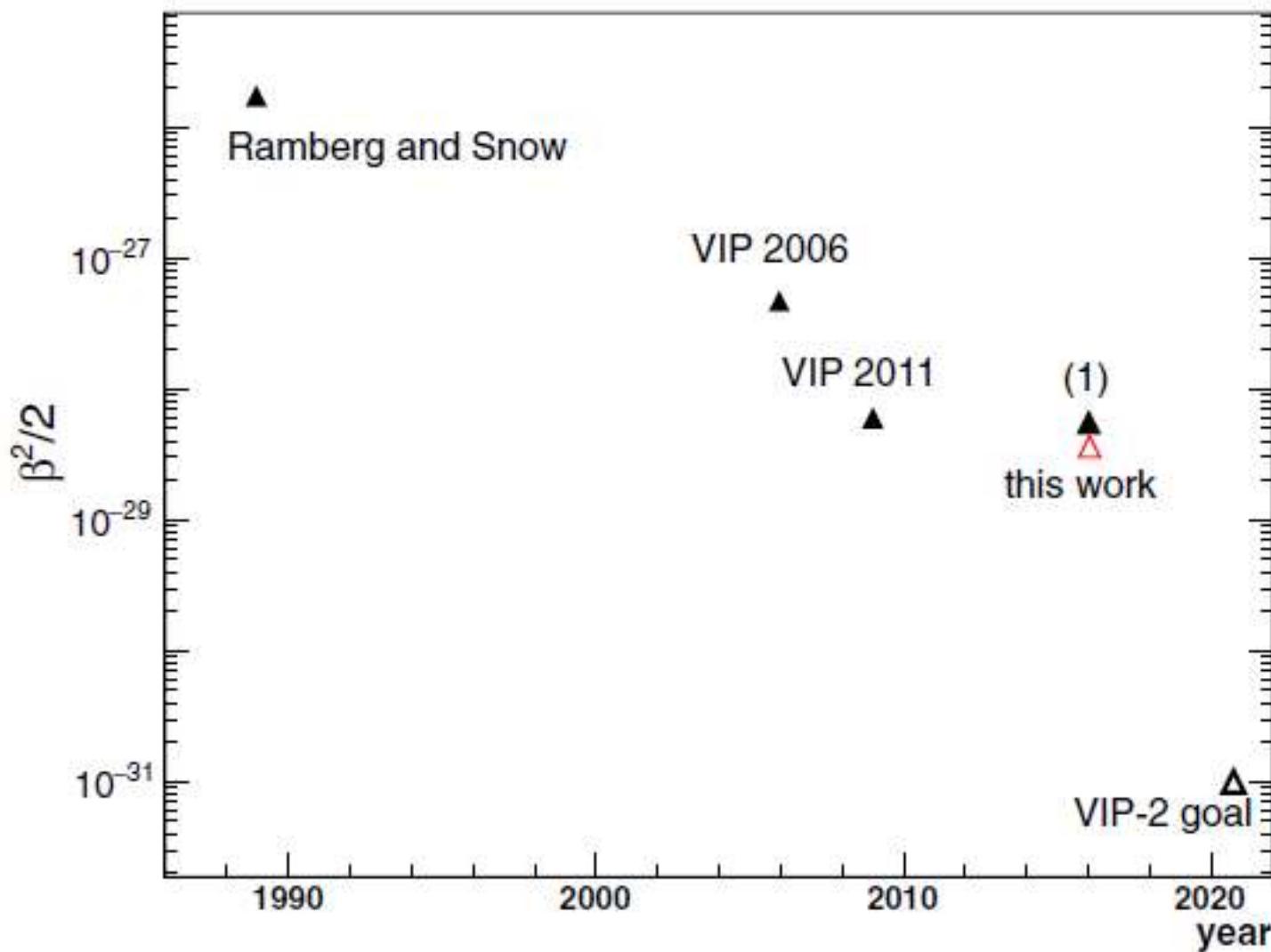
(a)



(b)

**Fig. 8** A global chi-square function was used to fit simultaneously the spectra with and without 100 A current applied to the copper conductor. The energy position for the expected PEP violating events is about 300 eV below the normal copper  $K_{\alpha 1}$  transition. The Gaussian function and the tail part of the  $K_{\alpha 1}$  components and the continuous background

from the fit result are also plotted. **a** The fit to the wide energy range from 3.5 keV to 11 keV, **b** the fit and its residual for the 7–11 keV range where there is no background coming from the calibration source. See the main text for details



**Fig. 10** Past results from PEP violation tests for electrons with a copper conductor, together with the result from this work and the anticipated goal of VIP-2 experiment. The result (1) is based on the same data set of this work, but using the spectra subtraction in the analysis

In the coming years we expect  
either to find a small violation or to  
be able to bound the probability  
that PEP is violated by electrons  
pushing it to  $10^{-31}$

# **Pauli Exclusion Principle violation:**

- as particle property
- space-time property

# VIP-2 underground experiment as a *Crash-Test* of Non-Commutative Quantum Gravity With Andrea Addazi and Antonino Marciao' (Fudan University – China)

Pauli Exclusion Principle (PEP) violations induced from non-commutative space-time can be searched VIP-2 experiment set-up. We show that the limit from VIP-2 experiments on non-commutative space-time scale  $\Lambda$ , related to energy dependent PEP violations, are severe:  $\kappa$ -Poincaré non-commutativity is ruled-out up to the Planck scale. In the next future  $\theta$ -Poincaré will be probed until the Grand-Unification scale! This highly motivates Pauli Exclusion Principle tests from underground experiments as a test of quantum gravity and space-time microscopic structure.

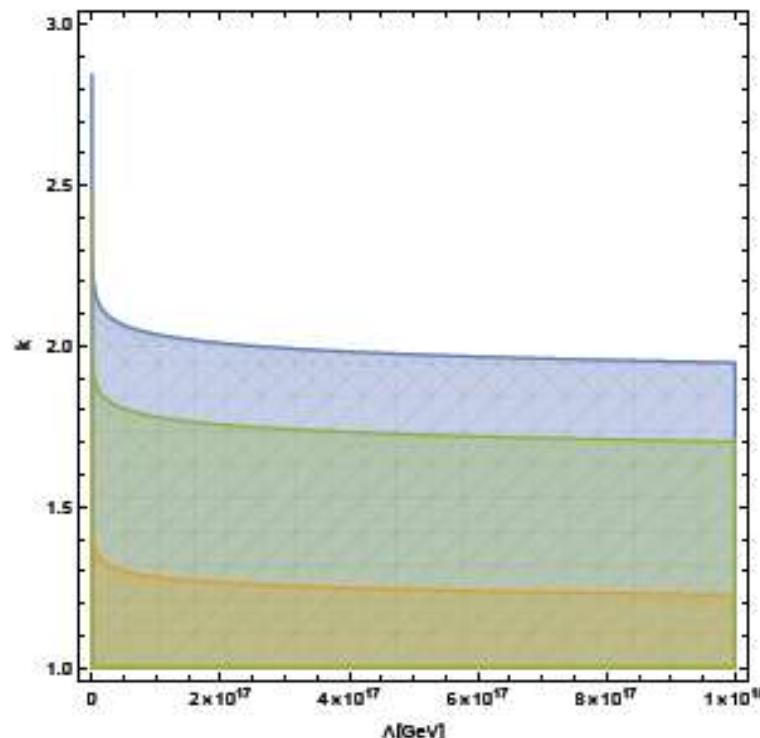


FIG. 1. Exclusion region plots, in the Planck scale window, of the non-commutative scale  $\Lambda$  (x-axis) and the energy powers  $k$  (y-axis) from Eq.1 are shown from present, next future and *futuristic* bounds from VIP-2 experiments  $\delta(E) < 10^{-29}, 10^{-40}, 10^{-45}$  (orange, green, blue respectively).

## On the Importance of Electron Diffusion in a Bulk-Matter Test of the Pauli Exclusion Principle

$$\beta^2/2 < 2.6 \times 10^{-40}.$$

**Search for a remnant violation of the Pauli Exclusion Principle in a Roman lead block**

$$\frac{1}{2}\beta^2 < 2.75 \cdot 10^{-44},$$

## Exclusion principle

# Putting the Pauli exclusion principle on trial

The exclusion principle is part of the bedrock of physics, but that hasn't stopped experimentalists from devising cunning ways to test it.

If we tightly grasp a stone in our hands, we neither expect it to vanish nor leak through our flesh and bones. Our experience is that stone and, more generally, solid matter is stable and impenetrable. Last year marked the 50th anniversary of the demonstration by Freeman Dyson and Andrew Lenard that the stability of matter derives from the Pauli exclusion principle. This principle, for which Wolfgang Pauli received the 1945 Nobel Prize in Physics, is based on ideas so prevalent in fundamental physics that their underpinnings are rarely questioned. Here, we celebrate and reflect on the Pauli principle, and survey the latest experimental efforts to test it.

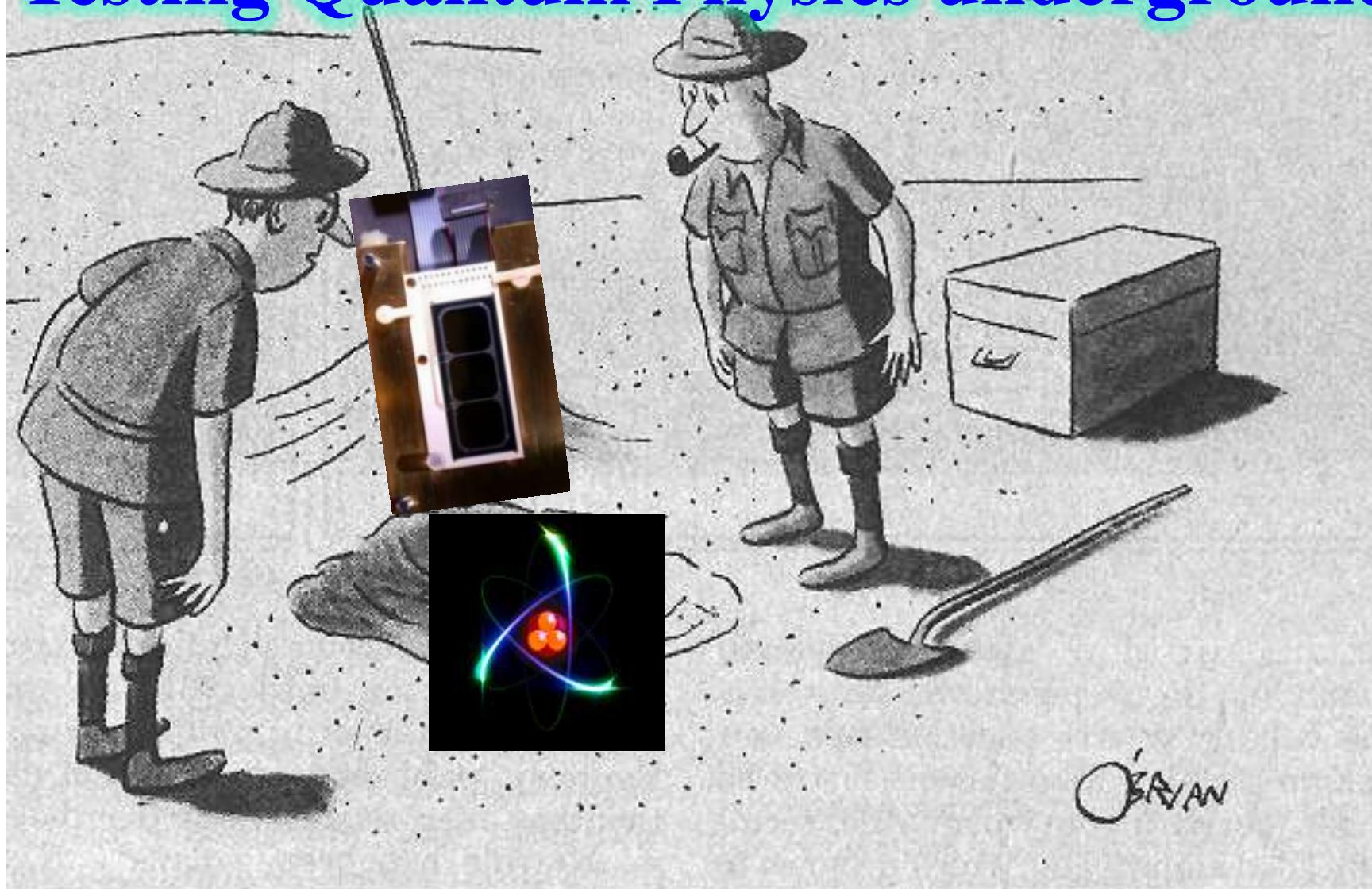
The exclusion principle (EP), which states that no two fermions can occupy the same quantum state, has been with us for almost a century. In his Nobel lecture, Pauli provided a deep and broad-ranging account of its discovery and its connections to unsolved problems of the newly born quantum theory. In the early 1920s, before Schrödinger's equation and Heisenberg's matrix algebra had come along, a young Pauli performed an extraordinary feat when he postulated both the EP and what he called "classically non-describable two-valuedness" – an early hint of the existence of electron spin – to explain the structure of atomic spectra.



PAULI ARCHIVE - PHO-0111

*Portrait of a young Pauli at Svein Rosseland's institute in Oslo in the early 1920s, when he was thinking deeply on the applications of quantum mechanics to atomic physics.*

# Testing Quantum Physics underground



*"This could be the discovery of the century. Depending, of course, on how far down it goes."*

# Acknowledgements



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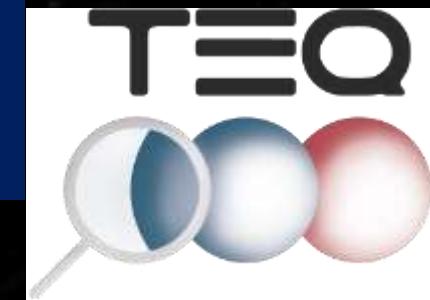


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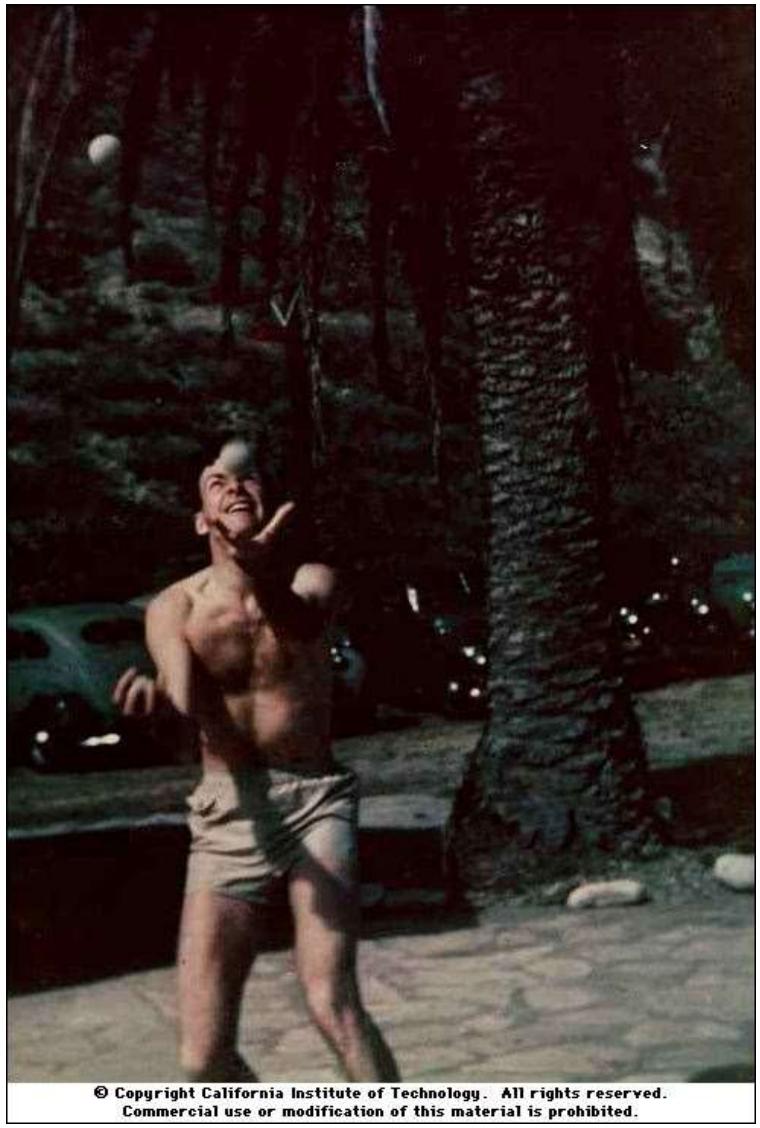
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Thank  
you

# Feynman



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the patterns of another  
till complex new ones are formed.  
They make others like themselves  
and a new dance starts.

Growing in size and complexity  
living things

masses of atoms  
DNA, protein

dancing a pattern ever more intricate.

Out of the cradle

onto dry land

here it is  
standing:

atoms with consciousness;

matter with curiosity.

Stands at the sea, wondering: I  
a universe of atoms  
an atom in the universe.

## 2. What does a violation of PEP mean?

*There are several not quite equivalent proofs of the symmetrization principle in the loose context of field theory.* Consider, e.g., the proof by G. Lüders and B. Zumino, Phys. Rev. **110** (1958) 1450). Its postulates are:

- I. The theory is invariant with respect to the proper inhomogeneous Lorentz group (includes translations, does not include reflections)
- II. Two operators of the same field at points separated by a spacelike interval either commute or anticommute (locality - microcausality)
- III. The vacuum is the state of lowest energy
- IV. The metric of the Hilbert space is positive definite
- V. The vacuum is not identically annihilated by a field

From these postulates it follows that (pseudo)scalar fields commute and spinor fields anticommute.

A necessary (though not sufficient) condition for violation is that at least one of the postulates is violated

I. The theory is **non-invariant** with respect to the proper inhomogeneous Lorentz group (includes translations, does not include reflections)

*Lorentz violation, would have far-ranging consequences and mark the appearance of new physics.*

II. Two operators of the same field at points separated by a spacelike interval **neither commute nor anticommute**

*This would entail a violation of locality (microcausality) at very short distances.*

III. The vacuum is **not** the state of lowest energy

*Unstable vacuum !!!*

IV. The metric of the Hilbert space is **not** positive definite

*States with negative norm !!!*

V. The vacuum is **identically annihilated** by some field

*Weird physics !!!*

*“Hopefully either violation will be found experimentally or our theoretical efforts will lead to understanding of why only bose and fermi statistics occur in Nature”*  
*(Greenberg)*

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**58158: Hunt for the “impossible atoms”: the quest for a tiny violation of the Pauli Exclusion Principle, *Implications for physics, cosmology and philosophy***

*is acknowledged*

# Spontaneous emission including nuclear protons

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When the emission of nuclear protons is also considered, the spontaneous emission rate is:

$$\frac{d\Gamma_k}{dk} = (N_P^2 + N_e) \frac{e^2 \lambda}{4\pi^2 a^2 m_N^2 k}$$

(the stochastic field is assumed to be coupled to the particle mass density)

provided that the emitted photon wavelength  $\lambda_{ph}$  satisfies the following conditions:

- 1)  $\lambda_{ph} > 10^{-15} \text{ m}$  (nuclear dimension)  $\rightarrow$  protons contribute coherently
- 2)  $\lambda_{ph} < (\text{electronic orbit radius})$   $\rightarrow$  electrons and protons emit independently  $\rightarrow$   
 $\rightarrow$  NO cancellation