The detection of Gravitational Waves: the dawn of a new era in the observation of the Universe

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Newton's law of Universal Gravitation

$$F = -G\frac{m \cdot m'}{r^2} \qquad V(r) = -G\frac{m \cdot m'}{r}$$

- In the solar system is almost always a good theory
- In the case of a body of mass m', rotating around a central body of mass m, with speed v', the Newton model is valid if: $|V(r)| << m'c^2 \Rightarrow r >> G\frac{m}{c^2}$ and $v' << c$

If m is the Sun:

$$m \equiv M_{\Theta} \approx 2\times10^{30}\,\mathrm{kg}$$
$$G \approx 6.67\times10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$$
$$c \approx 3\times10^{8}\,\mathrm{m\,s^{-1}}$$

⇒

$$r >> G\frac{m}{c^2} \approx 1.5\,\mathrm{km}$$

Almost always true

From Newton to Einstein

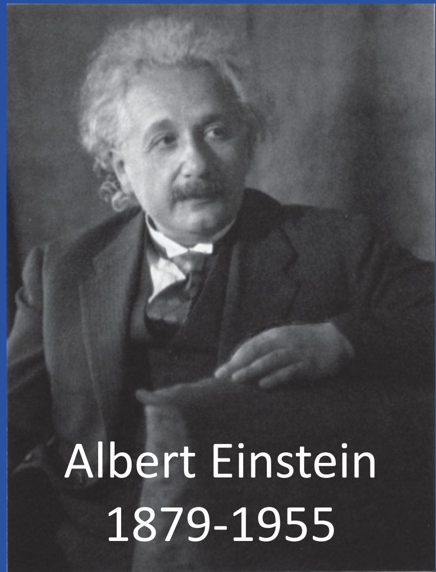
- The main critical point of the Newton's gravity is the "action at a distance"
  - Conflict with the electro-magnetism $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$
  - From Galilean to Lorentz transformations
  - Special theory of the relativity (1905)
    - 4 dimensions (flat) space-time
      - $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$
      - $\eta_{\mu\nu}$ Minkowski metric tensor
  - General Relativity (2015)

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

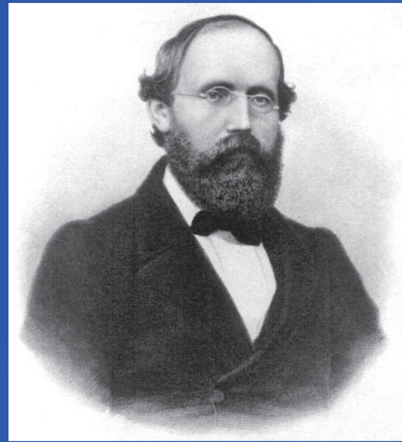
General Relativity

Theory of the gravitation

Carl Friedrich Gauss
1777-1855



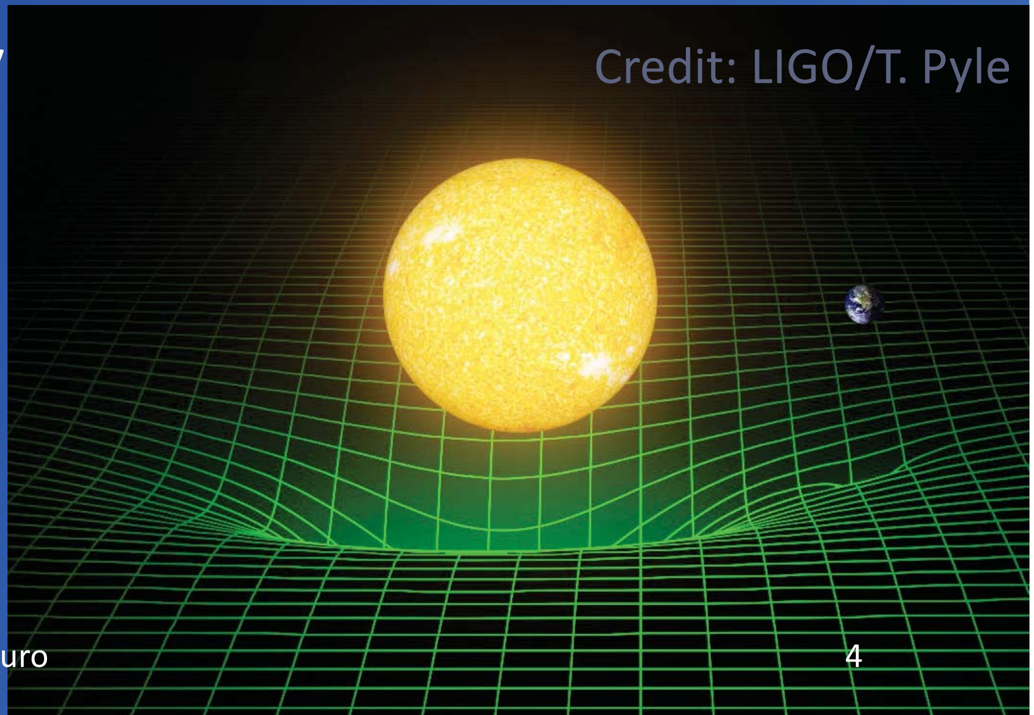
Albert Einstein
1879-1955



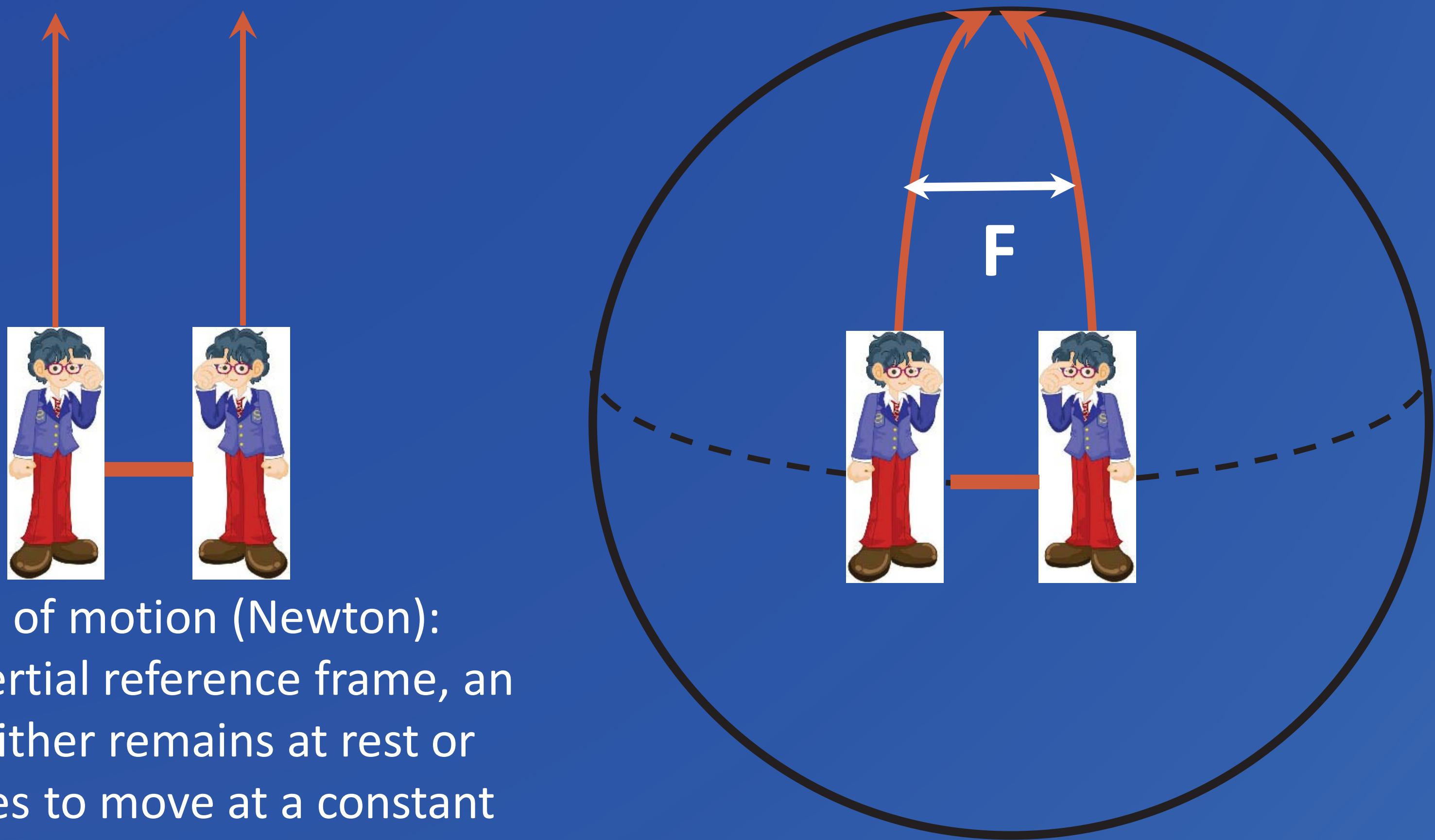
Bernhard Riemann
1826-1866

1915 – GR is “the” gravitation theory where space-time is no more flat, but curved by the presence of masses:

> “Mass tells space-time how to curve, and space-time tells mass how to move." (John Wheeler)

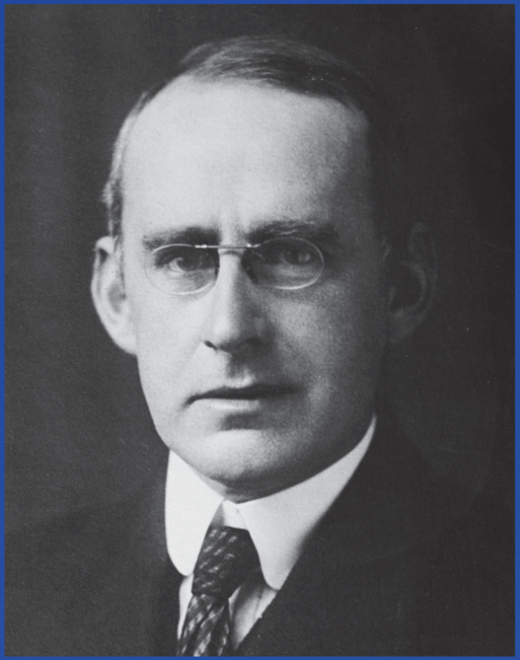


Curved geometry = Force?

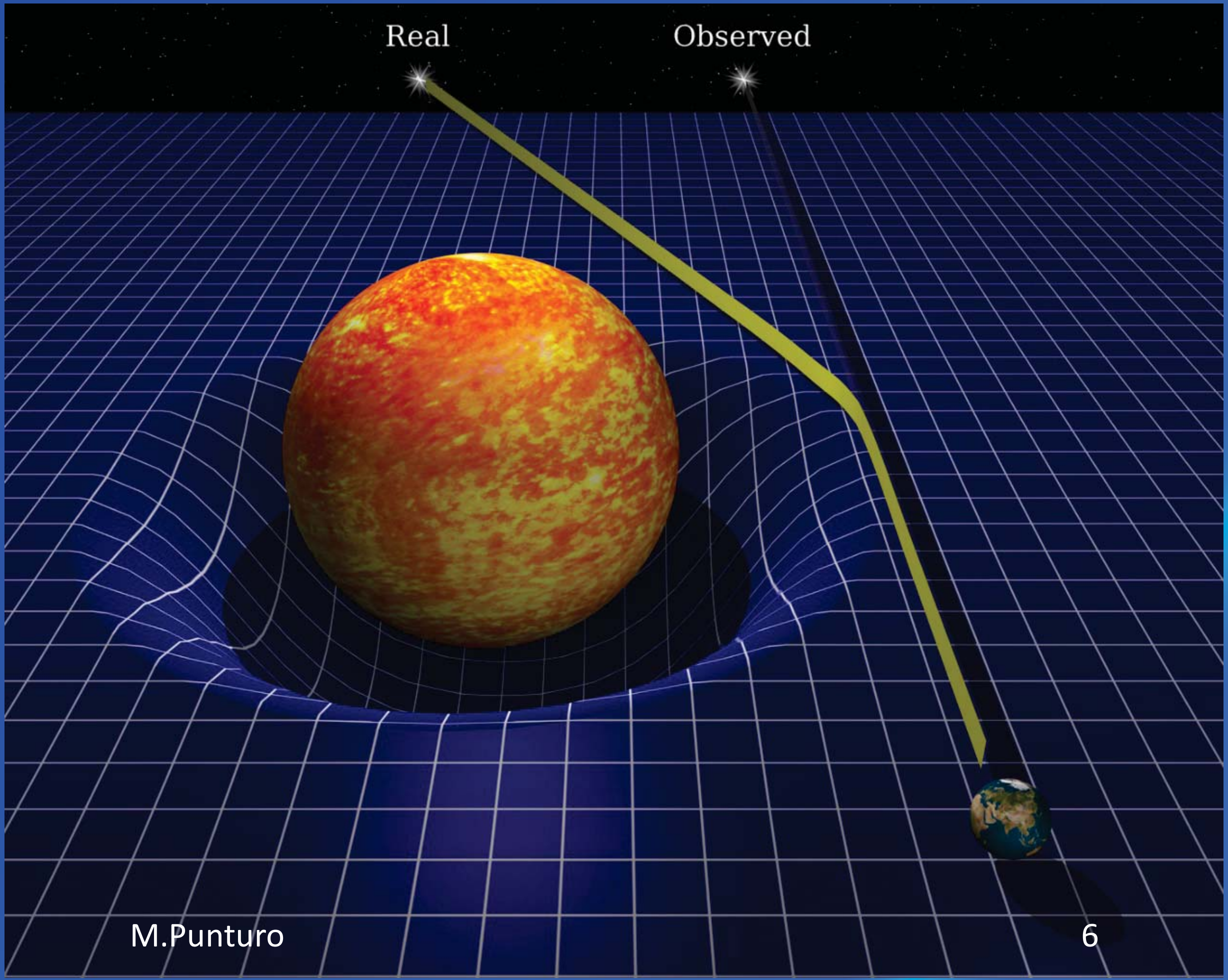


First law of motion (Newton):
In an inertial reference frame, an object either remains at rest or continues to move at a constant velocity (and direction), unless acted upon by a net force

1919 – Sun eclipse

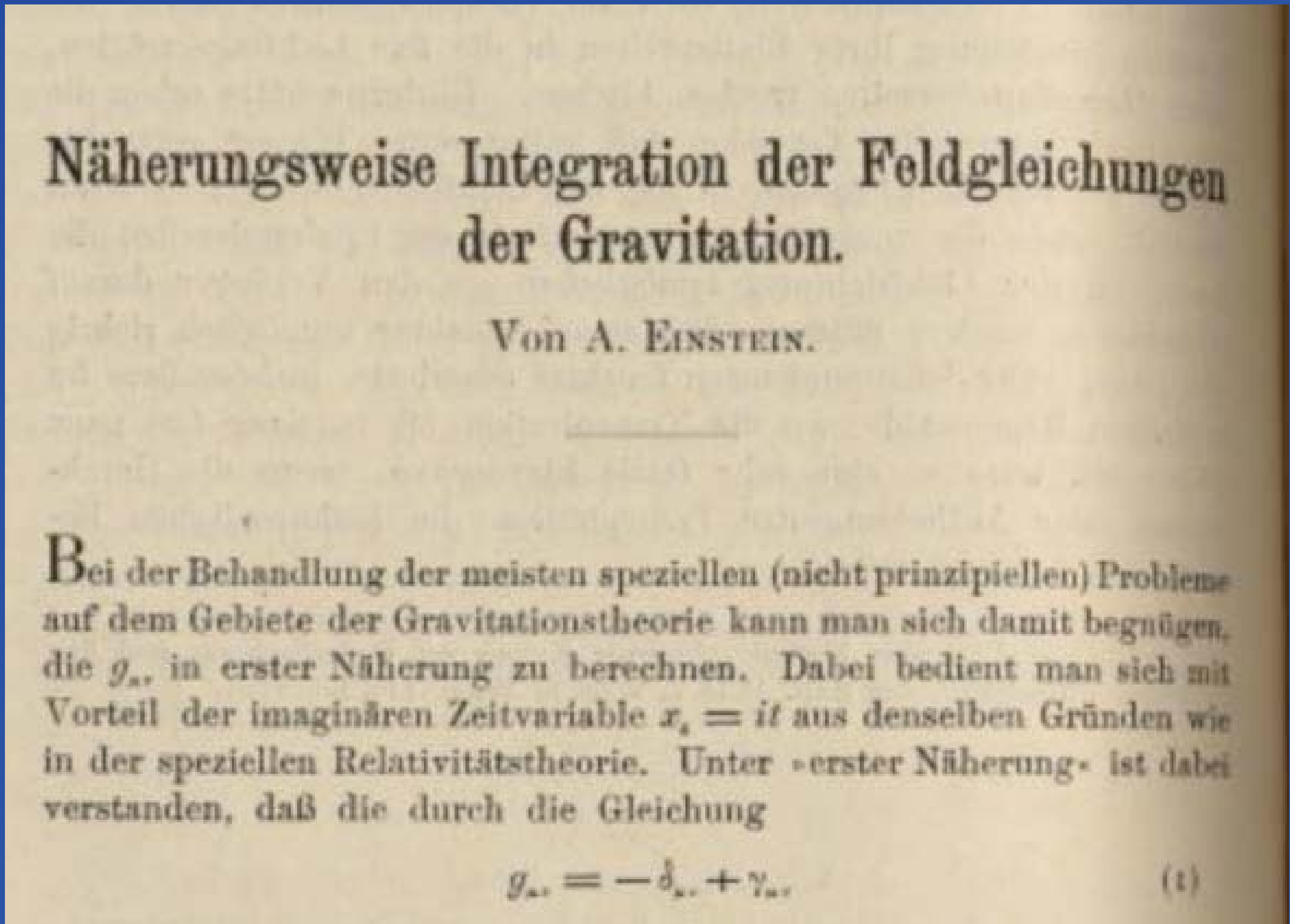


Arthur Eddington
1882-1944



Prediction of GW

- 1916: one year after the publication of the GR, Einstein published the key paper predicting the existence of GW

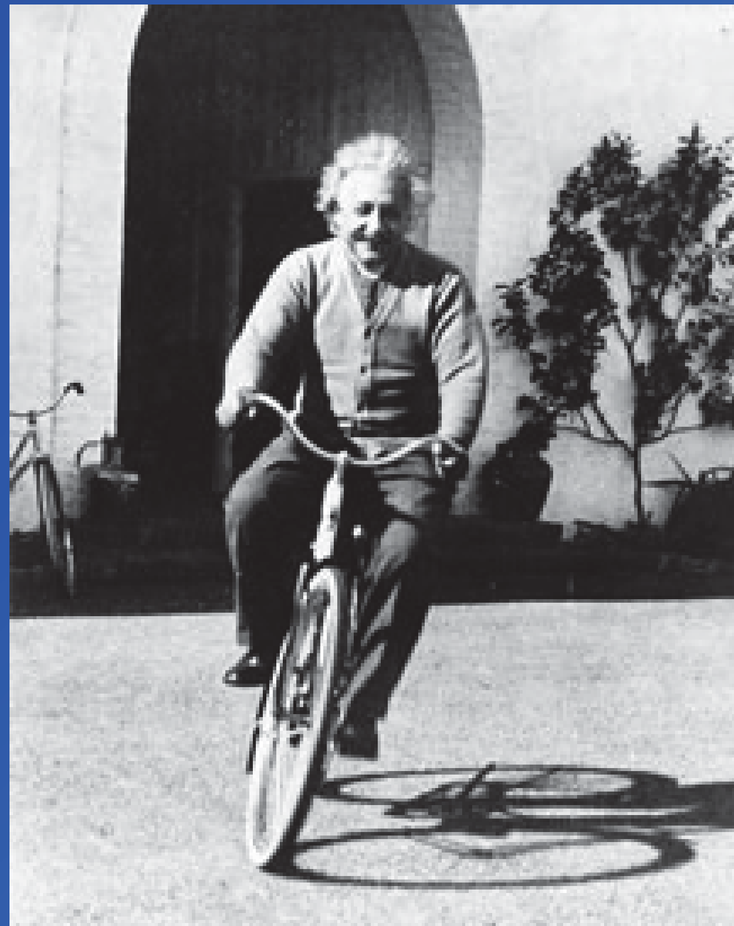


Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \quad (1)$$



Approximate integration of the field equations of gravitation

Einstein's field equation

- In the Einstein's General Relativity theory of the gravitation, the field equation can be written as

Curvature tensor → $$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ ← Energy-impulse Tensor

$T_{\mu\nu}$ represents the source of the deformation: "the mass"

$G_{\mu\nu}$ represents the effect of the deformation: "the geometry"

- Equations highly non linear, to be handled numerically, but they can be resolved in case of weak field (far from the masses that are generating the curvature)

GW in General Relativity

- In GR the distance can be generalised to a curved space time introducing the metric tensor $g_{\mu\nu}$:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$$

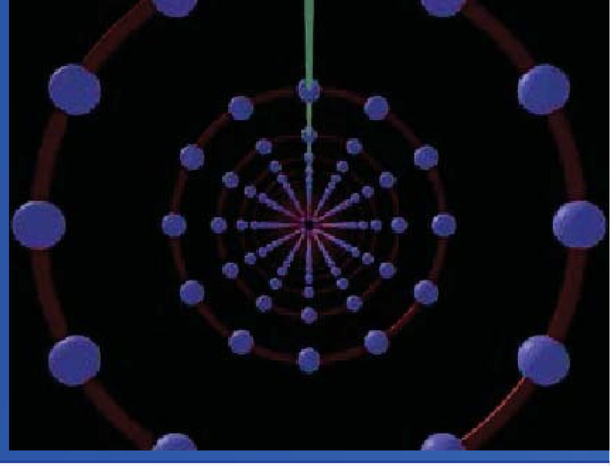
Related to the curvature of the space-time

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Far from the big masses Einstein field equation admits (linear approximation) wave solution (small perturbation of the background geometry)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \text{with} \left| h_{\mu\nu} \right| << 1 \quad \Rightarrow \quad \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0$$

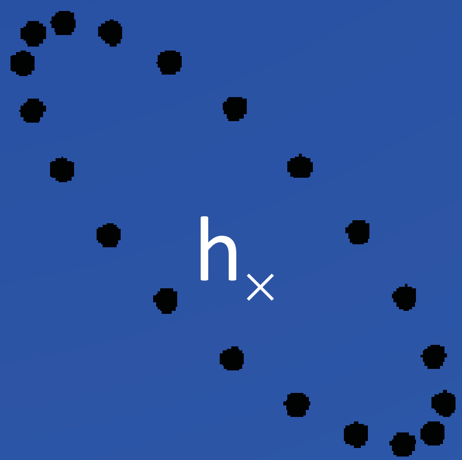
Gravitational Waves

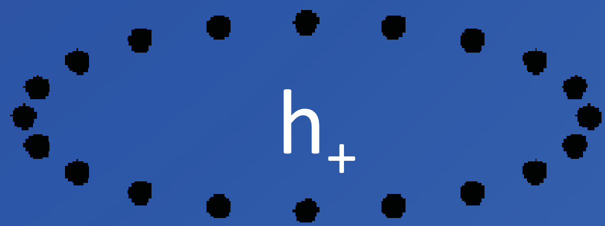


- Gravitational waves are a perturbation of the space-time geometry
- They present (in GR) two polarizations

$$\mathbf{h}(z,t) = e^{i(\omega t - kz)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- The effect of GWs on a mass distribution is the modulation of the reciprocal distance of the masses





$$\frac{\Delta l}{l} \approx h(t)$$

Let quantify the "deformation"

- Should we expect this?

- Coupling constant (fundamental interactions)

strong	*e.m.*	*weak*	*gravity*
0.1	1/137	10^{-5}	10^{-39}

GW emission: very energetic events but almost no interaction

- Or "space-time" rigidity (Naïf):

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \Rightarrow \frac{8\pi G}{c^4} = 4.8 \cdot 10^{42}\, N$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad \Rightarrow \quad Y_{Steel} \approx 2 \times 10^{11}\, Pa$$

- Very energetic phenomena in the Universe could cause only faint deformations of the space-time

GW emission

$$\partial_\lambda \partial^\lambda h^{\mu\nu}\left(t,\vec{x}\right) = -\kappa T^{\mu\nu}\left(t,\vec{x}\right)$$

Emission of GW

- Solving the field equation, considering a certain distribution of mass, it is possible to see that the amplitude of the emitted wave is given by

$$h_{\mu\nu} = \frac{2G}{c^4} \cdot \frac{1}{r} \ddot{Q}_{\mu\nu}$$

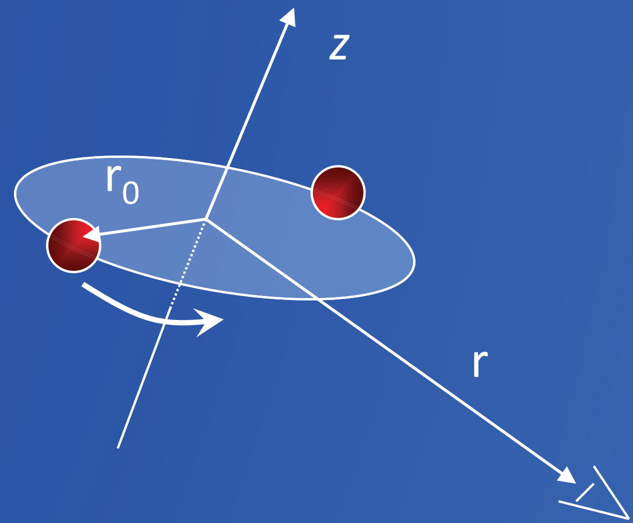
Where $Q_{\mu\nu}$ is the quadrupolar moment of the GW source

and r is the distance between the detector **and the** GW source

- Only sources with quadrupolar moment in the mass distribution can emit GW
- Why not monopole?
  - Mass/energy conservation law
- Why not dipole?
  - Mass has only one charge sign
  - Forbidden by the conservation of the linear momentum

Emission of GW

- Let consider a system of two point-like stars
- In the Newtonian approximation
  - No spin, circular orbits, no energy loss



$$2\frac{\Delta L}{L} \approx \begin{cases} h_{11} = -h_{22} = \frac{8G}{rc^4} M r_0^2 \Omega_{orb}^2 \cos 2\Omega_{\mathrm{orb}} t \\ h_{12} = h_{21} = -\frac{8G}{rc^4} M r_0^2 \Omega_{orb}^2 \sin 2\Omega_{\mathrm{orb}} t \end{cases}$$

$$Q^{kl} = 2Mr_0^2 \begin{pmatrix} \cos^2[\Omega_{orb}(t-r/c)] - \frac{1}{3} & \frac{1}{2}\sin[2\Omega_{orb}(t-r/c)] & 0 \\ \frac{1}{2}\sin[2\Omega_{orb}(t-r/c)] & \sin^2[\Omega_{orb}(t-r/c)] - \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{6} \end{pmatrix}$$

Useful to compute the order of magnitude of the expected space-time strain. For 2 NS at 10Mpc:

$$\left.\begin{aligned} M &= 1.4 M_{\Theta} = 1.4 \cdot \left(2 \cdot 10^{30} kg\right) \\ r &= 10\mathrm{MPc} = 10^7 \cdot \left(3 \cdot 10^{16} m\right) \\ r_0 &= 50 km \end{aligned}\right\} \Rightarrow \begin{cases} f_{GW} \approx 390 Hz \\ h \approx 2 \cdot 10^{-21} \end{cases}$$

Emission of GW

- In effect, if the binary system emits GW must radiate energy
- At the first order we obtain:

$$\tilde{M} = \mu^{\frac{3}{5}} \cdot M_T^{\frac{2}{3}}$$

$$M_T = M_1 + M_2$$

$$\mu = \frac{M_1 M_2}{M_T}$$

$$\begin{cases} h_{\oplus}(t,r,i) = 4\dfrac{1+\cos^2 i}{2}\dfrac{G\tilde{M}^{\frac{5}{3}}}{rc^4}\Omega^{\frac{2}{3}}(t)\cdot\cos[2\Omega(t)\cdot t] \\ h_{\otimes}(t,r,i) = 4(\cos i)\dfrac{G\tilde{M}^{\frac{5}{3}}}{rc^4}\Omega^{\frac{2}{3}}(t)\cdot\sin[2\Omega(t)\cdot t] \end{cases}$$

$$\Omega(t) = \frac{1}{8}\left(\frac{G\tilde{M}^{\frac{5}{3}}}{5c^5}\right)^{-\frac{3}{8}}(t_c - t)^{-\frac{3}{8}}$$

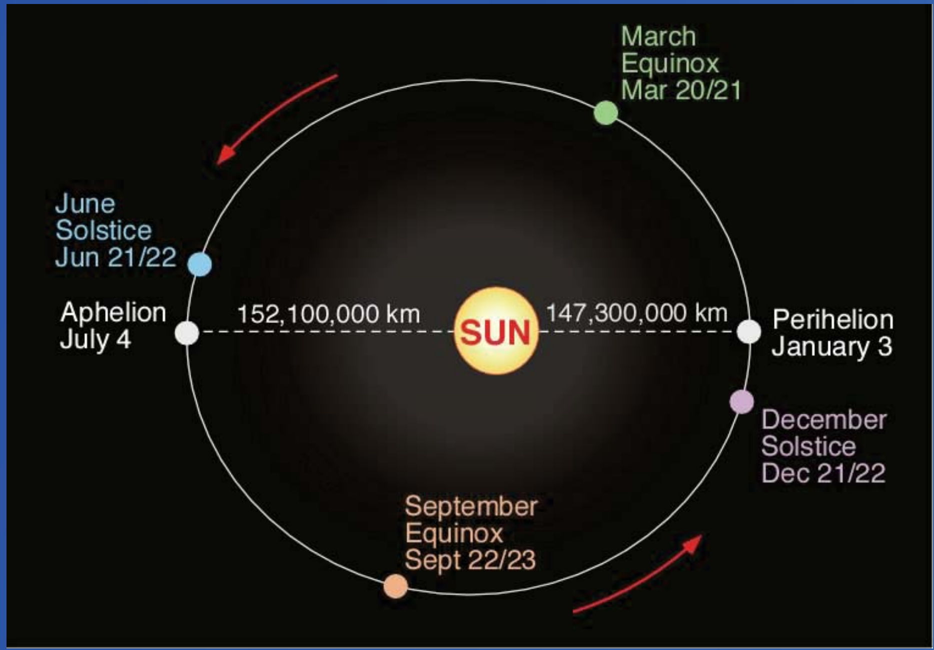
- More refined evaluation is obtained by increasing the post-Newtonian order of the series (on v/c)
- But the physics is clear:
  - BNS are standard candles (sirens):
    - The amplitude of the signal depends only by the distance
    - The (chirp) mass can be determined by the frequency sweep

Emission of GW

- Let suppose to have again a system of 2 coalescing neutron stars, located in the Virgo cluster (r~10Mpc):

$$h \approx 10^{-21} - 10^{-22}$$

$$\left.\begin{aligned} \delta L &\approx \frac{h}{2} \cdot L_0 \\ L_0 &\approx 10^{11} m \end{aligned}\right\} \Rightarrow \delta L \approx 10^{-10} m$$



Size of one atom in the Earth-Sun distance

$$\left.\begin{aligned} \delta L &\approx \frac{h}{2} \cdot L_0 \\ L_0 &\approx 10^{3} m \end{aligned}\right\} \Rightarrow \delta L \approx 10^{-18} - 10^{-19} m$$

Extremely challenging for any kind of detectors

Energy emitted through GW

$$-\frac{dE}{dt} = \frac{G}{45c^5} \dddot{Q}^{kl} \dddot{Q}^{kl}$$

$$\frac{G}{45c^5} \approx 10^{-55} \mathrm{m}^{-2} \mathrm{s}^{3} \mathrm{kg}^{-1}$$

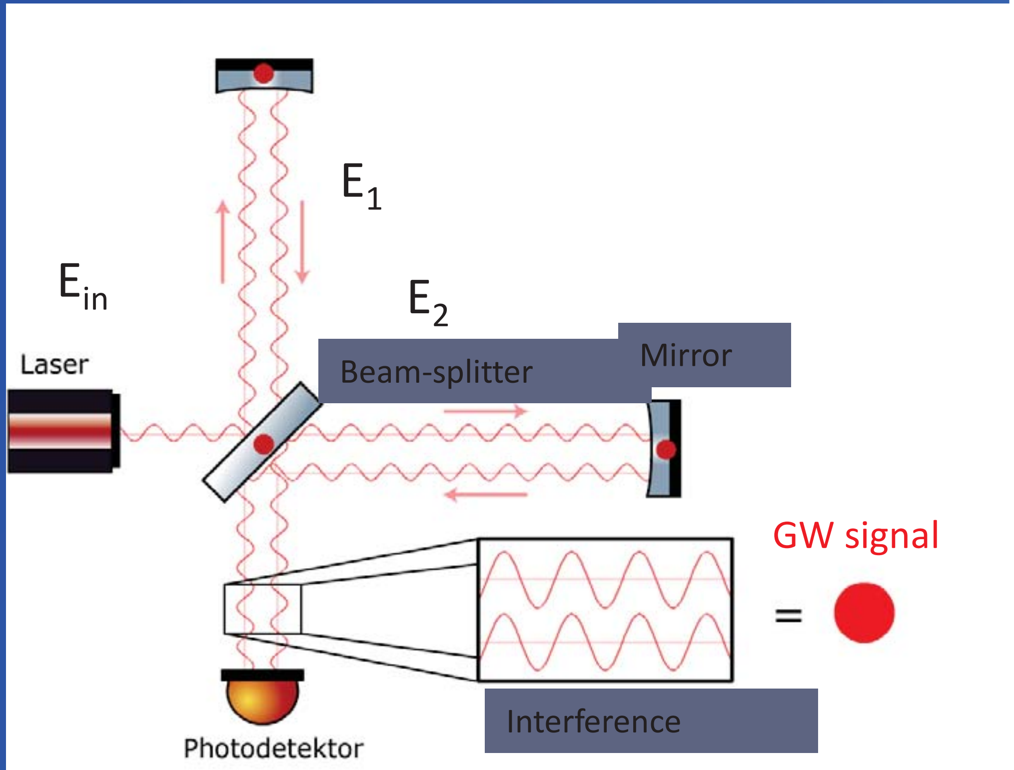
- It is impossible to think to realise a synthetic source of GW

Detection principles

- The quadrupolar nature of the GW makes the Michelson interferometer a “natural” GW detector

$$\delta L \approx \frac{h}{2} \cdot L_0$$

$10^2 \leq L_0 \leq 10^4$ m in terrestrial detectors



$$E_{out}(t) = E_1(t) + E_2(t) =$$

$$= \frac{E_{in}}{2}\{\cos[\omega t - kL_1(t)] + \cos[\omega t - kL_2(t)]\} =$$

Interference term

$$= E_{in} \cdot \cos\left[k\frac{L_2(t) - L_1(t)}{2}\right] \cdot \cos\left[\omega t - k\frac{L_2(t) + L_1(t)}{2}\right]$$

$$P_{out}(t) = \frac{P_{in}(t)}{2}\left\{1 + \cos\left[2\frac{2\pi n(t)}{\lambda_{laser}(t)} \cdot L \cdot h(t)\right]\right\}$$

ΔP Power fluctuations

Δn Index fluctuations

} Noise sources

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$$\phi_{GW} = \frac{4\pi}{\lambda} L_0 \cdot h$$

GW signal

18

Power fluctuation

- Power fluctuation? Shot noise!

$$\frac{\sigma_N}{N} = \frac{1}{\sqrt{N}} \quad \Rightarrow \quad \tilde{\phi}_{shot} = \sqrt{\frac{2\hbar\omega}{P}}$$

$$\tilde{h}_{shot} = \frac{\lambda}{4\pi}\frac{1}{L}\tilde{\phi}_{shot} = \frac{1}{2L}\sqrt{\frac{\hbar\lambda c}{\pi P}}$$

To allow the GW detection, the shot noise should be smaller than the expected signal ($h \sim 10^{-21}$-10^{-22})

It doesn't work!

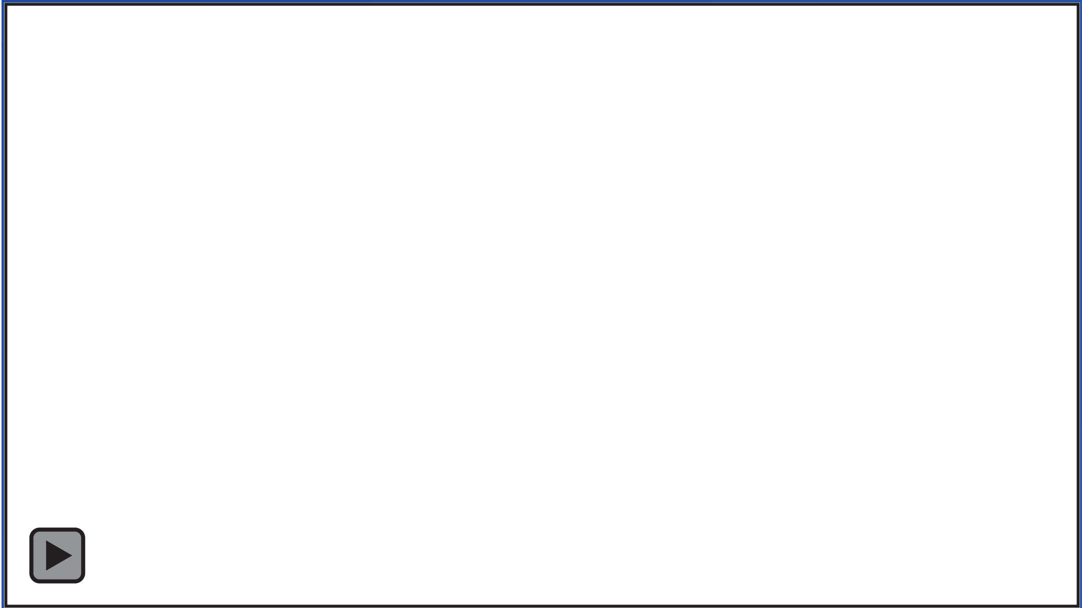
Try the intuitive numbers: $P \leq 100W$, $L \sim 10^3 m$: → $h_{shot} \sim 10^{-20}$

It works!

If we try $P \sim 1kW$, $L \sim 10^5 m$: → $h_{shot} \sim 10^{-23}$

Fabry-Perot cavities

- We need a “trick” to build ~100km long detectors on the Earth



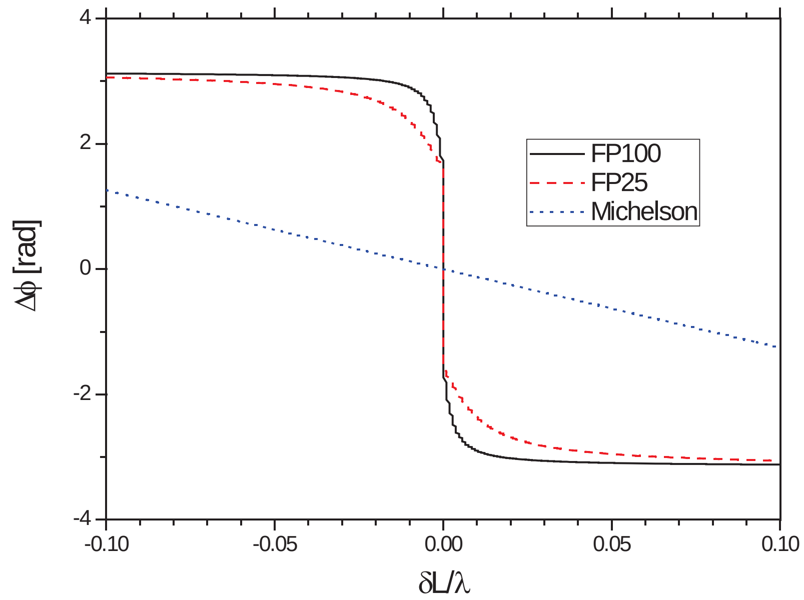
Effective length:

$$L' = L_0 \times \frac{2F}{\pi}$$

$$I(t) = I_0 \cdot e^{-\frac{t}{\tau_s}}, \quad \tau_s \approx \tau_{rt} \frac{F}{2\pi}, \quad F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

$$\frac{\partial \phi_{FP}}{\partial \Delta L_{opt}} = \frac{8F}{\lambda} = \frac{2F}{\pi} \cdot \frac{\partial \phi_{Mich}}{\partial \Delta L_{opt}}$$

- **Fabry-Perot cavities:** amplify the length-to-phase transduction
- **Higher finesse** $\rightarrow$ higher *df/dL*
- *Drawback: works only at resonance*



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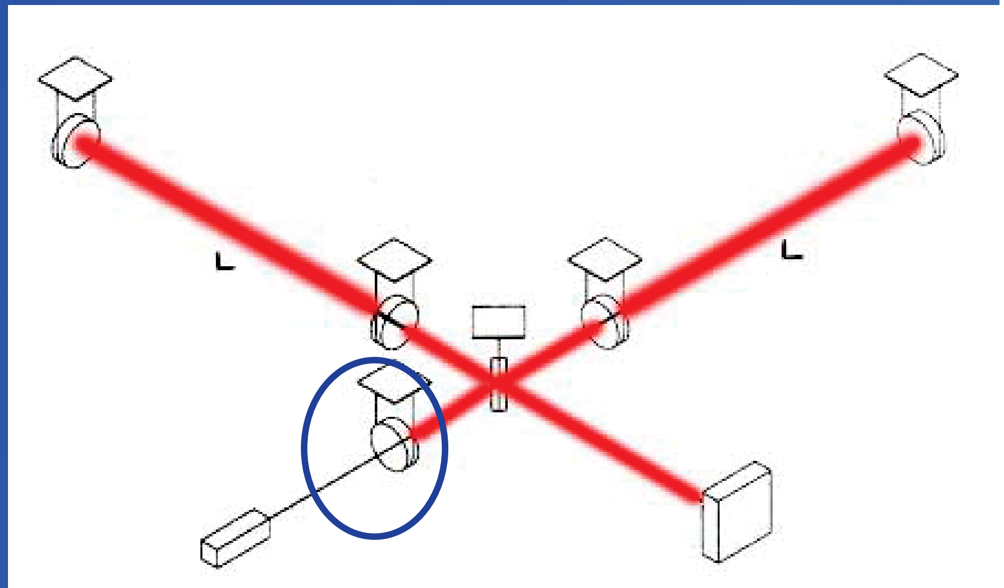
Power recycling

- We need a "trick" to realize a 1000W CW laser

- GW interferometers work near the dark fringe:
  - Huge power wasted at the input port:
    - Recycle it



P_{eff}= *Recycling factor* $\cdot P_{in}$

$\Rightarrow$ 20 W $\rightarrow$ ~1 kW

Shot noise reduced by a factor ~7

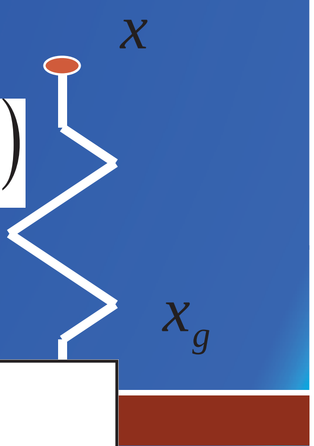
One more cavity to be controlled

Seismic noise

- The soil is continuously vibrating because of:
  - Natural seismic activity
  - Volcans
  - Wind, ocean waves
  - Human and industrial activities

$$x_{seism}(f) \approx \begin{cases} 10^{-7}\, m/\sqrt{Hz} & f < 0.1 \div 0.5 Hz \\ \dfrac{10^{-7}}{f^2}\, m/\sqrt{Hz} & f > 0.1 \div 0.5 Hz \end{cases}$$

- We need to filter it: $m\ddot{x} = -k\left(x - x_g\right) \quad \Rightarrow -\omega^2 x = -\omega_0^2\left(x - x_g\right)$



$$\frac{x(\omega)}{x_g(\omega)} = \frac{\omega_0^2}{\omega_0^2 - \omega^2} = \frac{f_0^2}{f_0^2 - f^2}$$

$$\omega_0 = \sqrt{\frac{g}{l}}$$

Under vacuum and giant!


VIRGO
INFN
Istituto Nazionale
di Fisica Nucleare

Terrestrial Detectors

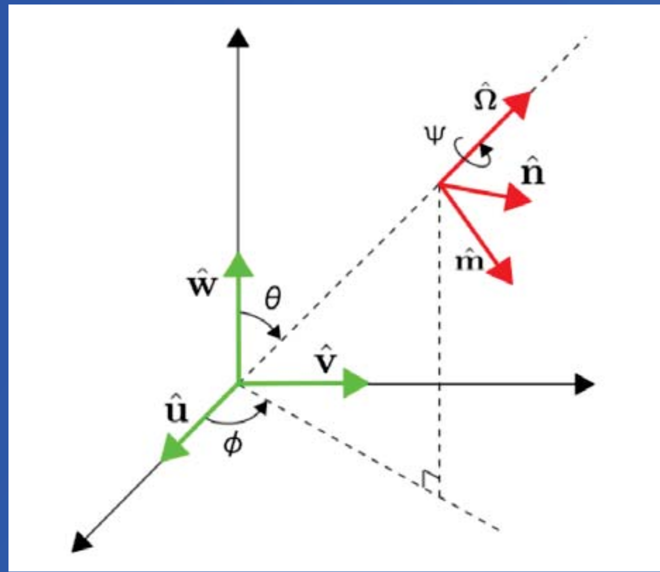
Advanced detectors 2015-2025

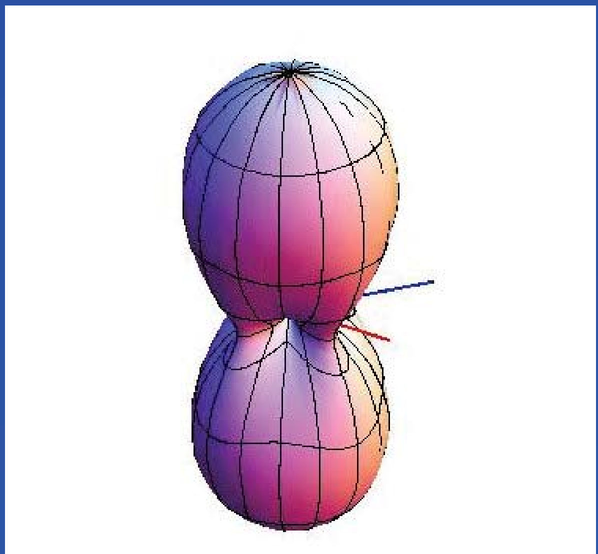
Antenna pattern

- An L shaped detector sees a linear combination of the two polarisations

$$h(t) = \sum_A h_A(t - \hat{\Omega}\cdot x) F^A(\hat{\Omega}, \psi).$$

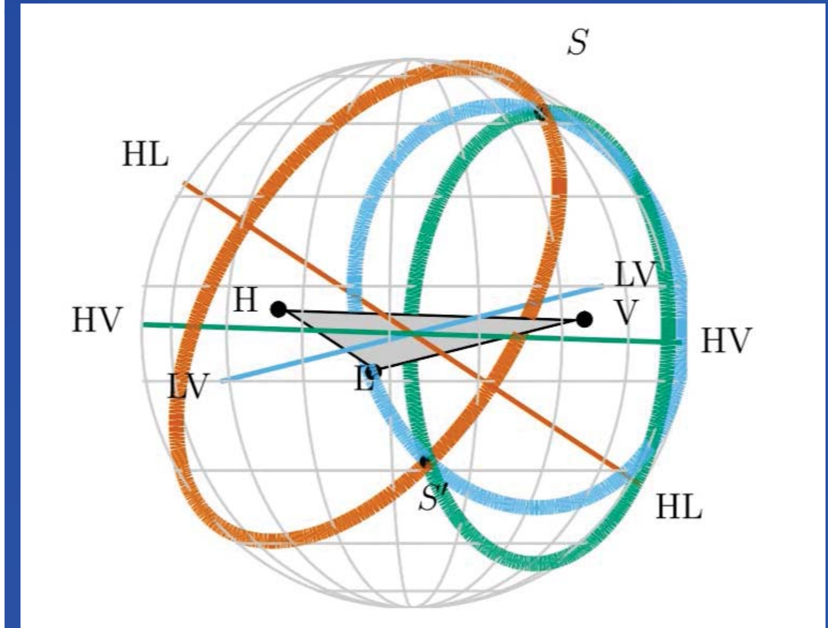
$$\begin{aligned} F^+(\theta,\phi,\psi) &= \frac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi, \\ F^\times(\theta,\phi,\psi) &= -\frac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi - \cos\theta\sin 2\phi\cos 2\psi, \end{aligned}$$



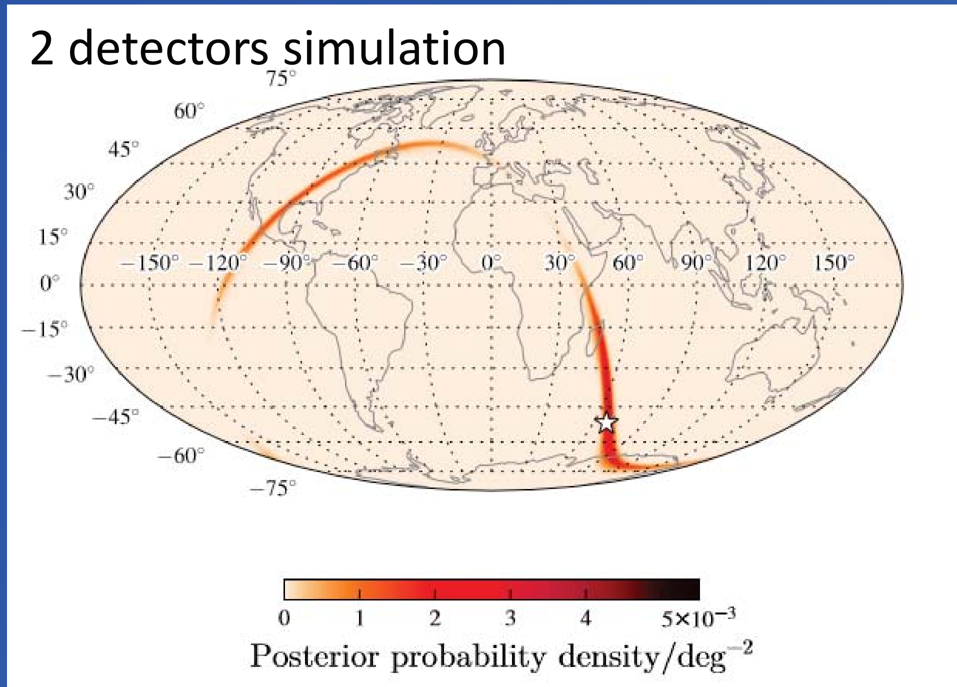


Sky localization

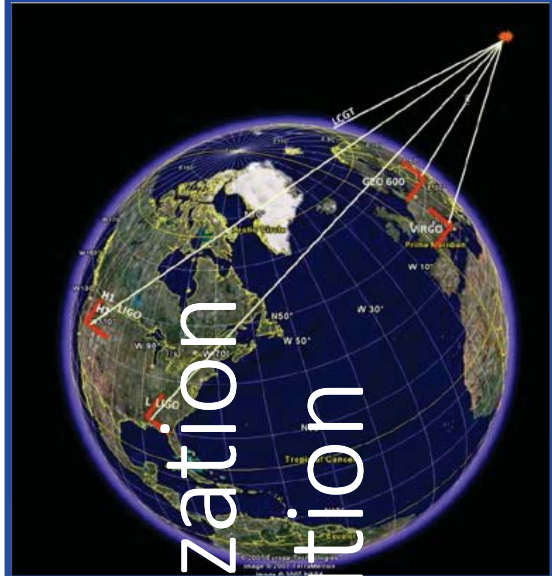
- To localise the GW source we need a network (≥3) of detectors (5 parameters→2Δt, 3 Amplitudes) and a good SNR:

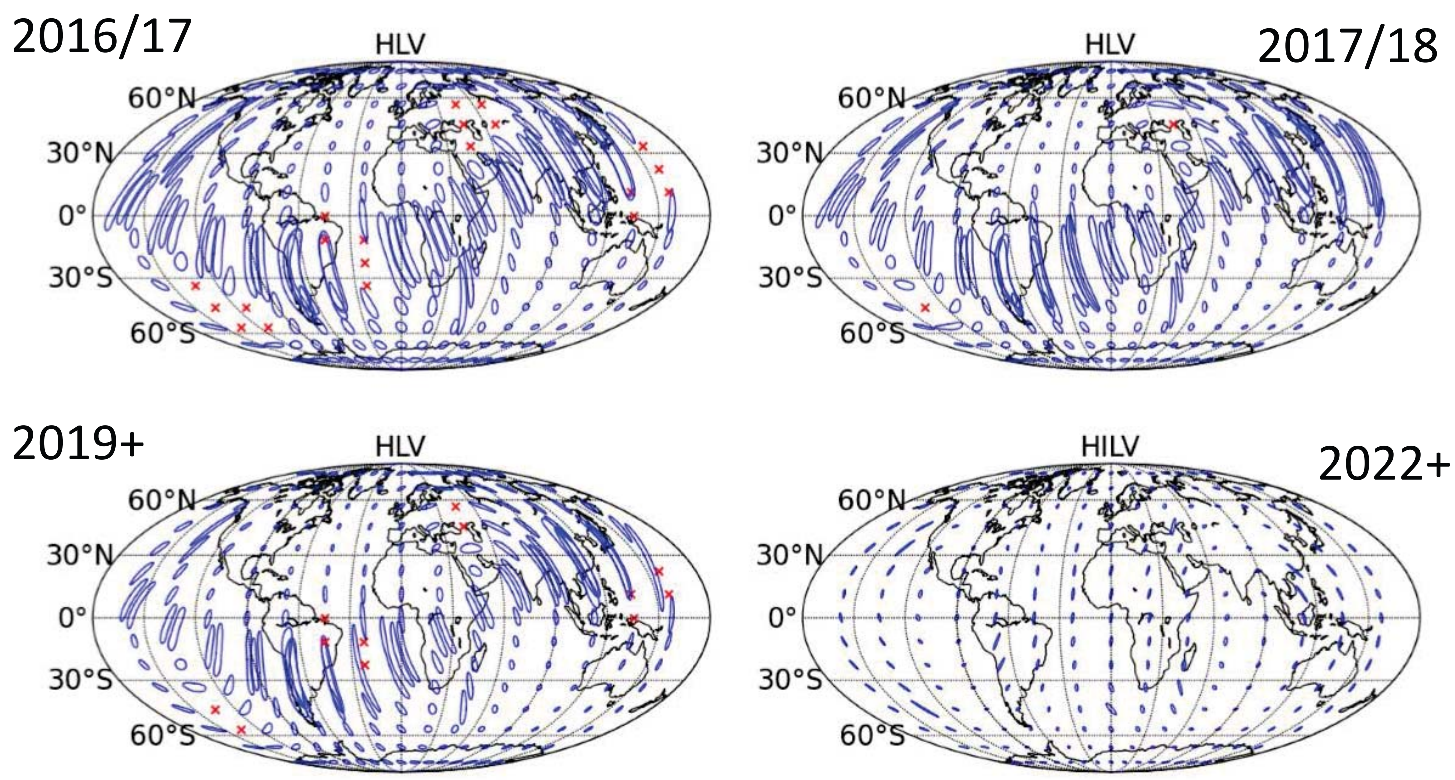


- 2 detectors: a circles
  - Improvement by phase relationship and amplitude measurement
- 3 detectors: 2 points



Sky localization evolution





Epoch			2015 – 2016	2016 – 2017	2017 – 2018	2019+	2022+ (India)
Estimated run duration			4 months	6 months	9 months	(per year)	(per year)
Burst range/Mpc		LIGO	40 – 60	60 – 75	75 – 90	105	105
		Virgo	—	20 – 40	40 – 50	40 – 80	80
BNS range/Mpc		LIGO	40 – 80	80 – 120	120 – 170	200	200
		Virgo	—	20 – 60	60 – 85	65 – 115	130
Estimated BNS detections			0.0005 – 4	0.006 – 20	0.04 – 100	0.2 – 200	0.4 – 400
90% CR	% within	5 deg^2	< 1	2	*> 1 – 2*	*> 3 – 8*	*> 20*
		20 deg^2	< 1	14	*> 10*	*> 8 – 30*	*> 50*
		median/deg^2	480	230	—	—	—
searched area	% within	5 deg^2	6	20	—	—	—
		20 deg^2	16	44	—	—	—
		median/deg^2	88	29	—	—	—

GW detection

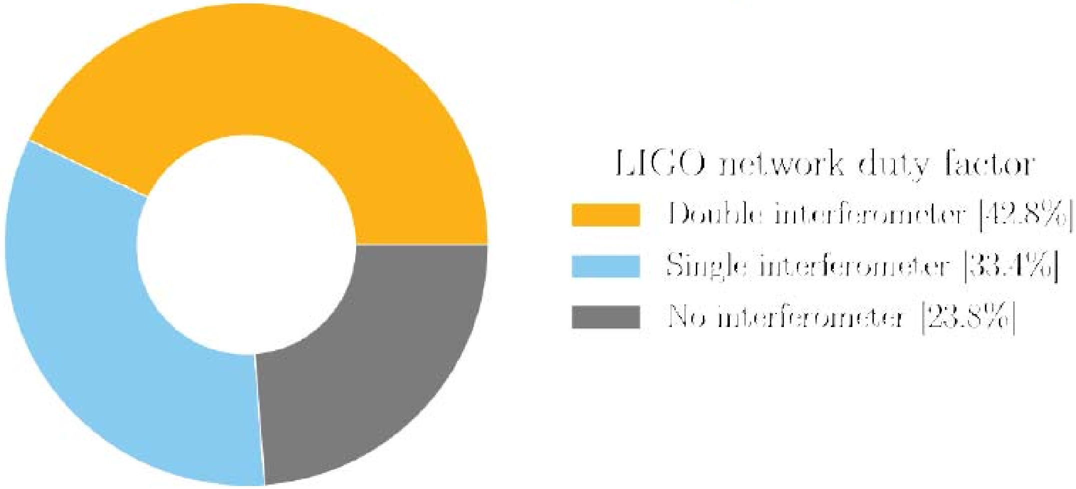

LIGO Hanford (Washington)

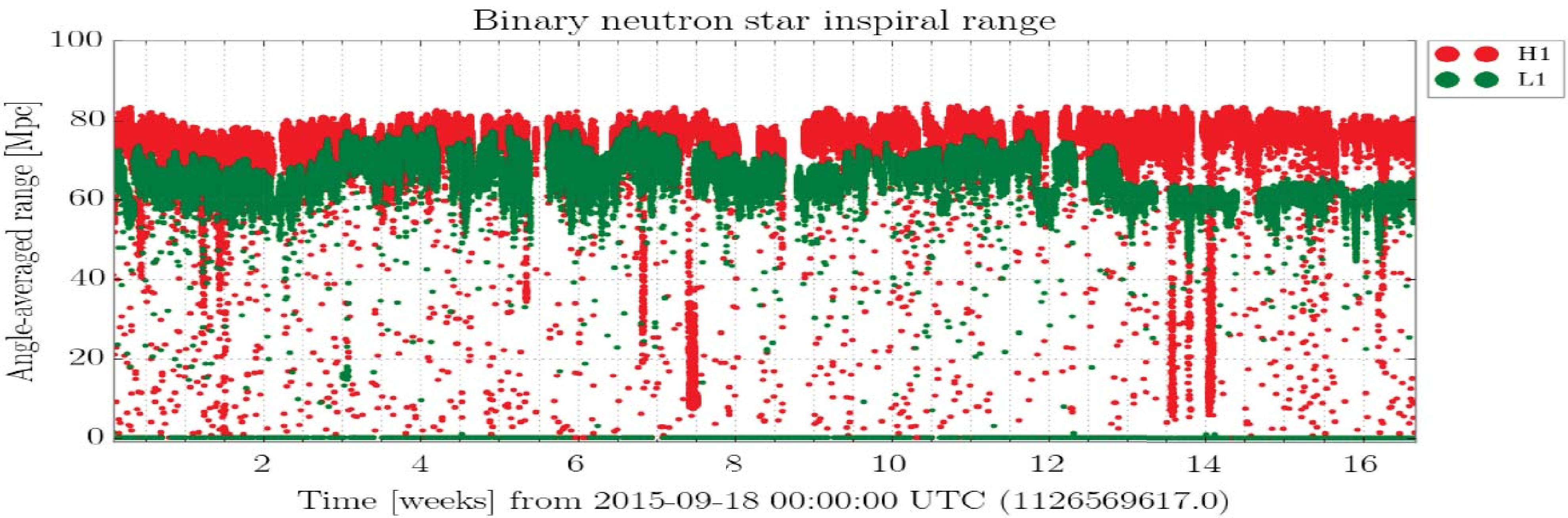

HANFORD
MIT
3030 km
(10ms)
CIT
LIVINGSTON


LIGO Livingston (Luisiana)

O1 in a nutshell

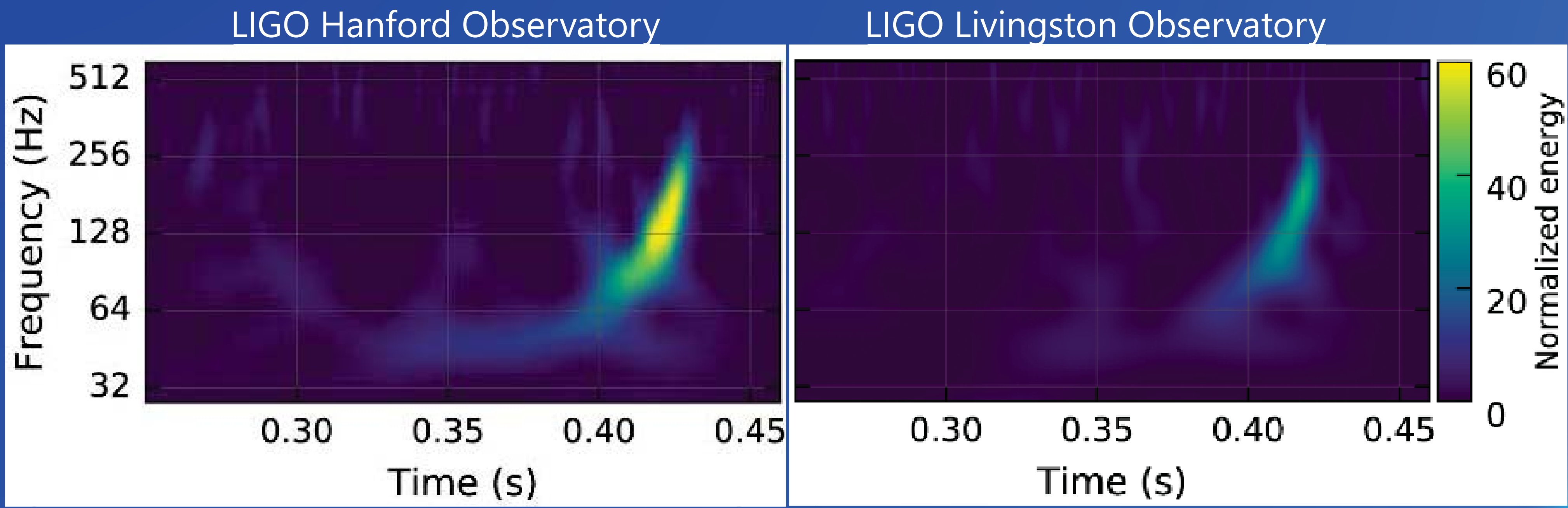
- Official dates : 18th of September 2015 to 12th of January 2016
- Dates with very good confidence : from the 12th of September to the 15th of January 2016
- H1 livetime : 62.6 %
- L1 livetime : 55.3 %





14 Settembre, 2015 – 11:50:45 CET

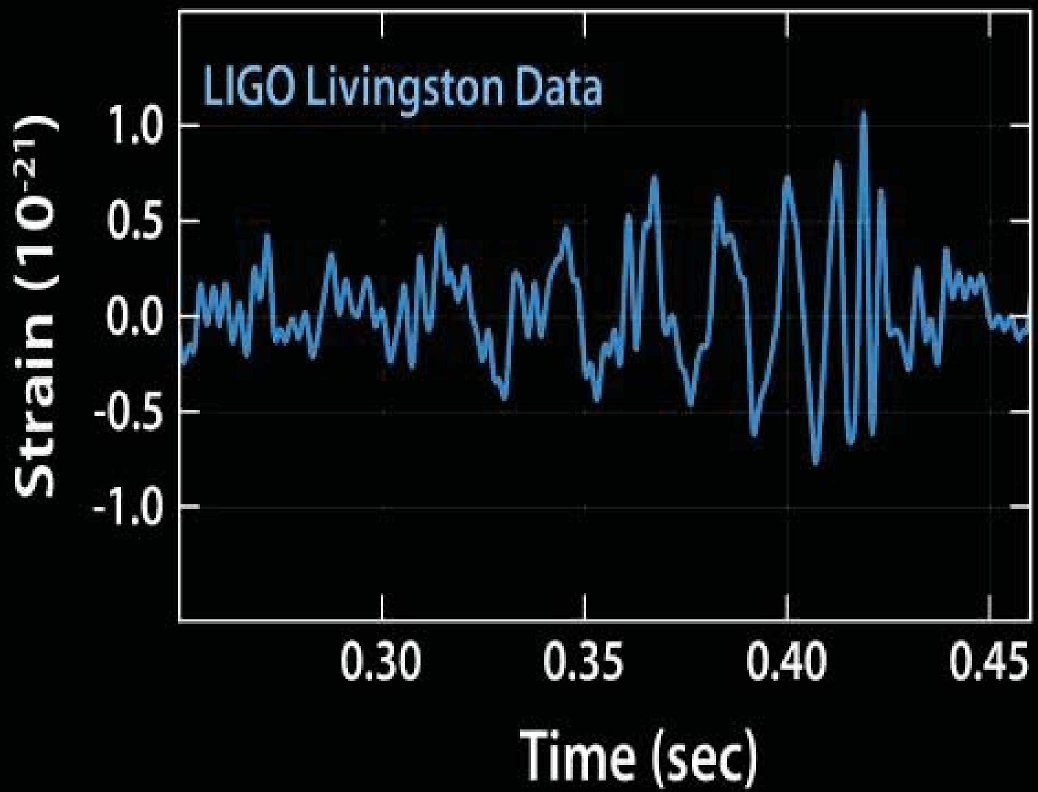
September 14, 2015 – 11:50:45 CEST



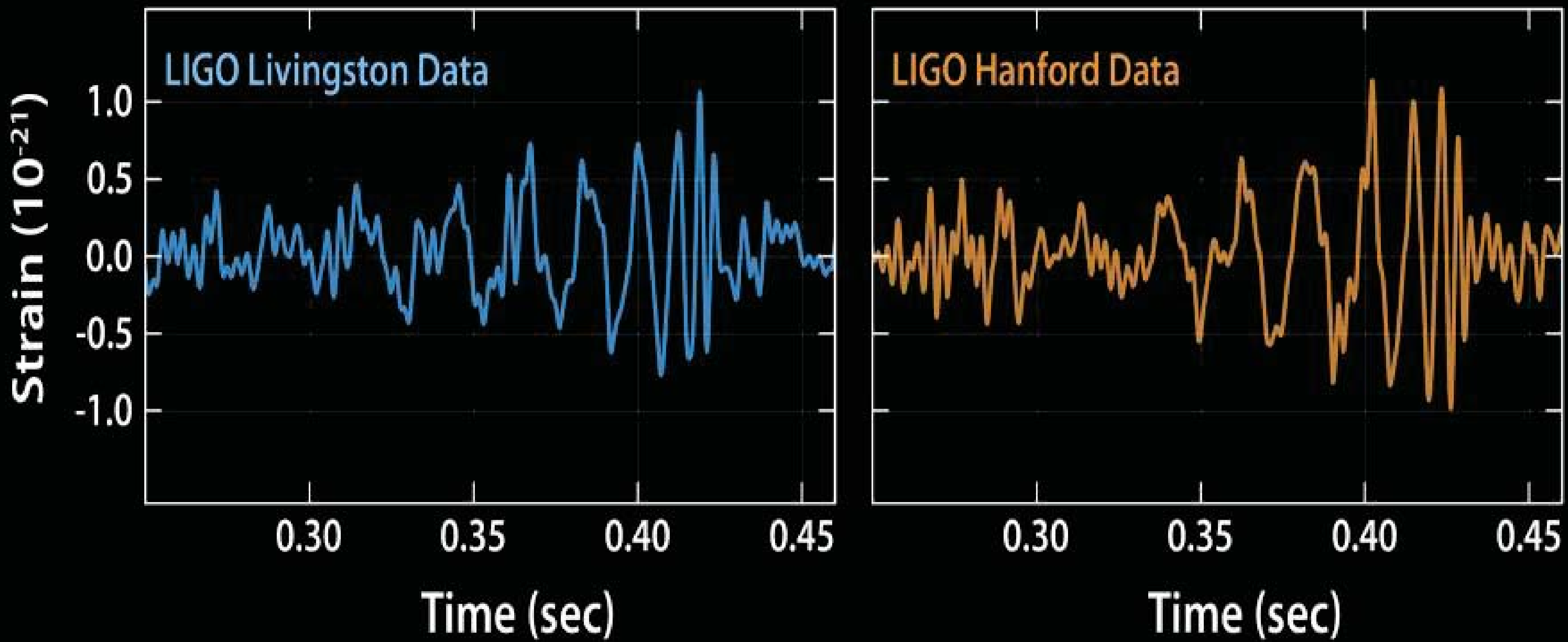
Initial detection made by a low latency searches for generic GW transients: **Coherent WaveBurst**

Reported within 3 minutes after data acquisition

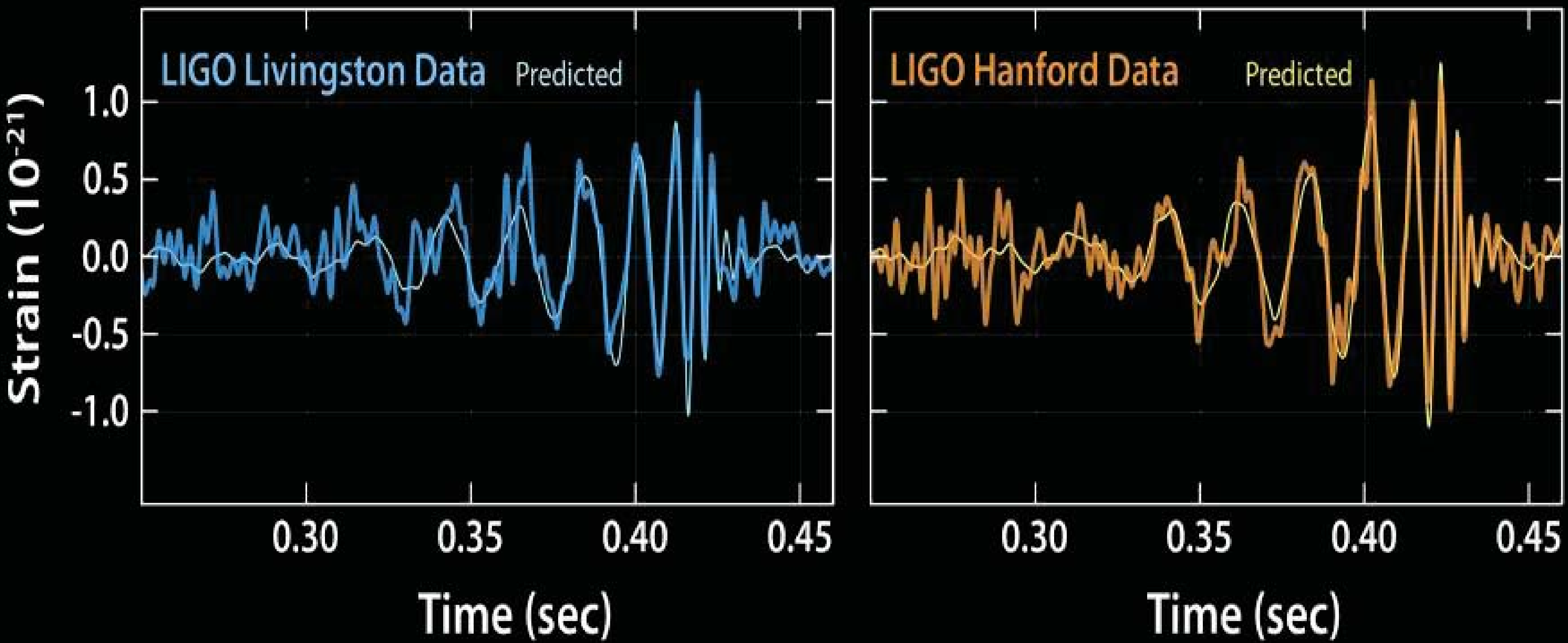
Large signal, well above background noise (35-350Hz band-pass filter)



We saw a very similar waveform in the LIGO Hanford detector, only 7 milliseconds later.
We see a few cycles increasing in amplitude and frequency, and the signal settles down back to the noise– all in a small fraction of a second.
The detected distortion of space time was very small, a part in 10^21. The distance changes we measured in LIGO's arms are 4 parts in a thousandth of a proton diameter: tiny!

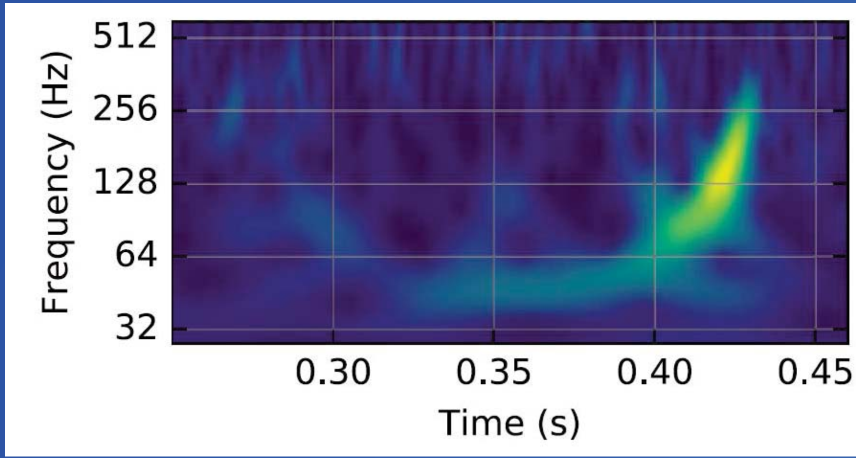


Numerical relativity waveform prediction for a system with parameters consistent with those recovered from GW150914



Binary system: BBH

- In 0.2s the GW signal sweeps, in 8 cycles, from 35 to 150Hz
- Ok it is a binary system:
  - BNS, NS-BH or BBH?



- The coalescing phase can be used to determine the chirp mass:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{M^{1/5}} \simeq \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

$$\mathcal{M} \simeq 30 M_\odot$$

$$M = m_1 + m_2 \text{ is } \gtrsim 70 M_\odot$$

Schwarzschild radii of the resulting body: $2GM/c^2 \gtrsim 210\ \text{km}$

BNS, NS-BH or BBH?

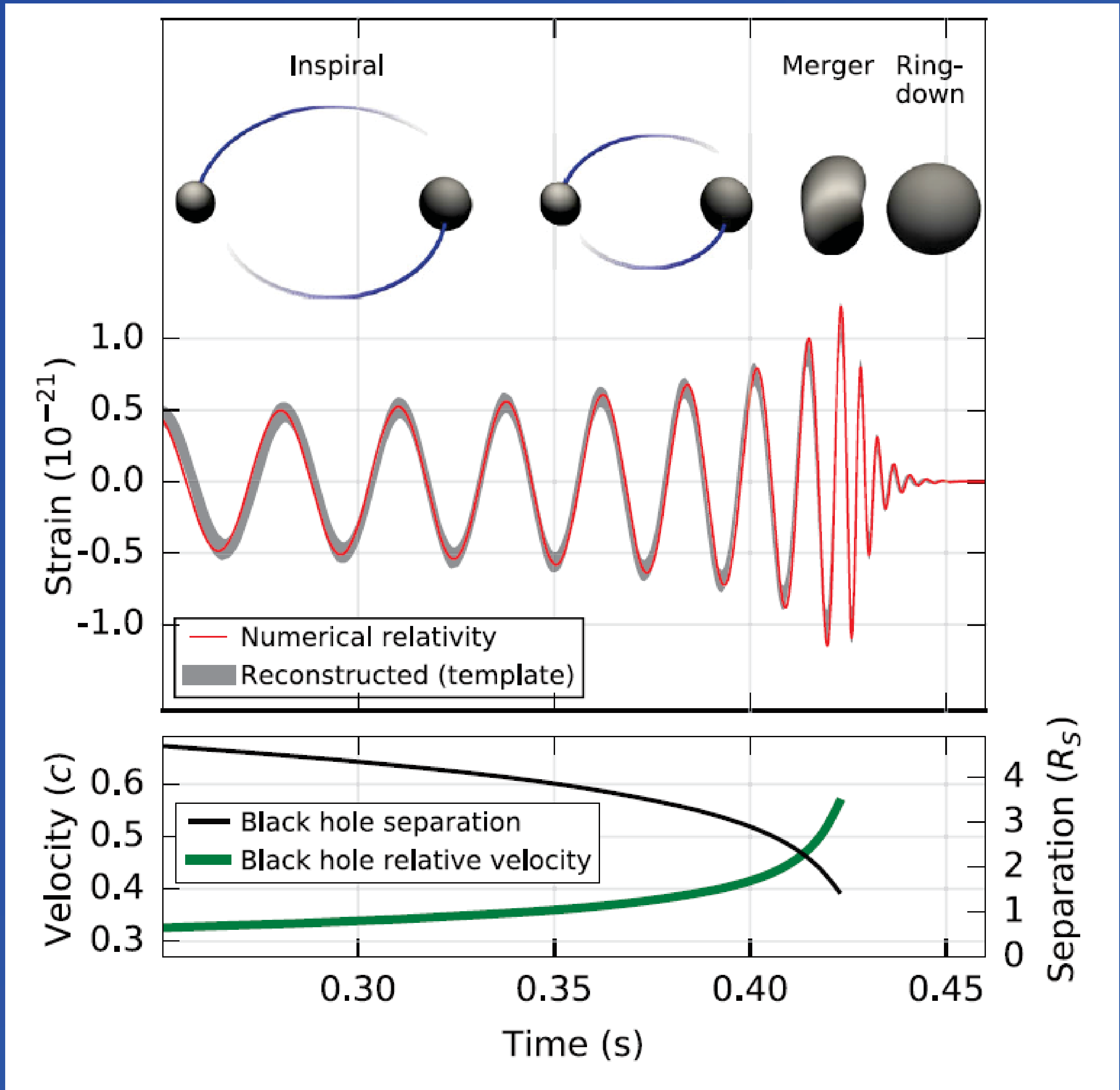
- They should be very close and compact objects because the Keplerian separation should be
  - $R = \left[\frac{GM}{(2\pi \times 75Hz)^2}\right]^{1/3} \sim 350km$
  - Two NS: not enough total mass
  - NS-BH, the BH should be very massive → impossible to reach 75Hz
  - BBH !
- The mass ratio (and the spins) of the two coalescing BH is determined through the next orders of the PN expansion in the templates

Source Parameters for GW150914

- Use numerical simulations fits of black hole merger to determine parameters, we determine total energy radiated in gravitational waves is $3.0 \pm 0.5\ M_o\, c^2$
- The system reached a peak ~3.6 $\times 10^{56}$ ergs, and the spin of the final black hole < 0.7 (not maximal spin)

Primary black hole mass	$36^{+5}_{-4}\, M_\odot$
Secondary black hole mass	$29^{+4}_{-4}\, M_\odot$
Final black hole mass	$62^{+4}_{-4}\, M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift, z	$0.09^{+0.03}_{-0.04}$

Ultra-Relativistic collision

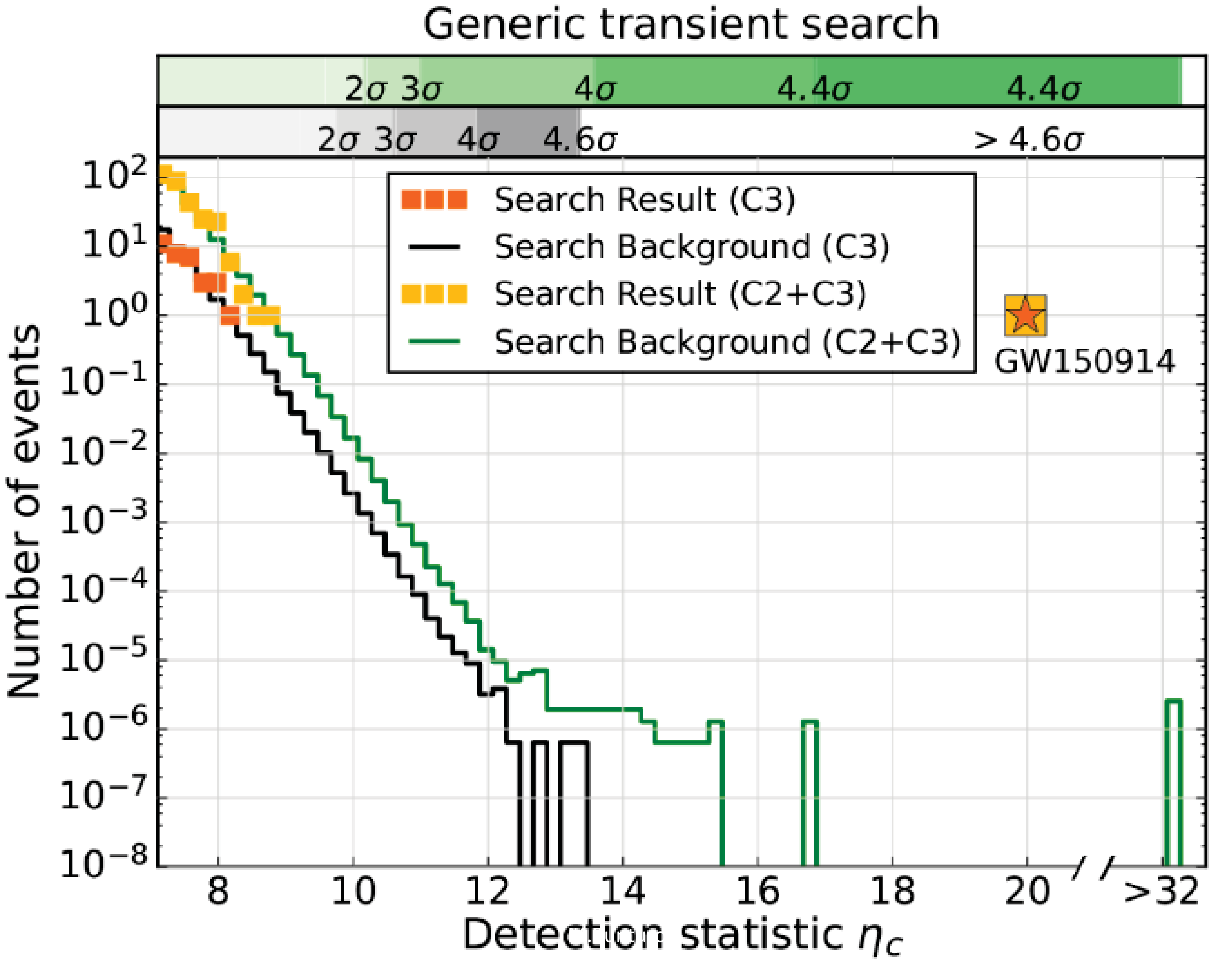


$$\frac{v}{c} = \left(\frac{GM\pi f}{c^3} \right)^{\frac{1}{3}}$$

$$R_S = \frac{2GM}{c^2}$$

Two analysis methods

- Generic transient search: cWB
  - Based on a search of excess of energy in time-frequency for signals up to 1kHz and duration up to few seconds
  - Running online and detecting the events few minutes after it occurred
  - Reconstruct waveform in both detectors using multi-detector maximum likelihood method
  - Detection statistics (~SNR): $\eta_c = \sqrt{\frac{2E_c}{1+E_n/E_c}}$
    - E_c = dimensionless coherent signal energy by cross correlating the two reconstructed waveforms and E_n is residual noise energy
  - Restricting to events with f increasing with time, GW150914 is the strongest event in the search with $\eta_c = 20$
  - Yields false alarm rate < 1 per 22500 years
  - Probability of background event during data run $< 2\times10^{-6}$ or $> 4.6\,\sigma$.



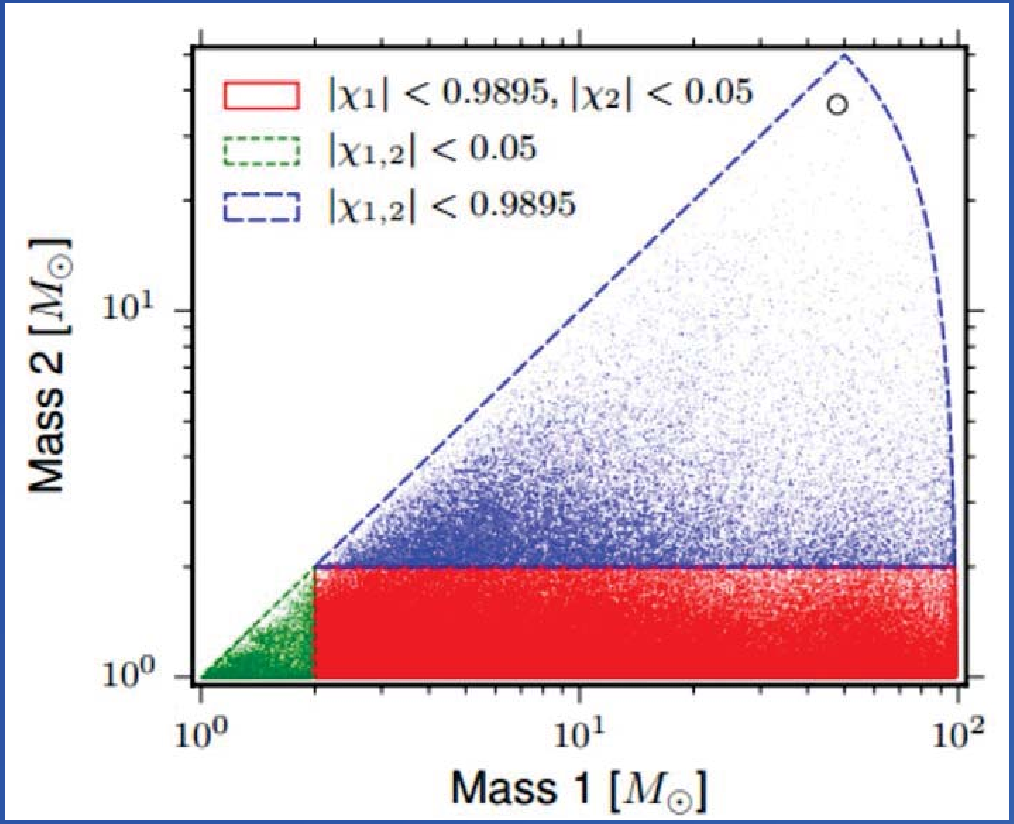
Generic transient search
2σ 3σ 4σ 4.4σ 4.4σ
2σ 3σ 4σ 4.6σ > 4.6σ
Search Result (C3)
Search Background (C3)
Search Result (C2+C3)
Search Background (C2+C3)
GW150914
Number of events
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
8
10
12
14
16
18
20
>32
Detection statistic η_c

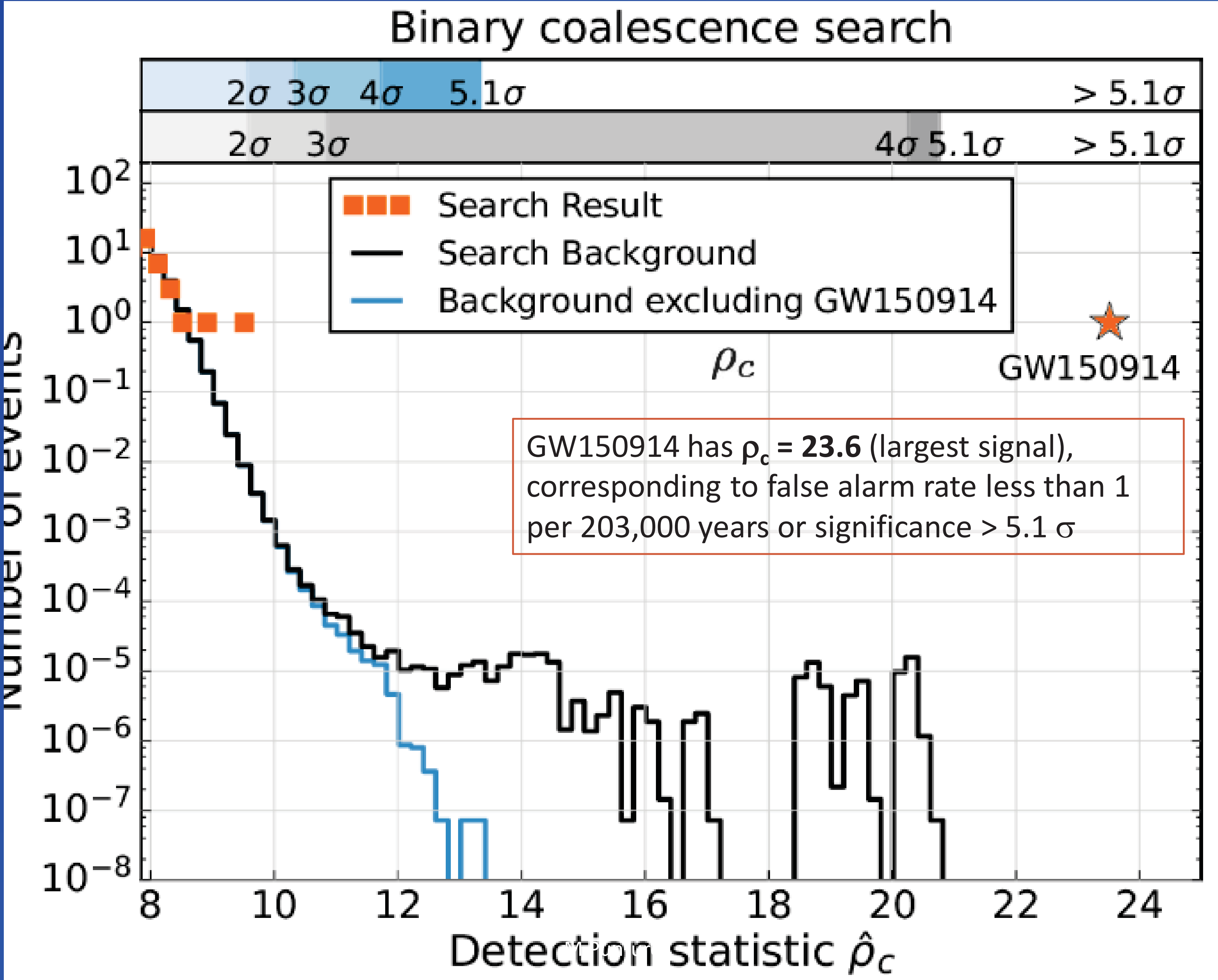
Template based transient search

- Two template banks, based on two different methods to build the templates
  - EOBNR, based on Effective-One-Body formalism , that combines weak field PN approximation with strong field effects, calibrated and corrected by NR simulation
  - IMRPhenom: inspiral–merger–ringdown phenomenological formalism based on extending frequency-domain PN expressions and hybridizing PN and EOB with NR waveforms.
- Two DA pipelines, sharing the same banks
  - PyCBC, GstLAL

Template bank

- Component masses 1 to 99 solar masses; total mass, up to 100 solar masses and dimensionless spin < 0.99
- ~250,000 wave forms are used to cover the parameter space



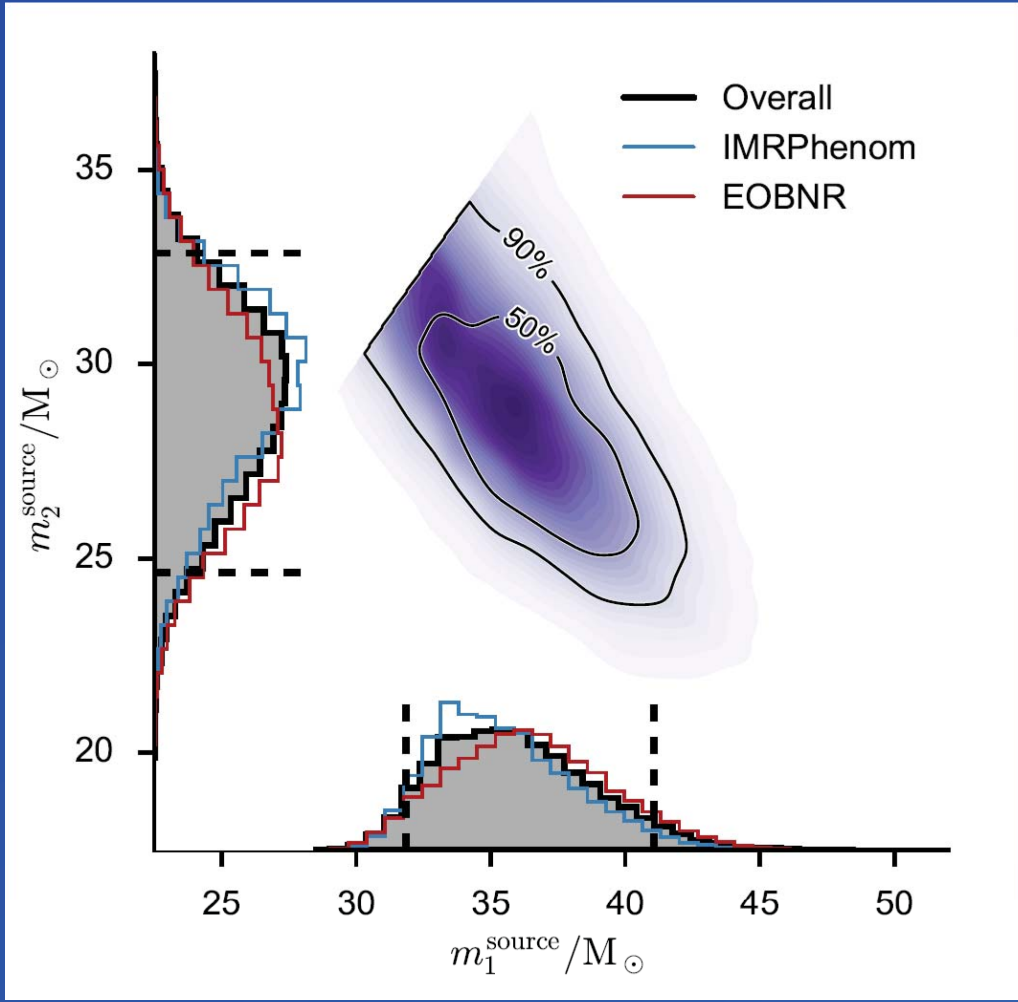


GW150914 has ρ_c = **23.6** (largest signal), corresponding to false alarm rate less than 1 per 203,000 years or significance > 5.1 σ

Parameters estimation

- Using the template based search it is possible to reconstruct the parameters of the coalescing BHs

Masses

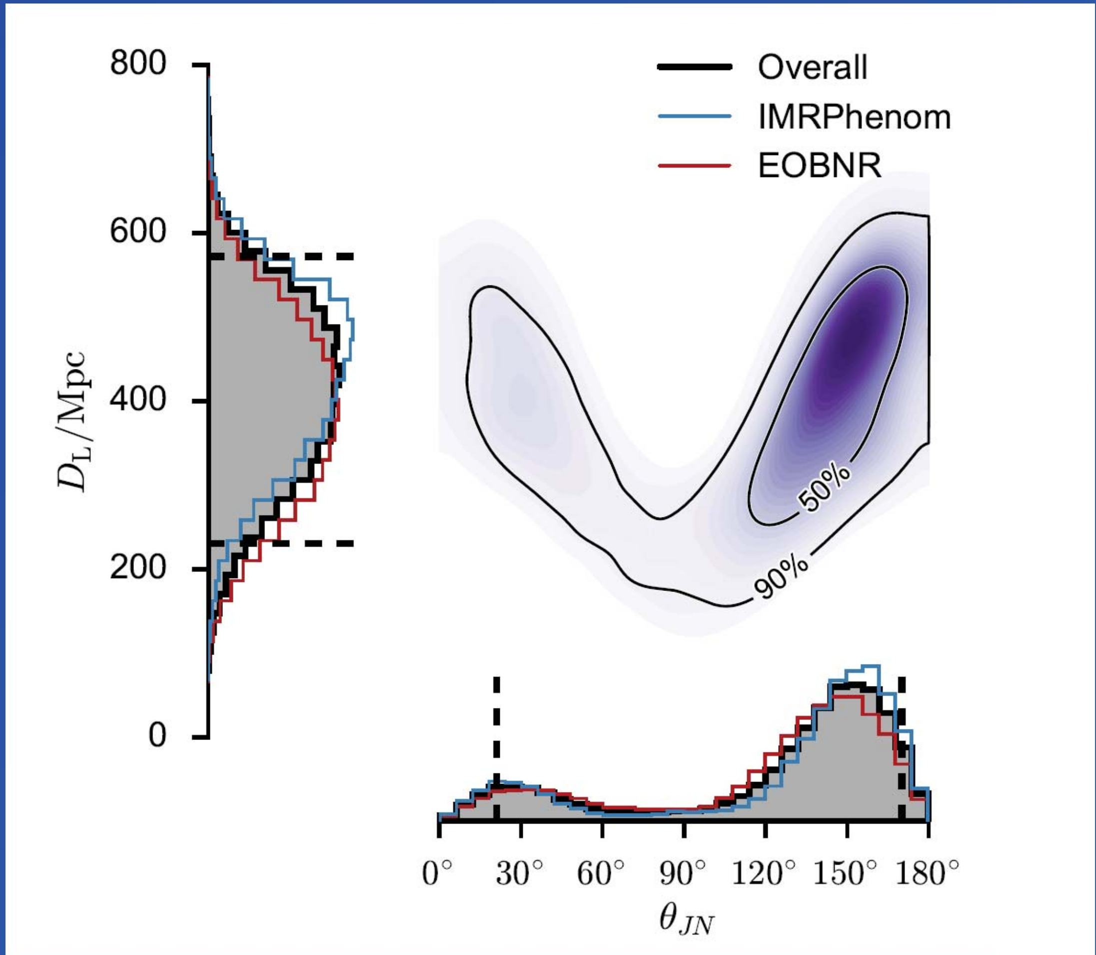


arXiv:1602.03839

Parameters estimation

- Using the template based search it is possible to reconstruct the parameters of the coalescing BHs

Luminosity distance



Binary inclination angle

arXiv:1602.03839

Implications of GW150914

Astrophysics implications
Key facts

- Binary black holes do exist!
  - **Form** and **merge** in time scales accessible to us
  - Predictions previously encompassed $[0 – 10^3]$ / Gpc^3 / yr
  - Now (full O1) we exclude lowest end: **rate** $\in [9, 240]$ **Gpc^{-3} yr^{-1}.**
- Masses ($M > 20\ M_\odot$) large compared with *known* stellar mass BHs
- Progenitors are
  - Likely **heavy**, $M > 60\ M_\odot$
  - Likely with a **low metallicity**, $Z < 0.25\ Z_\odot$
- Measured redshift $z \sim 0.1$
- **Low metallicity models can produce low-z mergers at rates consistent with our observation**

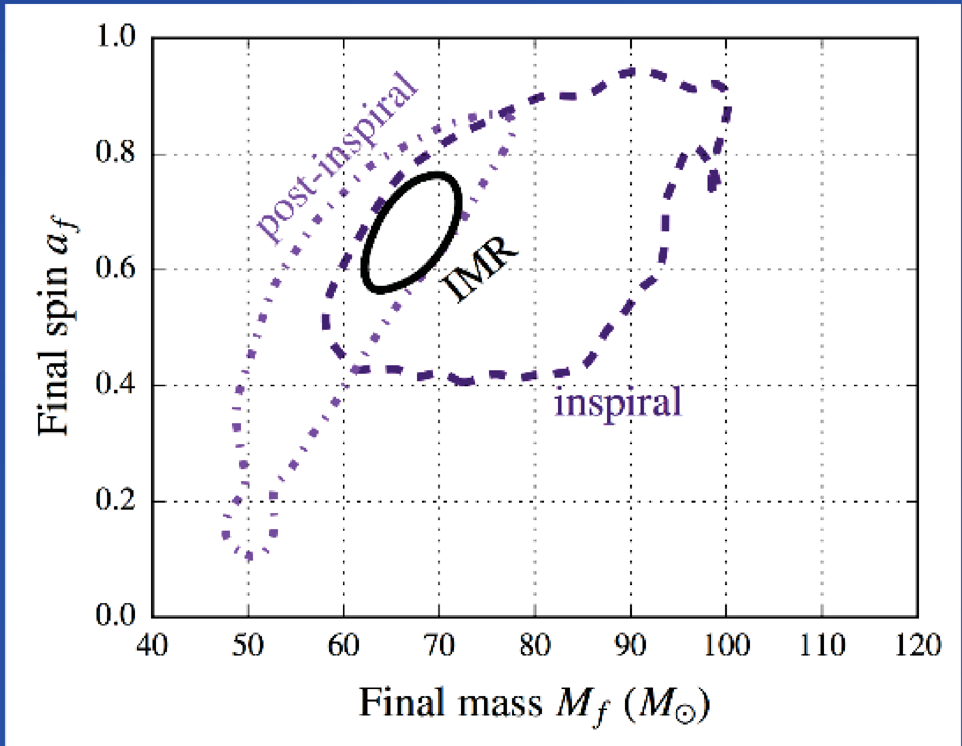
ApJL, 818, L22, 2016

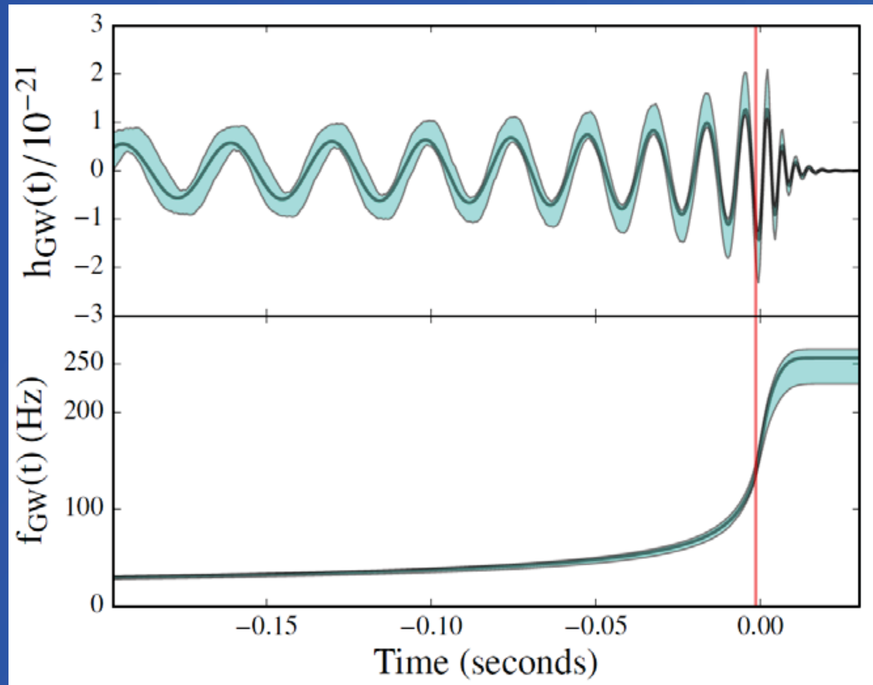
Testing GR

- Most relativistic binary know today : J0737-3039
  - Orbital velocity $v/c \sim 2 \times 10^{-3}$
- GW150914 : Higly disturbed black holes
  - Non linear dynamics
- Access to the properties of space-time
  - Strong field, high velocity regime testable for the first time $v/c \sim 0.6$
- Tests :
  - Waveform internal consistency check
  - Deviation of PN coefficients from General Relativity
  - Bound on graviton mass
- All tests are consistent with predictions of General Relativity

arXiv:1602.03841

Waveform internal consistency





1. Predict final black hole mass and spin from the inspiral signal
2. Predict final black hole mass and spin from the ring-down phase
3. Compare to check consistency of GR in different regimes

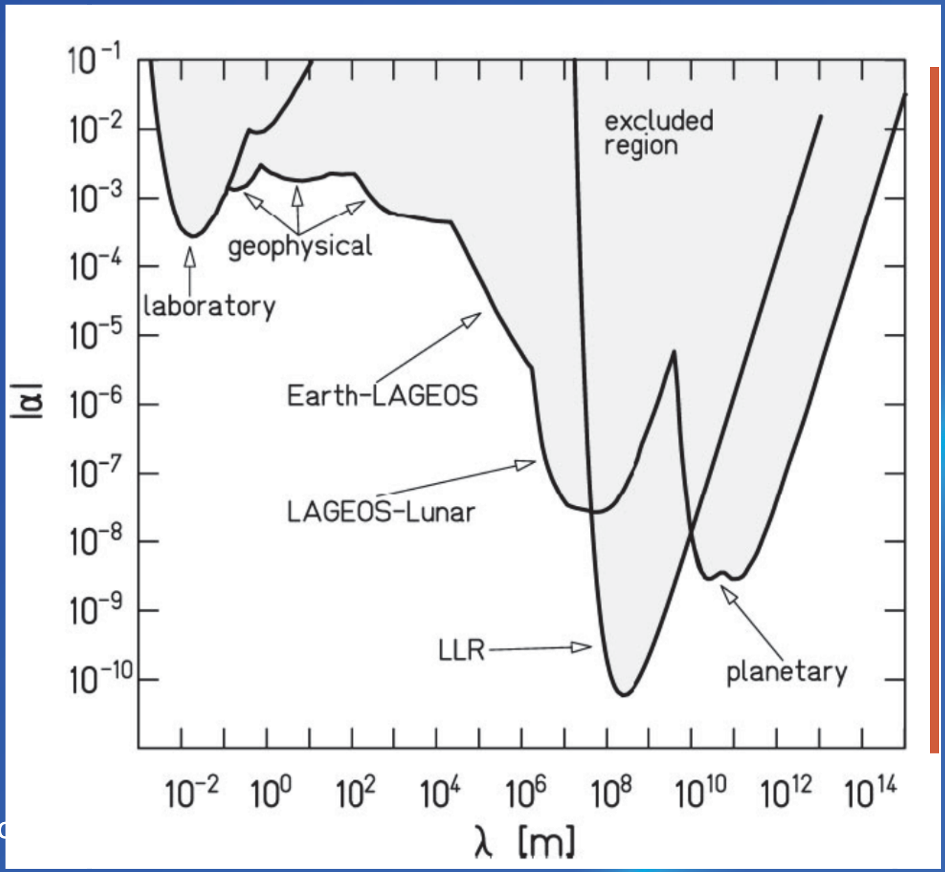
arXiv:1602.03841

Testing the mass of the graviton

- In GR the GW travel at the light speed
  - Introducing the graviton it corresponds to state that it is massless
- But if $v_{GW} < c$ the graviton should have mass m_g and then we can write: $E^2 = p^2c^2 + m_g^2c^4$
- The Compton wavelength related to m_g is: $\lambda_g = h/(m_g c)$
- Then, the graviton travel speed is Energy (frequency) dependent→dispersion:
  - $\frac{v_g^2}{c^2} \equiv \left(\frac{cp}{E}\right)^2 = 1 - \left(\frac{hc}{\lambda_g E}\right)^2$
- Further consequence is that the inverse r dependence op the Newtonian gravitational potential has a Yukawa-type correction:
  - $V(r) = \frac{GM}{r}\left[1 - \alpha \cdot exp(-r/\lambda_g)\right]$

Testing the mass of the graviton

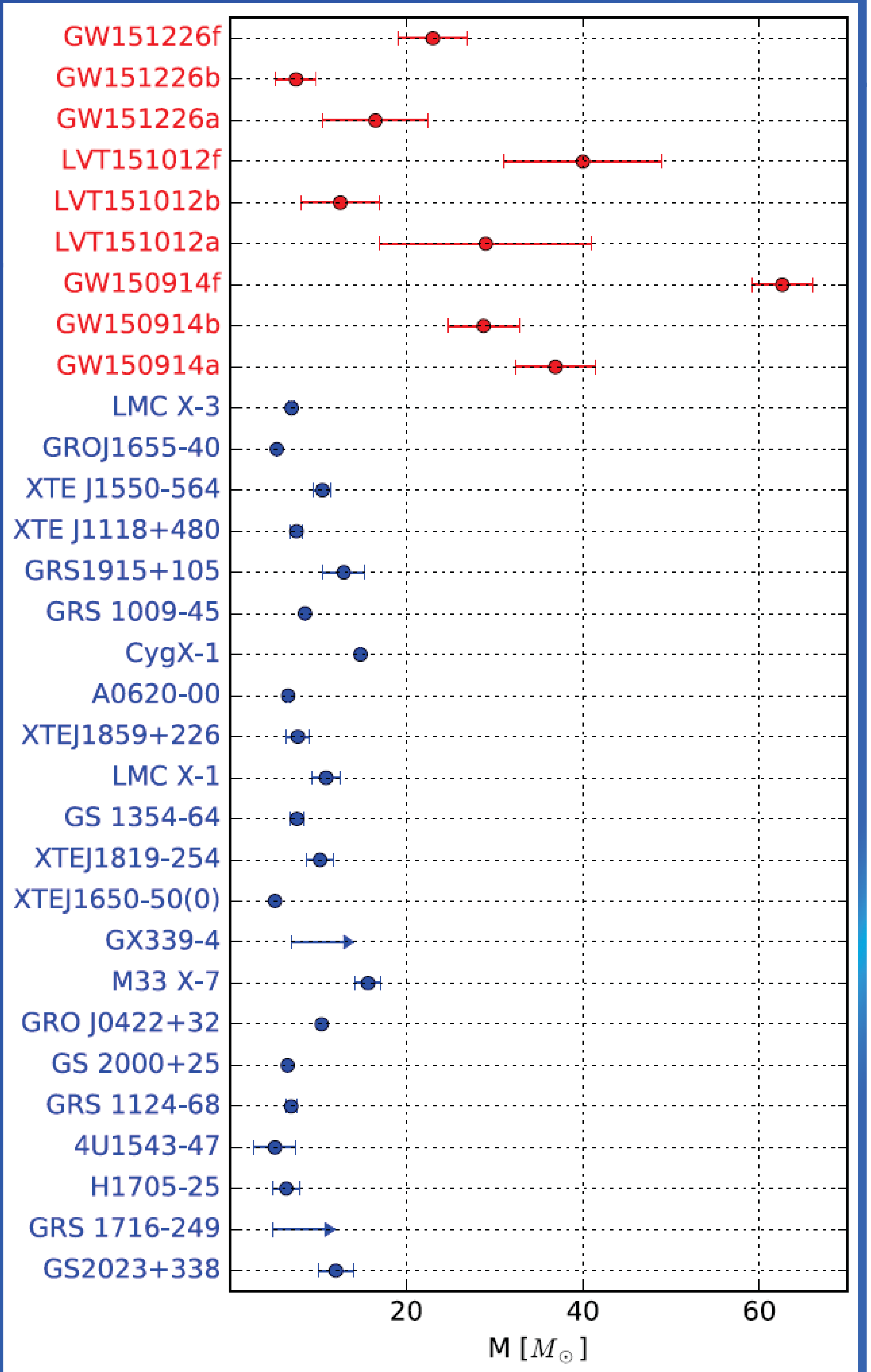
- The effect of the dispersion should add a 1PN phase term depending on λ_g.
- Introducing this term in the template has been tested the dispersion of the GW
- Results, upper limit: $m_g < 1.2 \times 10^{-22} eV/c^2$ at 90% confidence ($\lambda_g > 10^{13}$km)
  - 3 times better than Solar System bound
  - 3 orders of magnitude better than the one from binary-pulsar observations
  - less constraining than model-dependent bounds coming from the large-scale dynamics of galactic clusters and weak gravitational-lensing observations



GW151226

New “family” of BH?

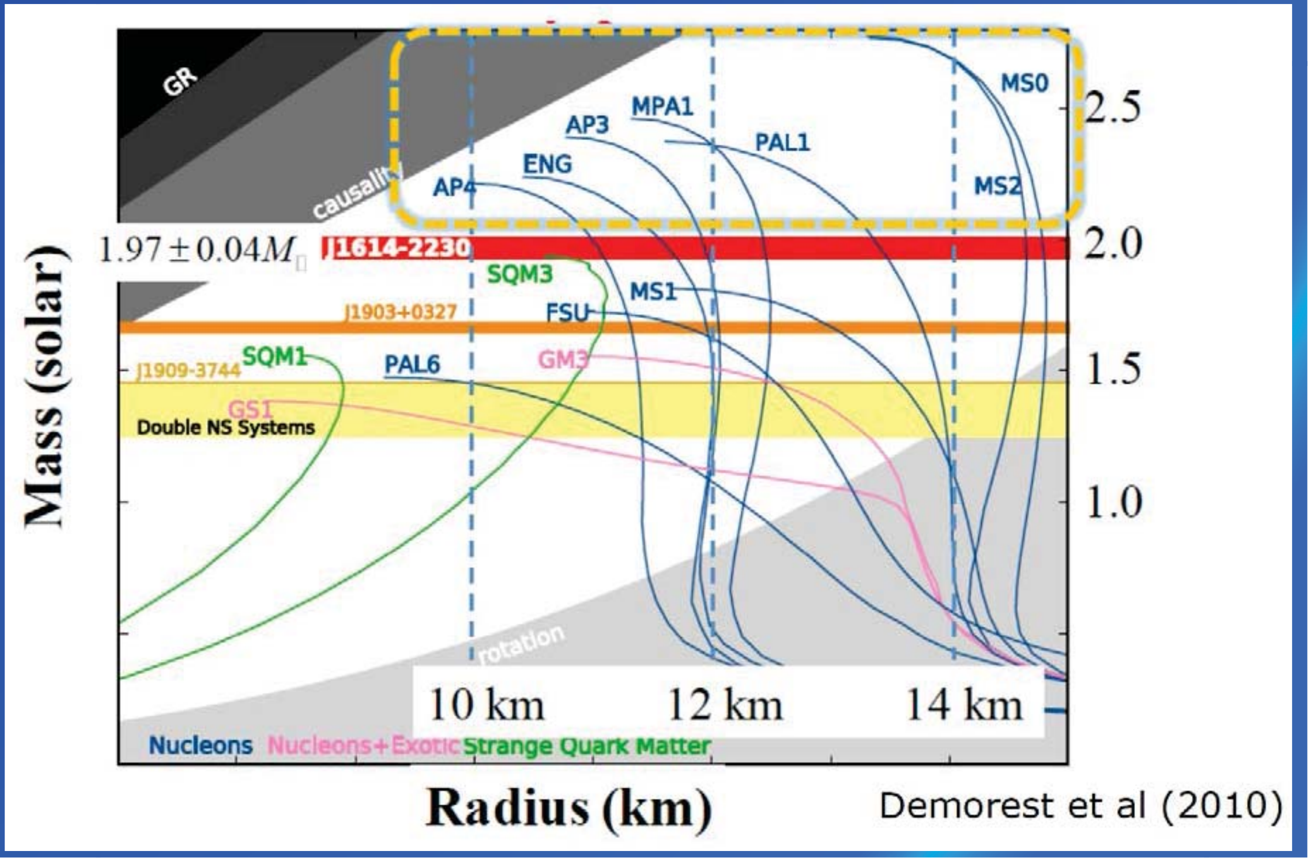
Event	GW150914	GW151226	LVT151012
Signal-to-noise ratio ρ	23.7	13.0	9.7
False alarm rate FAR/yr^{-1}	$< 6.0 \times 10^{-7}$	$< 6.0 \times 10^{-7}$	0.37
p-value	7.5×10^{-8}	7.5×10^{-8}	0.045
Significance	$> 5.3\,\sigma$	$> 5.3\,\sigma$	1.7σ
Primary mass $m_1^{\rm source}/{\rm M}_\odot$	$36.2^{+5.2}_{-3.8}$	$14.2^{+8.3}_{-3.7}$	23^{+18}_{-6}
Secondary mass $m_2^{\rm source}/{\rm M}_\odot$	$29.1^{+3.7}_{-4.4}$	$7.5^{+2.3}_{-2.3}$	13^{+4}_{-5}
Chirp mass $\mathscr{M}^{\rm source}/{\rm M}_\odot$	$28.1^{+1.8}_{-1.5}$	$8.9^{+0.3}_{-0.3}$	$15.1^{+1.4}_{-1.1}$
Total mass $M^{\rm source}/{\rm M}_\odot$	$65.3^{+4.1}_{-3.4}$	$21.8^{+5.9}_{-1.7}$	37^{+13}_{-4}
Effective inspiral spin $\chi_{\rm eff}$	$-0.06^{+0.14}_{-0.14}$	$0.21^{+0.20}_{-0.10}$	$0.0^{+0.3}_{-0.2}$
Final mass $M_{\rm f}^{\rm source}/{\rm M}_\odot$	$62.3^{+3.7}_{-3.1}$	$20.8^{+6.1}_{-1.7}$	35^{+14}_{-4}
Final spin $a_{\rm f}$	$0.68^{+0.05}_{-0.06}$	$0.74^{+0.06}_{-0.06}$	$0.66^{+0.09}_{-0.10}$
Radiated energy $E_{\rm rad}/({\rm M}_\odot c^2)$	$3.0^{+0.5}_{-0.4}$	$1.0^{+0.1}_{-0.2}$	$1.5^{+0.3}_{-0.4}$
Peak luminosity $\ell_{\rm peak}/({\rm erg\,s^{-1}})$	$3.6^{+0.5}_{-0.4} \times 10^{56}$	$3.3^{+0.8}_{-1.6} \times 10^{56}$	$3.1^{+0.8}_{-1.8} \times 10^{56}$
Luminosity distance $D_{\rm L}/{\rm Mpc}$	420^{+150}_{-180}	440^{+180}_{-190}	1000^{+500}_{-500}
Source redshift z	$0.09^{+0.03}_{-0.04}$	$0.09^{+0.03}_{-0.04}$	$0.20^{+0.09}_{-0.09}$
Sky localization $\Delta\Omega/{\rm deg}^2$	230	850	1600



Next discovery potential

Fundamental Physics with NS

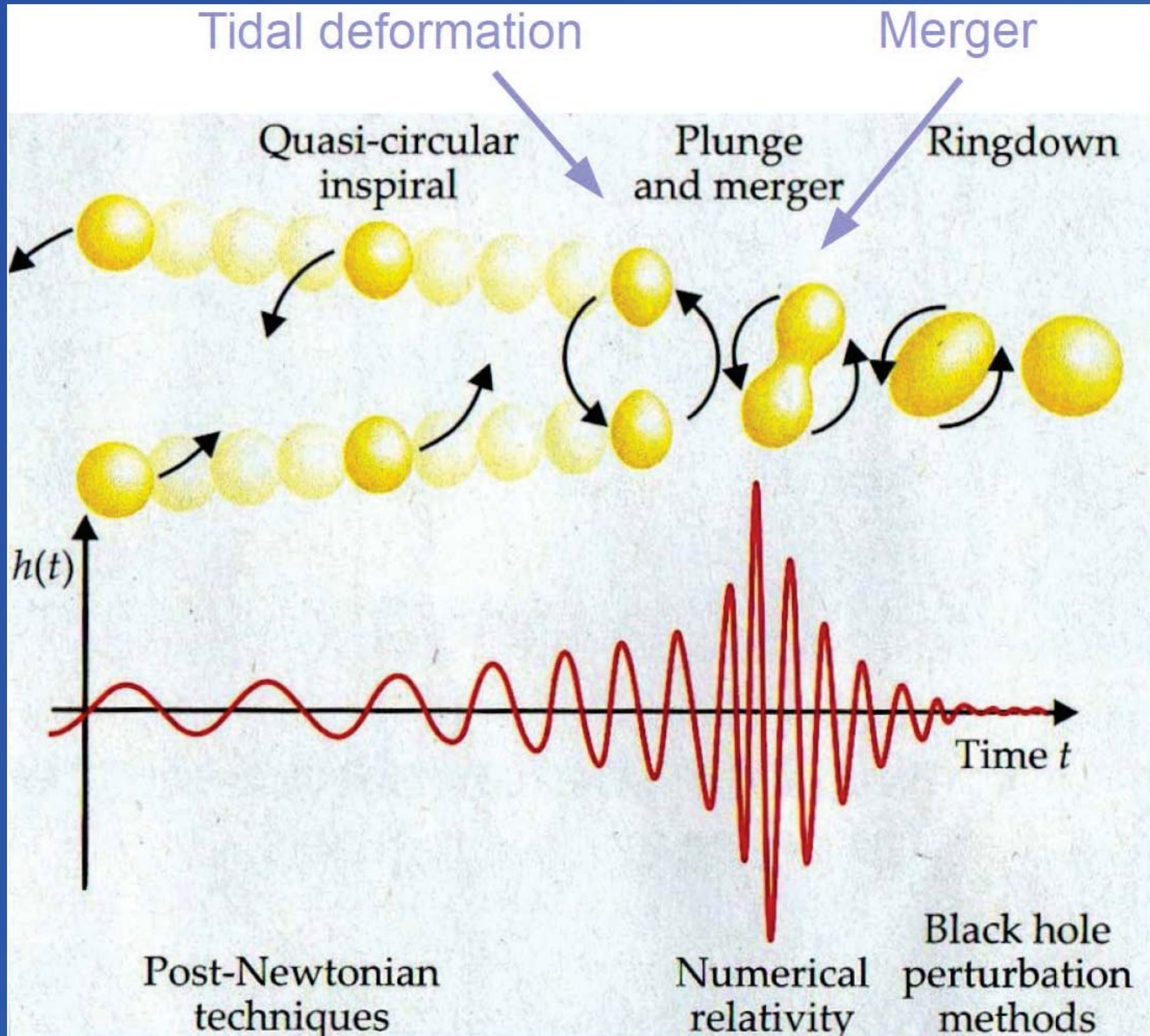
- What is the matter in the extreme conditions of a NS? ($\rho > \rho_c \sim 10^{15} g/cm^3$)
  - Nucleons (n,p,e,μ)?
  - Hyperon rich matter?
  - Quark Matter? Deconfined quarks?
- Plethora of possible EOS of a NS:



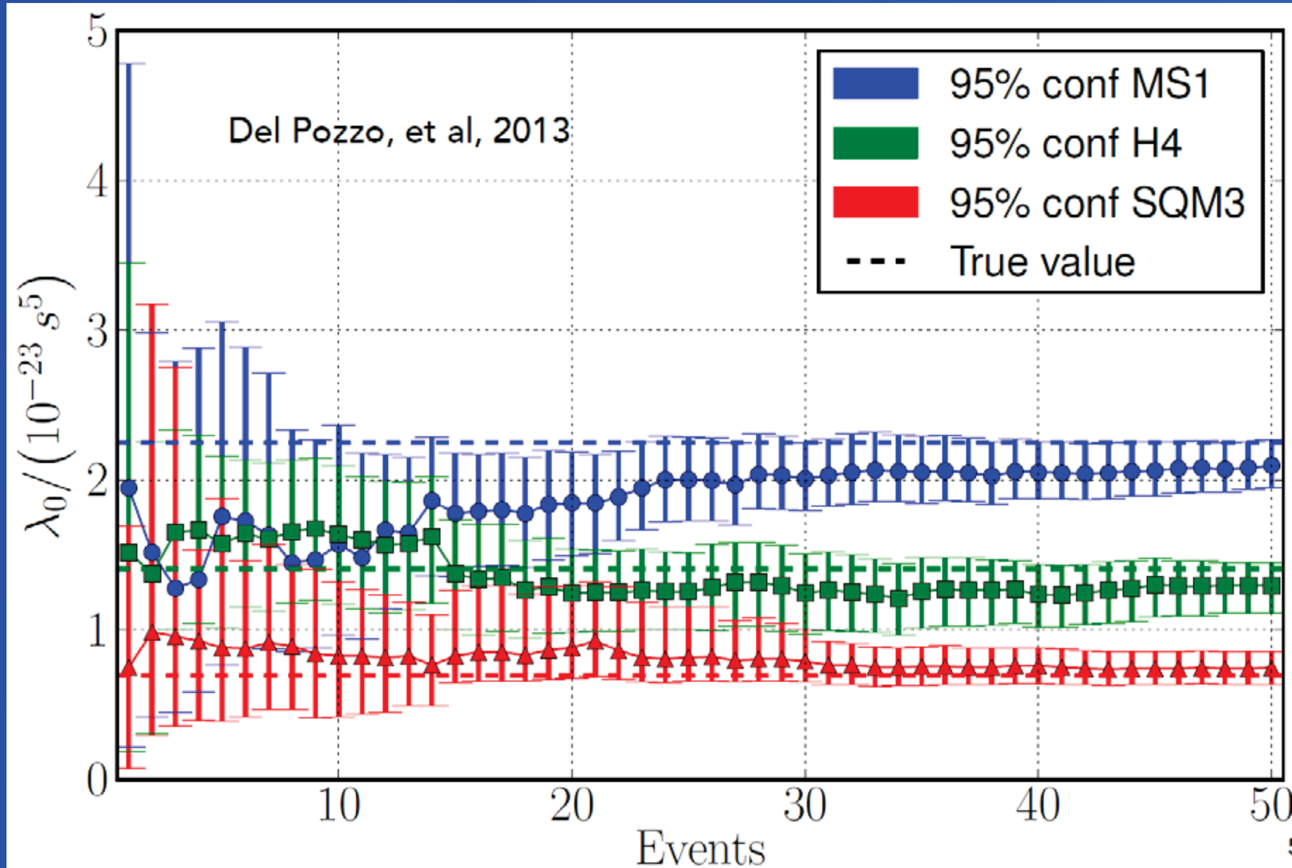
M.Punturo

Observing BNS deformation

- GW observation from NS coalescing binaries could constrain the EOS:
  - Tidal deformation λ of the NS under external field is related to its EOS and it affects the binary orbital evolution
  - Crucial: high SNR



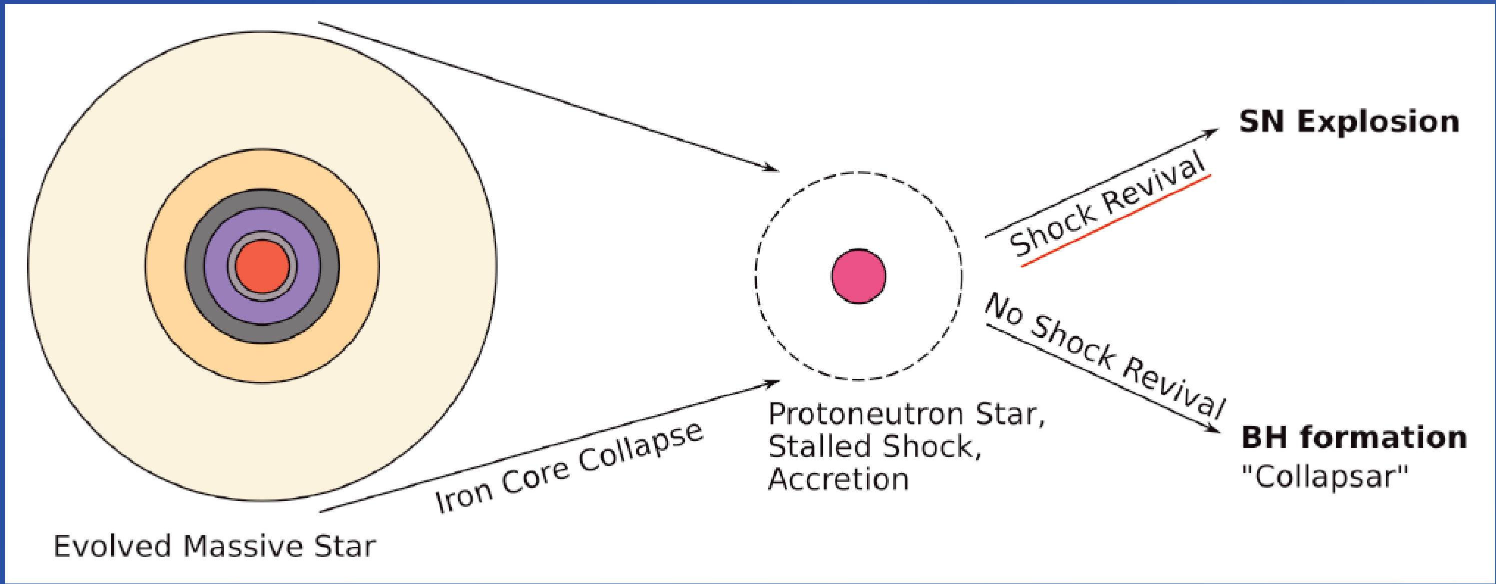
Discriminating NS EOS



- Possible in 2G – 2G+ - 3G
- EM counterpart:
  - GRB emission:
    - Confinement of some of the parameters (BH-NS coalescence)
    - Info on the external crust
  - High energy Neutrino emission:
    - Info on the nuclear processes in the Hyper-massive NS
    - Info on the shockwave

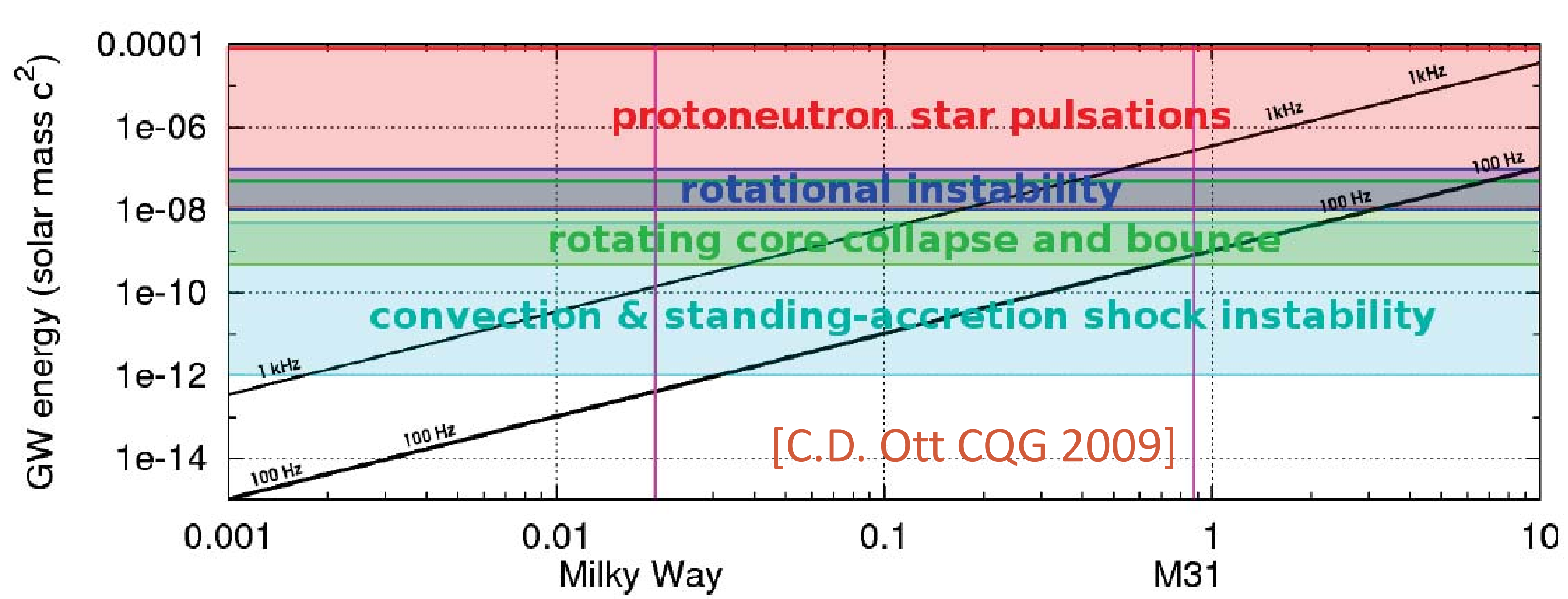
Supernova Explosions

- Mechanism of the core-collapse SNe still unclear
  - Shock Revival mechanism(s) after the core bounce TBC



- GWs generated by a SNe should bring information from the inner massive part of the process and could constrains on the core-collapse mechanisms
- But, quantity of energy emitted in GW quite unpredicted and small
  - Advanced detectors expected to be sensible within the galaxy (few events per century)
    - 3G (ET)

Stellar explosions - SNe

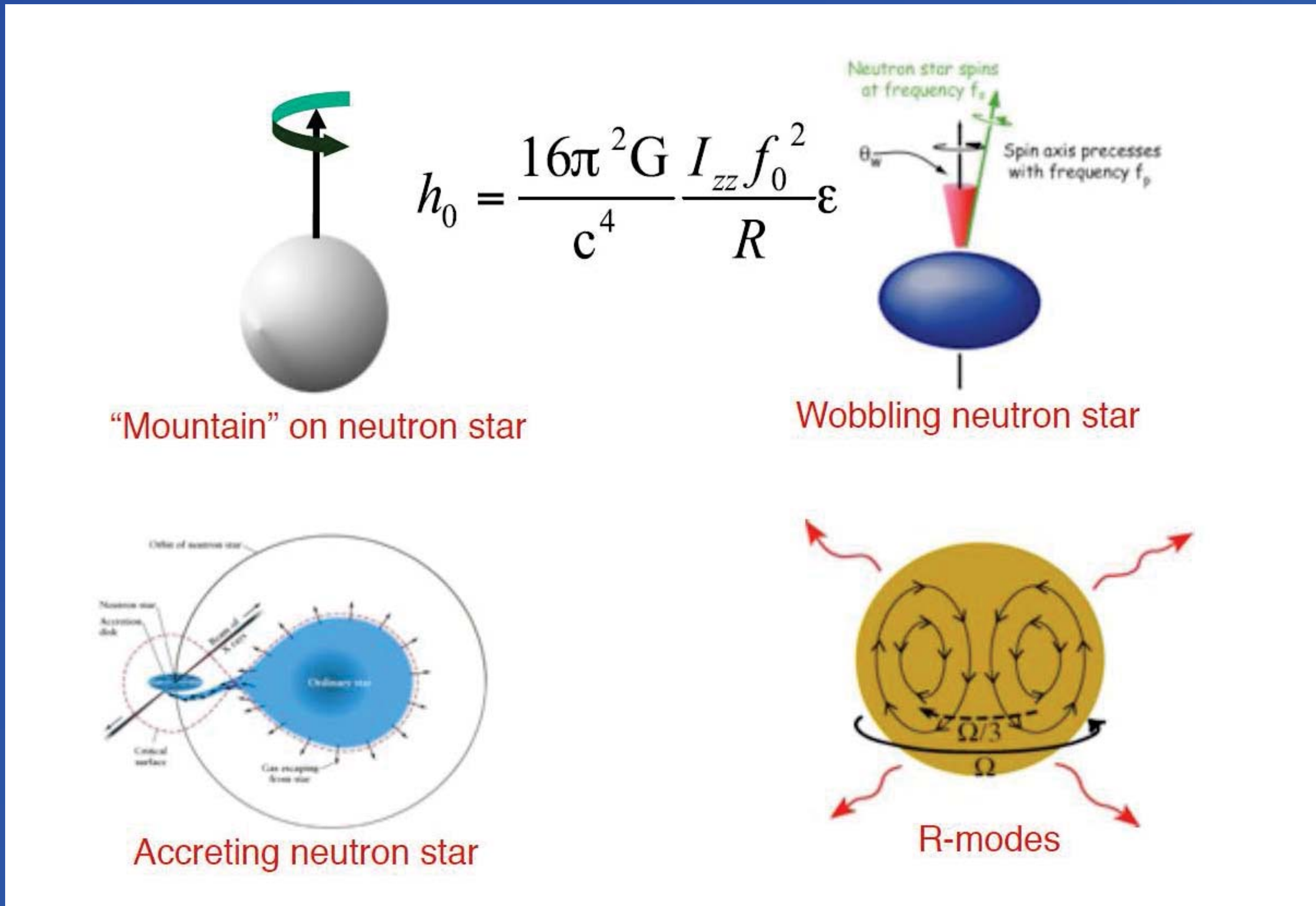


ET scenario

- Low energy neutrino detection can contribute to the determination of the explosion mechanism, but is there overlap in the detection range?
  - Mton neutrino detectors?
- Coincident/afterglow E.M. observations

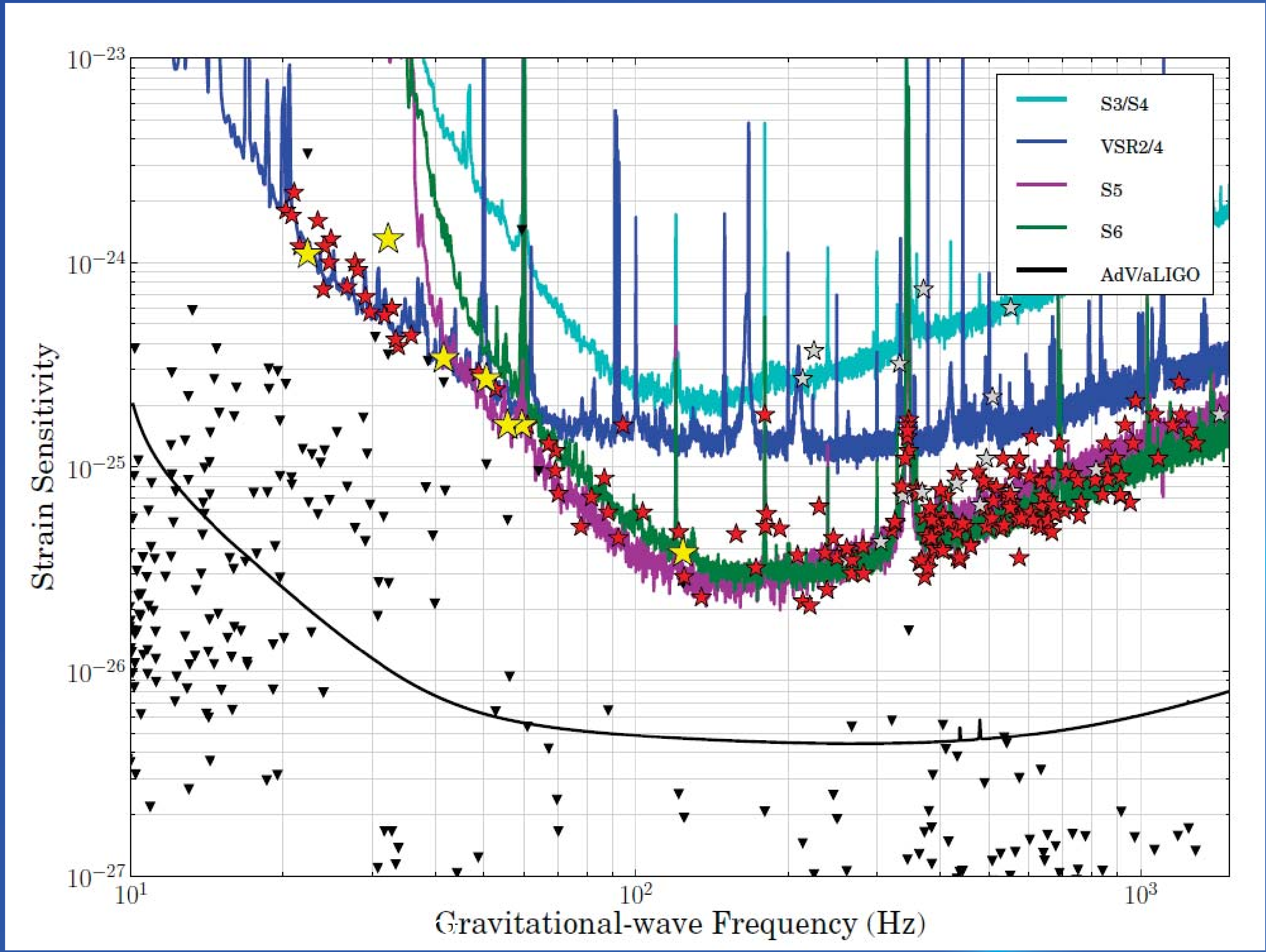
Radiation from isolated NS

- An Isolated NS can emit through different mechanisms
- To reveal the GW emitted by a NS could constrain its EOS

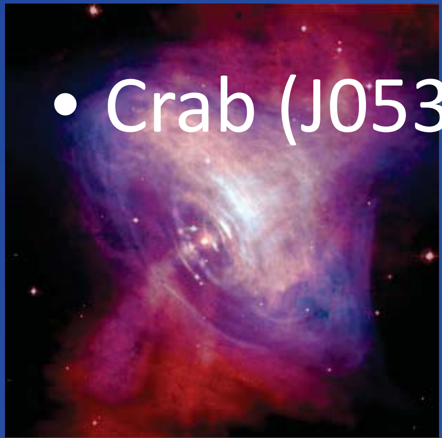


Continuous Waves from Isolated NS

Upper limits on the gravitational wave strain amplitude for 195 pulsars using data from the LIGO S3, S4, S5 and S6, and Virgo VSR2 and VSR4 runs. The triangles show the spin-down limits for a selection of these stars. The curves give estimates of the expected sensitivity of the runs and also future runs with Advanced LIGO and Advanced Virgo.



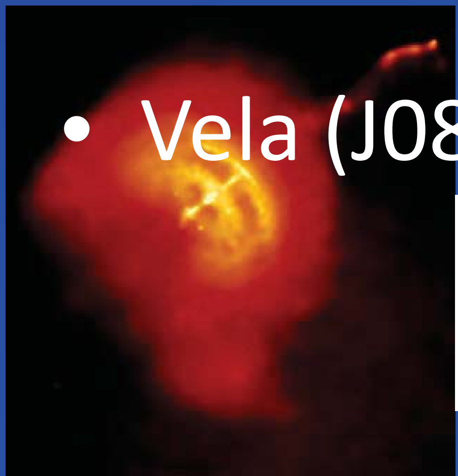
Low frequency pulsars



- Crab (J0534+2200)

$$f_{GW} \approx 59.44 Hz$$

$$\frac{\dot{E}_{GW}}{\dot{E}} \leq 1\% \quad ; \quad \varepsilon \leq 8\times 10^{-4}$$



- Vela (J0835−4510)

$$f_{GW} \approx 22.39 Hz$$

$$\frac{\dot{E}_{GW}}{\dot{E}} \leq 10\% \quad ; \quad \varepsilon \leq 6\times 10^{-4}$$

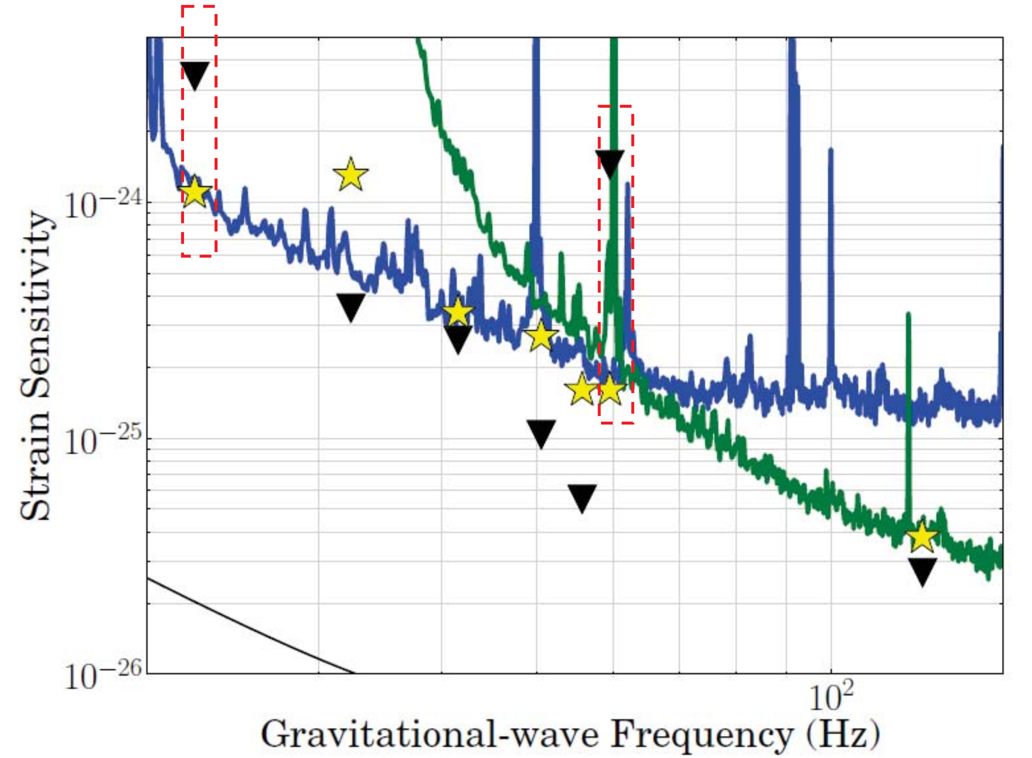


Figure 2. Zoomed version of Figure 1 focusing on the seven high interest pulsars.

Mountains in the NS?

$$\varepsilon \approx 10^{-8} \rightarrow H_{\text{mountain}} \approx mm$$

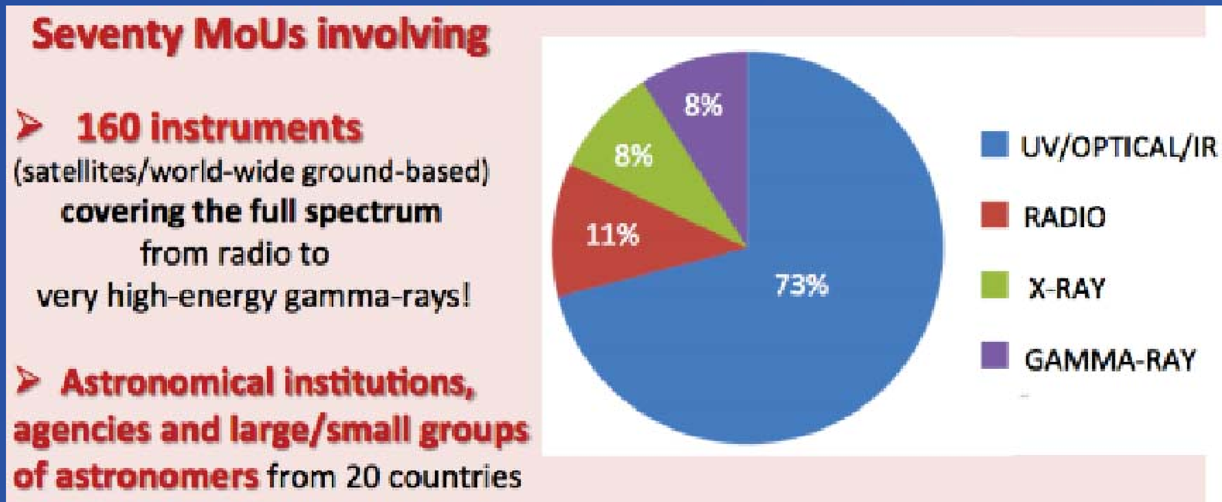
The Astrophysical Journal, 785:119 (18pp), 2014 April 20

Table 7
(Continued)

Pulsar	f_{rot} (Hz)	f_{gw} (Hz)	$\dot{f}_{rot}$ (Hz s^{-1})	d (kpc)	h_0^{sd}	Prior $h_0^{95\%}$	S6/VSR2,4 $h_0^{95\%}$	ε
J2124−3358	202.79	405.59	-4.4×10^{-16}	0.3	4.0×10^{-27} [a]	4.9×10^{-26}	3.9×10^{-26}	6.7×10^{-8}
J2129−5721	268.36	536.72	-1.5×10^{-15}	0.4	4.7×10^{-27} [a]	6.2×10^{-26}	5.2×10^{-26}	6.8×10^{-8}

Multimessenger: EM-follow-up

- LVC called for EM observers to join a follow-up program
  - LIGO and Virgo share *promptly* with astronomers interesting triggers; up to a few at current sensitivity
  - Provide limited directional information, promptly estimated



- Big participation to GW150914 observation:
  - 24 groups carried out observations
  - Challenging! Source location with large uncertainty ~ 600 deg^2

New paradigm

- The detection of GW opens a revolutionary paradigm in the observation of the universe:
  - GW detectors listen the universe, localising the source, giving information on the mass and dimension of the source, identifying the processes that involve large amount of asymmetric and accelerated matter
  - “Telescopes” follows the indication on the localisation and see the electromagnetic and astroparticle messages, describing what is occurring in the “external” shells and in the neighbouring of the catastrophic event
- Multi-messenger observation of the Universe

GW150914: EM-follow-up

- No significant (as expected) EM counterpart revealed by the EM partners
- Debated Fermi-GBM excess of energy
- In future, adding Virgo, will change the scenario:
  - In the design LIGO-Virgo network, GW150914 could have been localized to less than 20 deg^2.

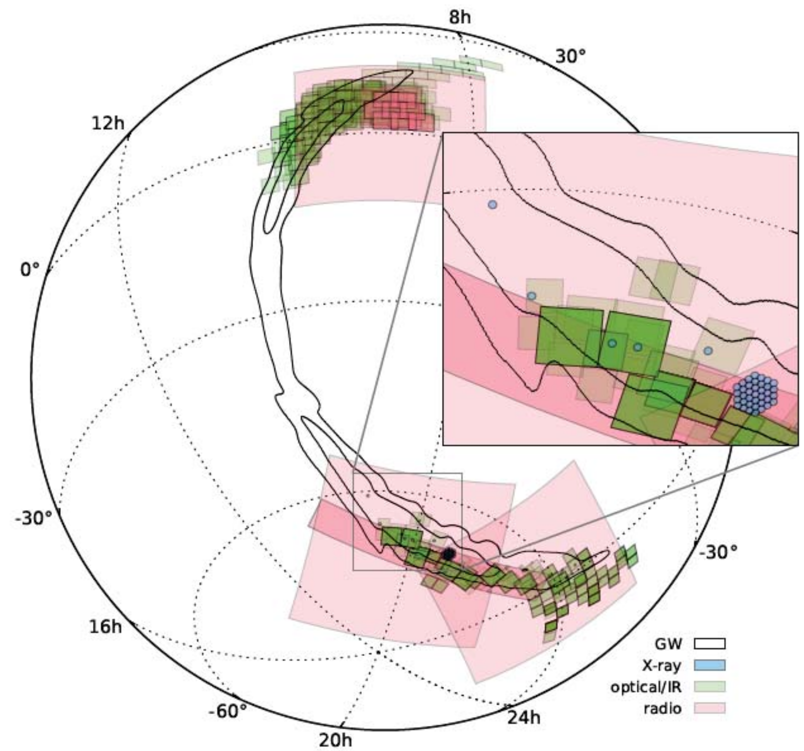
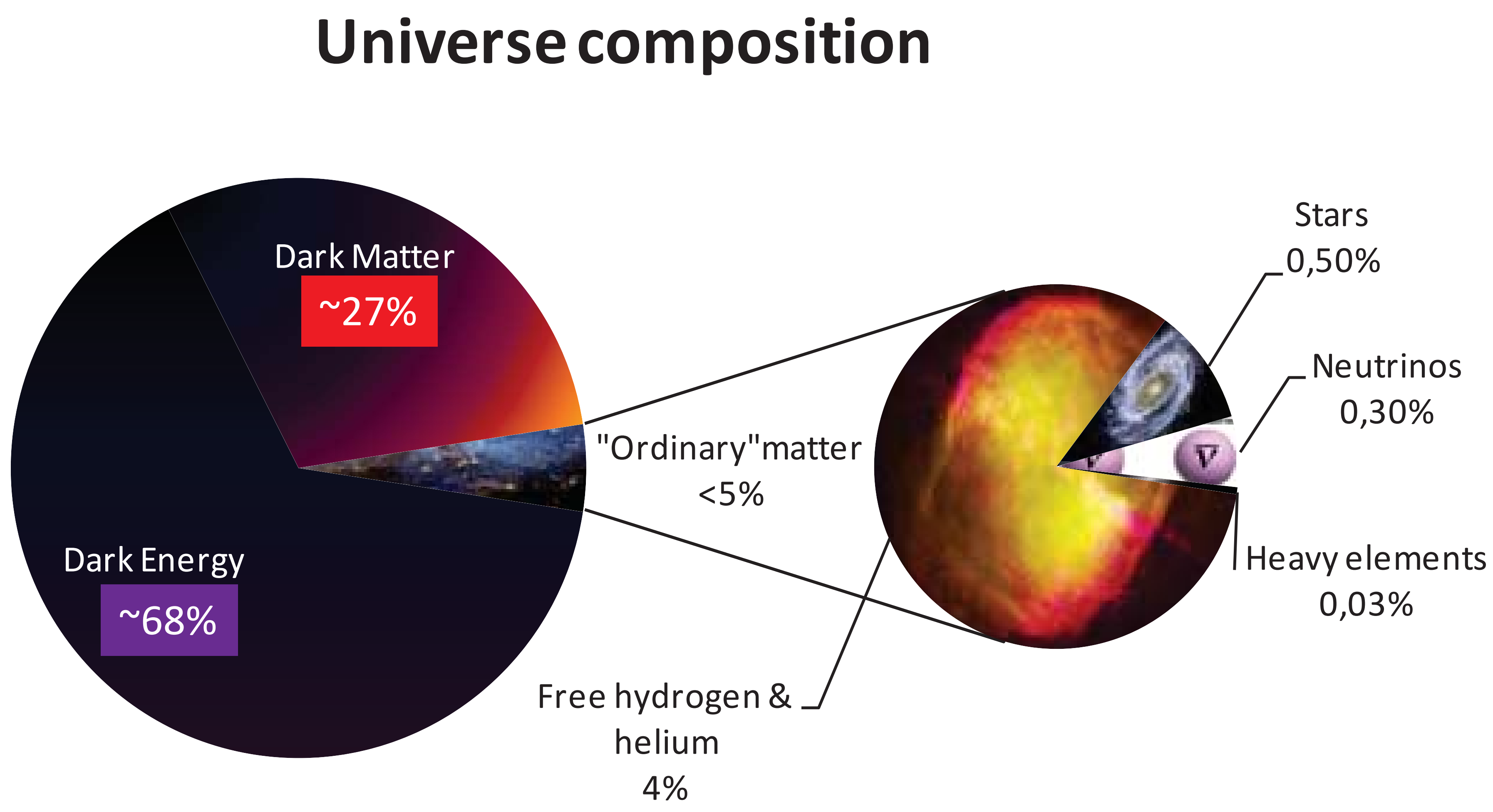


Figure 3. Footprints of observations in comparison with the 50% and 90% credible levels of the LIB GW localization. Radio fields are shaded red, optical/infrared fields are green, and X-ray Telescope (XRT) fields are blue circles. Where fields overlap, the shading is darker. The inset highlights the *Swift* observations consisting of a hexagonal grid and a selection of the *a posteriori* most highly ranked galaxies. The projection is the same as in Fig. 2.

What we know of the Universe?



GRB progenitors

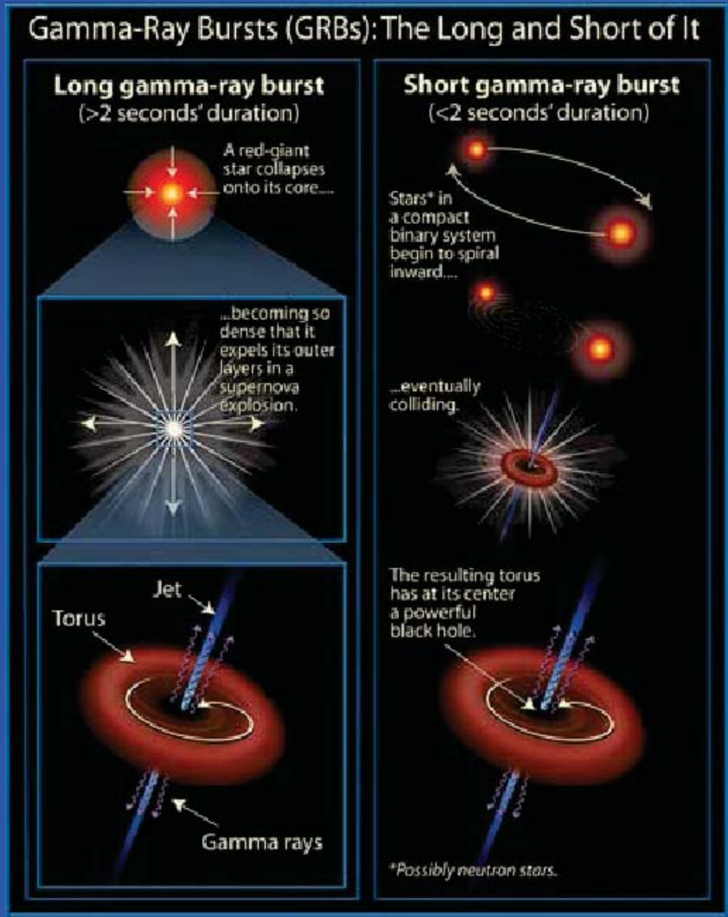
- The simultaneous detection of GW and sGRB from a coalescing BNS could confirm the nature of the progenitors of the GRB
- But it is possible to obtain also a measure of the cosmological model of the Universe

$$D_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{\left[\Omega_M (1+z)^3 + \Omega_\Lambda (1+z)^{3(1+w)}\right]^{1/2}}$$

GW

GRB

Affected by the Mass ambiguity due to the red-shift of the frequencies (in effect considering the merging phase it is possible to resolve the ambiguity)



Ω_M: total mass density
Ω_Λ: Dark energy density
H_0: Hubble parameter
w: Dark energy equation of state parameter

Possible in 3G (ET)

Extreme Gravity: do we need the darkness?

- Today we need more "matter" in the Universe to keep GR

$$\left(G_{\mu\nu} + G'_{\mu\nu}\right) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

To verify GR in condition of strong field could test the need of modified gravity theories

- But the Einstein field equations are the "simplest" solution in a curved space-time
- There are a series of alternative theories of the gravity (for example f(R) theories) that could introduce an extra-curvature stress-energy tensor

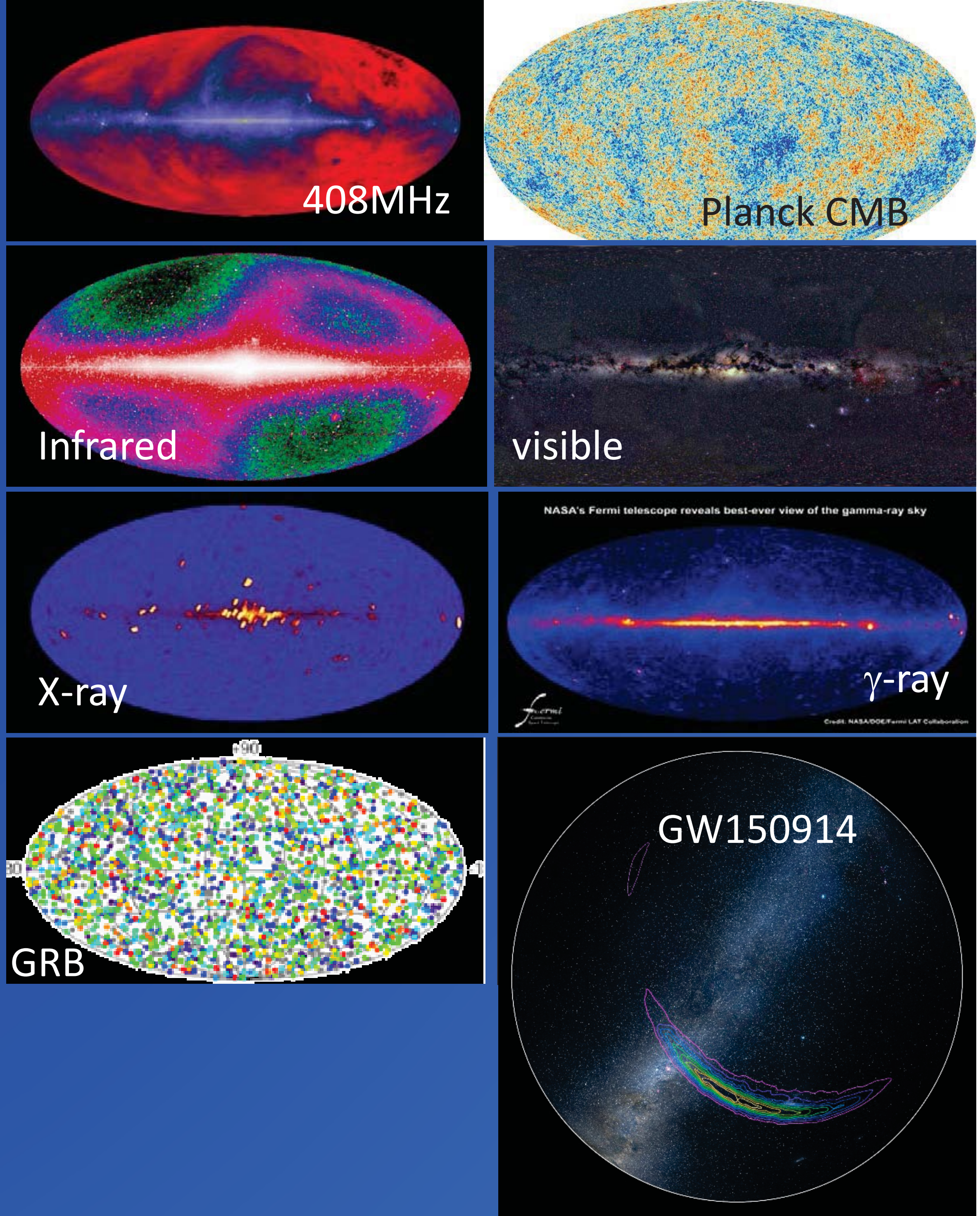
$$f'(R(g))\,R_{\mu\nu}(g) - \frac{1}{2}f(R(g))g_{\mu\nu} - \nabla_\mu\nabla_\nu f'(R(g)) + g_{\mu\nu}\Box f'(R(g)) = \kappa T_{\mu\nu}$$

Higher order Gravity (4^{th})!

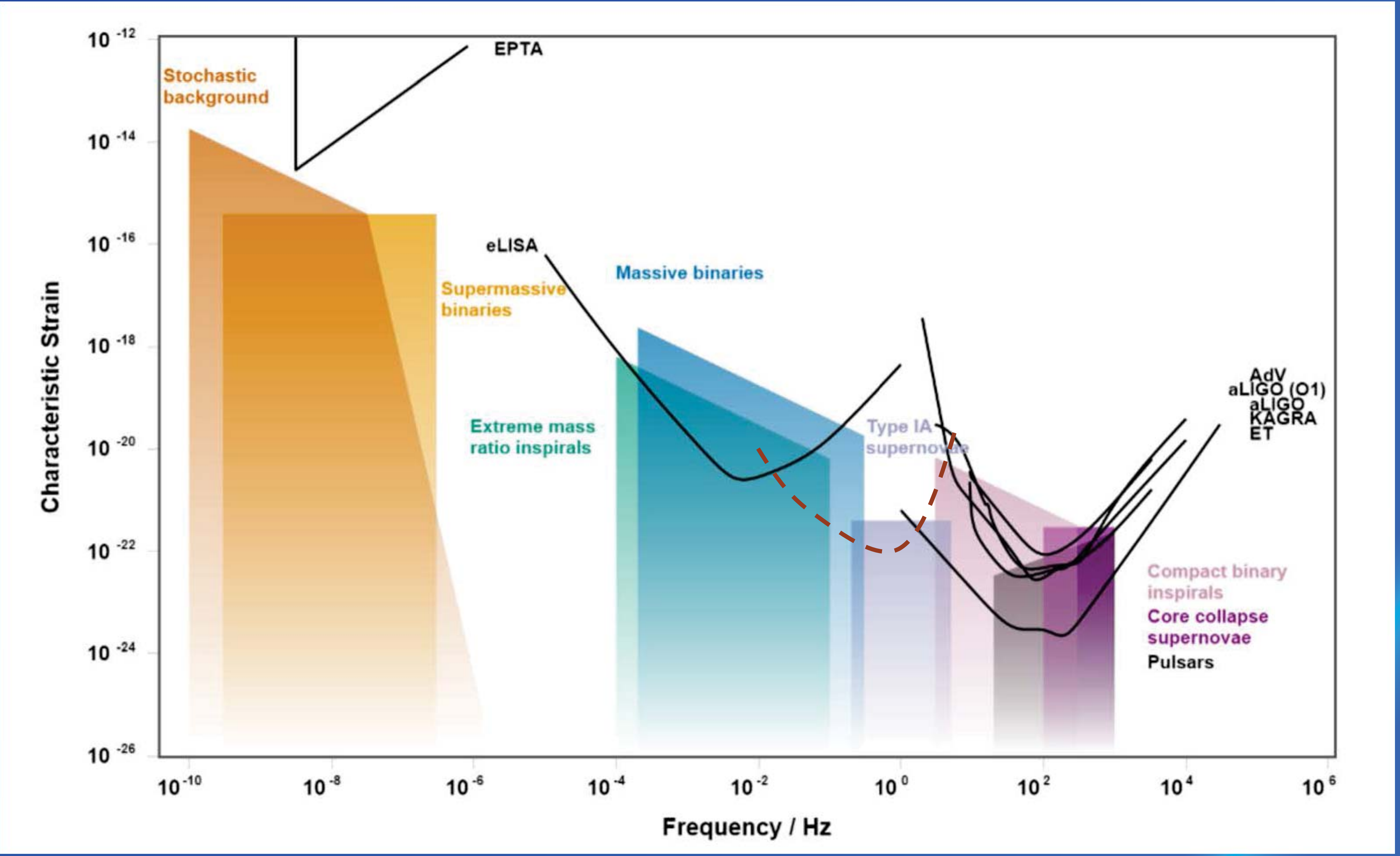
What Next?

GW astronomy

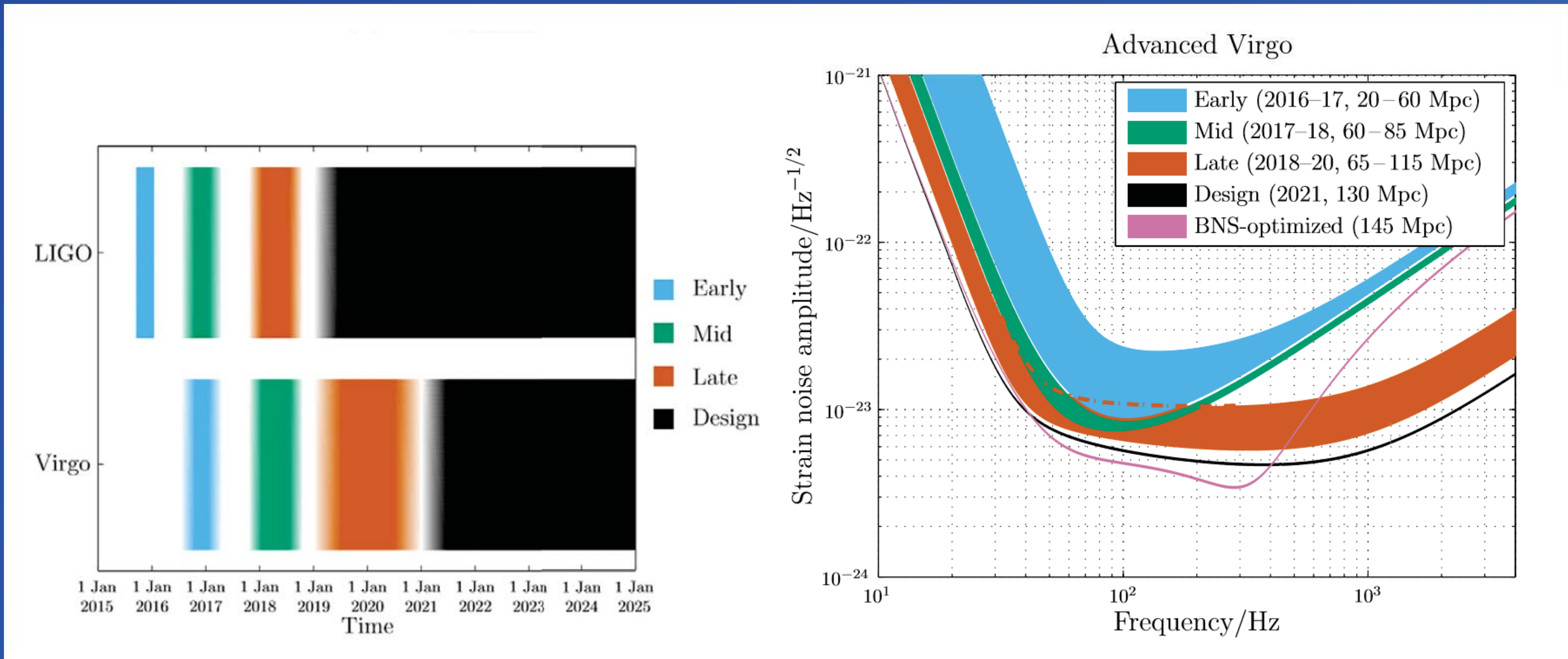
- E.M. astronomy covers all the frequency spectrum
- Thanks to GW150914 GW astronomy era has been open
- But the two key ingredients of e.m. astronomy need to be implemented:
    - To see more distant
    - To cover all the frequency range



GW sources and detectors



Short term evolution

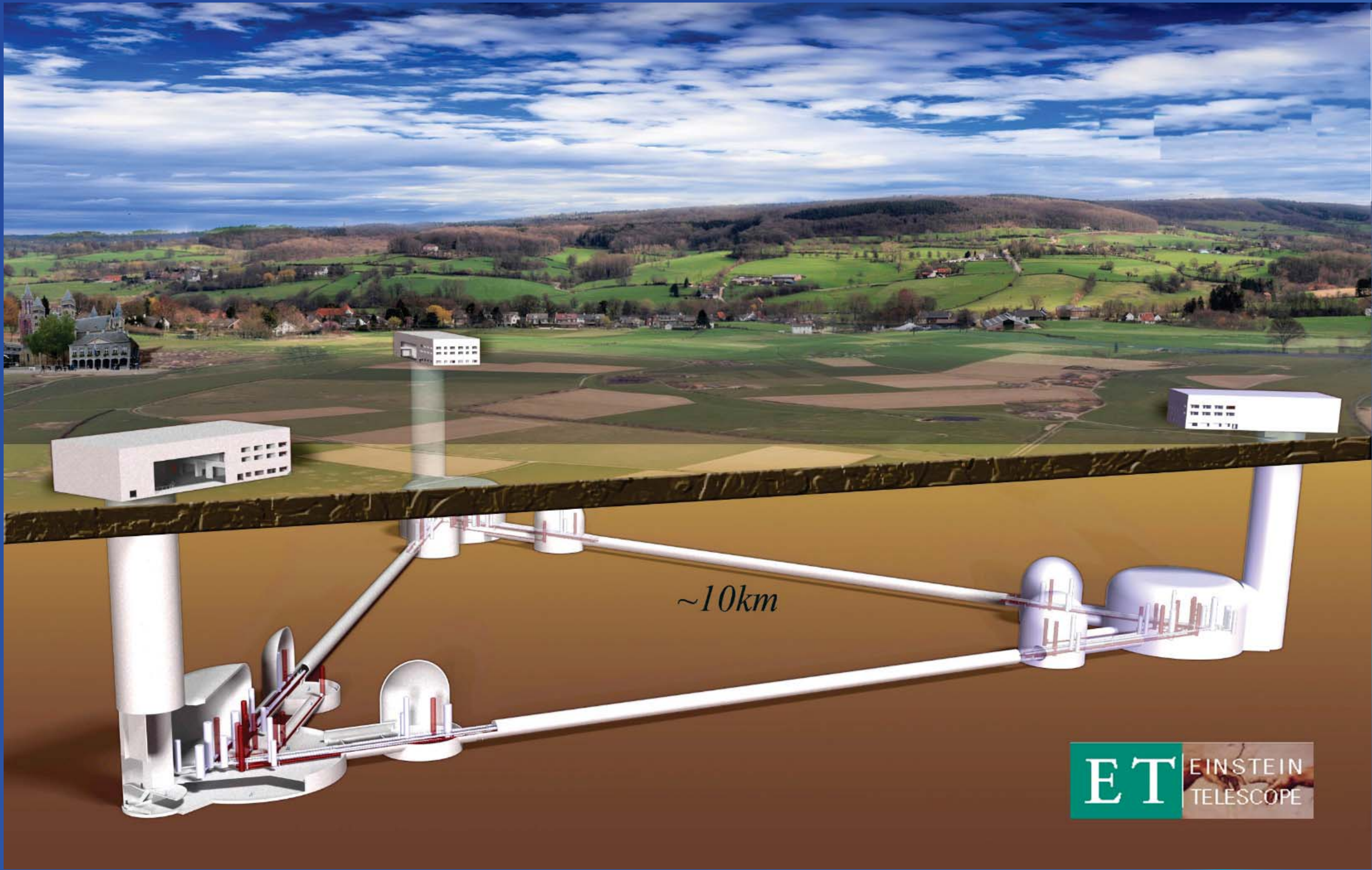


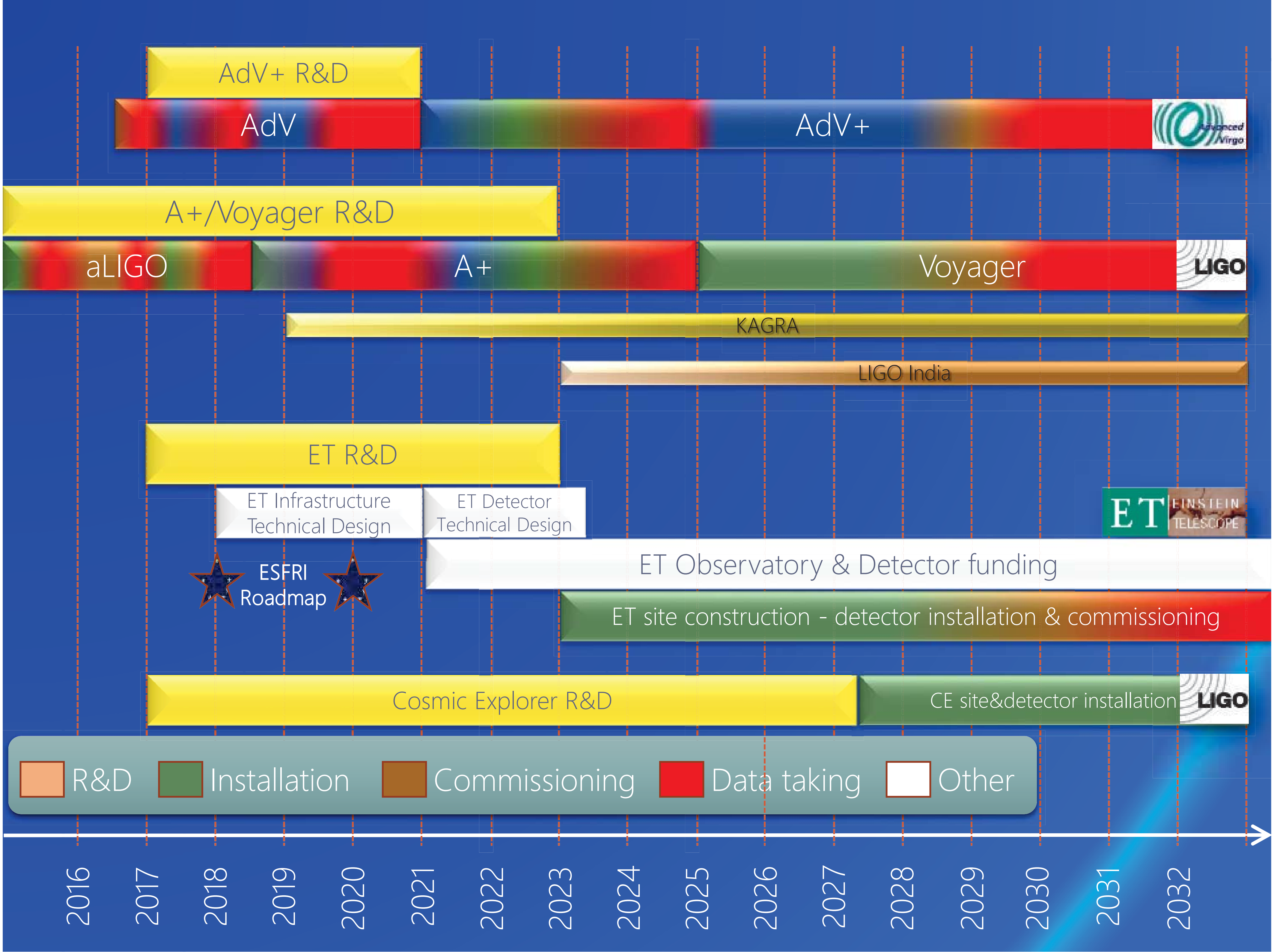
B.P.Abbott et al., Living Reviews in Relativity 19, 1 (2016)

Next Generation

- Advanced detectors will be upgraded to gain a factor 2/3 in the next decade, but the limit of the infrastructures are close
- New research infrastructure proposed in Europe
  - Einstein Telescope
    - Observatory vs Detector
    - Multi-detector facility
    - Capable to detect both the polarizations
    - 10 km arm, underground, cryogenic
    - Conceptual design study funded by EU in 2008-2011
- Cosmic Explorer/LIGO 3G quickly growing idea

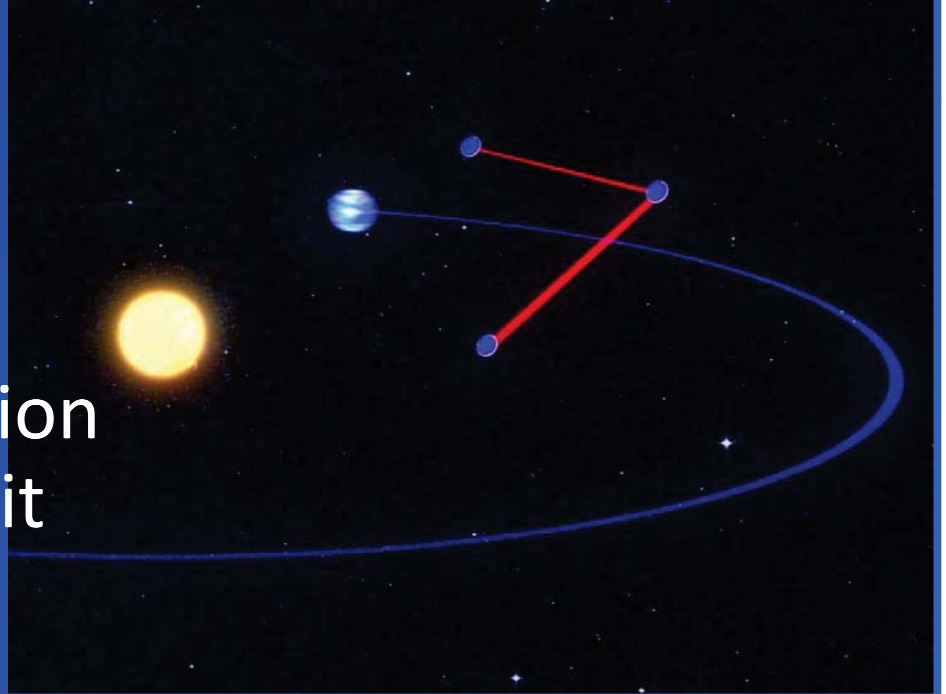
Einstein Telescope





AdV+ R&D
AdV
AdV+
A+/Voyager R&D
aLIGO
A+
Voyager
LIGO
KAGRA
LIGO India
ET R&D
ET Infrastructure Technical Design
ET Detector Technical Design
ET EINSTEIN TELESCOPE
ESFRI Roadmap
ET Observatory & Detector funding
ET site construction - detector installation & commissioning
Cosmic Explorer R&D
CE site&detector installation
LIGO
R&D
Installation
Commissioning
Data taking
Other
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032

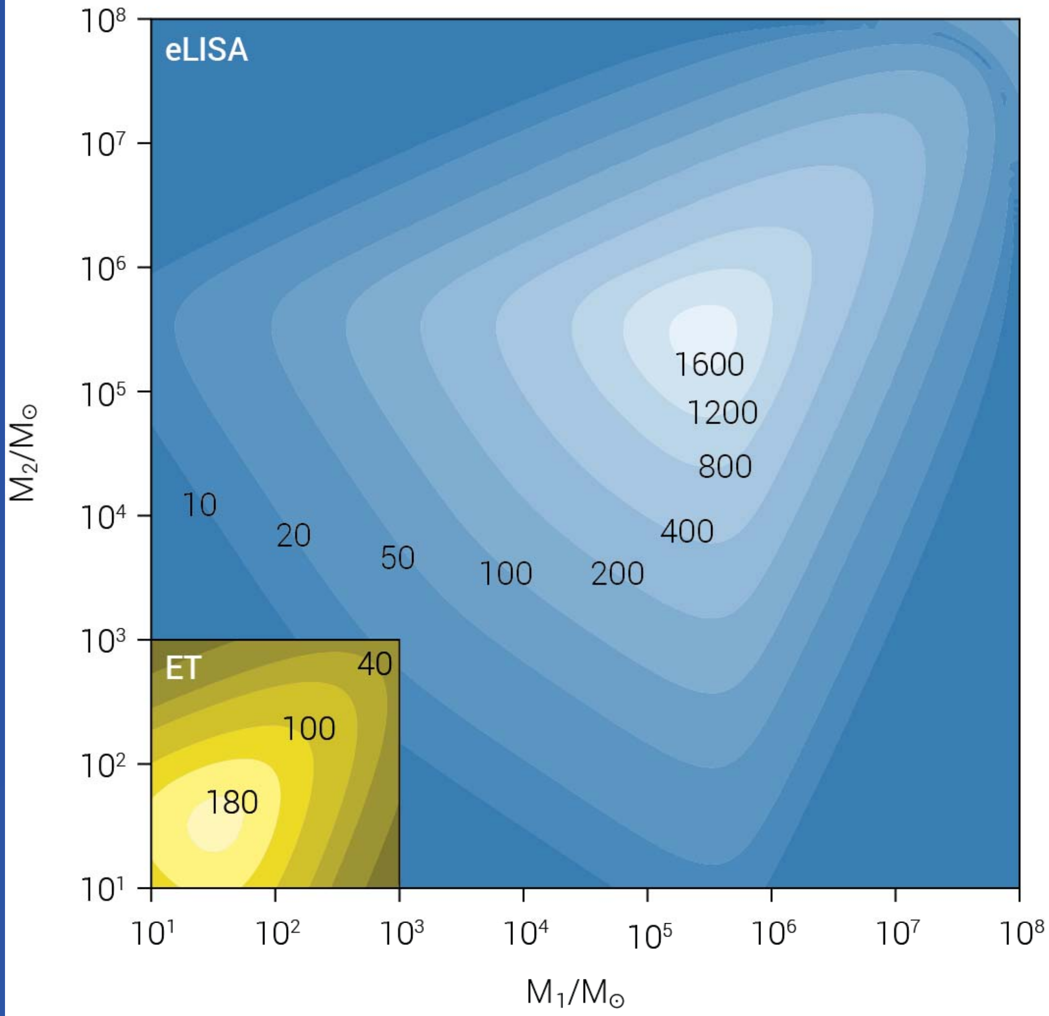
LISA

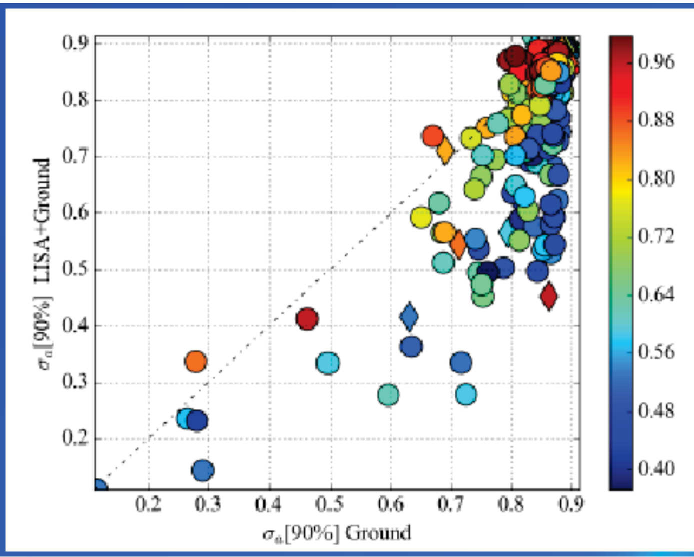


- LISA or now eLISA is a constellation of 3 satellites in heliocentric orbit
- It presents a 10^6 km arm lenght
- It is focused on 10^{-4}-10^{-1}Hz frequency range, corresponding to
  - signals produced by very massive objects:
    - Coalescences of ~$10^5 M_S$ black holes, internal modes
  - Signal of lighter object (stellar mass black holes, BNS, stars) far from the coalescence
- LISA will test the gravity in a very strong field regime
- It will have an astonishing SNR on its detections

eLISA-ET complementarities

- eLISA is scheduled to be launched in ≥2034 although an anticipation to ~2029 is under evaluation
- eLISA will be active in parallel to a rich network of 2G+/3G detectors
- They are complementary (different sources) and cooperative (same sources in different phases):
  - eLISA could indicate where stellar mass BHBH are coalescing years before the merging
  - Using eLISA constrained mass, mass ratio and location it is possible to improve the (other) parameter estimation (spin)





Salvatore Vitale
arXiv:1605.01037

Conclusions

- Gravitational wave detection is quite more than the final confirmation of GR, 100 years after Einstein's prediction
- It is the dawn of a new era,
  - where access to the internal astrophysical processes in cosmic catastrophes is finally possible
  - Where a new paradigm in the observation of the universe is defined

Nell'ultima frazione della collisione, la potenza emessa è pari a 100.000 miliardi di miliardi di soli

END

... spazio-tempo curvo (2)

- L'equazione di campo, che lega la causa della deformazione ($T_{\mu\nu}$, cioè la masso o l'energia) all'effetto della deformazione dello spazio-tempo è

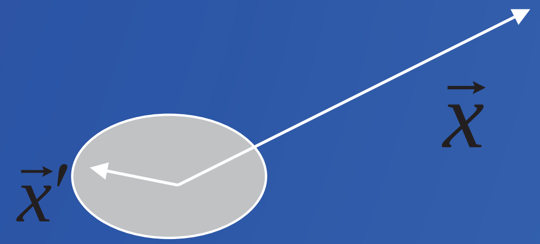
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

- Dove $R_{\mu\nu}$ è il tensore di Ricci (che nasce dalla contrazione del tensore di Rieman, legato alle deformazioni mareali) ed R la sua traccia. Il termine a sinistra dell'uguale è spesso indicato con un solo tensore deformazione $G_{\mu\nu}$.
- L'equazione di campo è un'equazione differenziale estremamente complicata, dove compaiono in maniera non lineare le derivate del tensore metrico
- Non c'è attualmente una soluzione analitica di tale equazione e si procede per risoluzione numerica approssimata (supercomputers!) o facendo la dovuta approssimazione lineare
- Per piccole deformazioni l'approssimazione lineare sembra la via giusta, ma sono piccole queste deformazioni?

Potenziale gravitazionale

- Consideriamo una distribuzione continua di massa, con densità *ρ(x')*. Supponiamo di valutare il potenziale gravitazionale ad una posizione ***x***, all'esterno della massa stessa:

$$\phi(\vec{x}) = -\int_V \frac{G \cdot \rho(x')}{|\vec{x} - \vec{x}'|} d^3x'$$



- Essendo ***x*** esterno alla massa, possiamo scrivere lo sviluppo in multipoli per il potenziale usando lo sviluppo in serie di Taylor di 1/|***x***-***x***'| in un intorno di ***x***'=0:

$$\frac{1}{|\vec{x} - \vec{x}'|} = \frac{1}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} = \frac{1}{r} + \sum_k \frac{x^k x'^k}{r^3} + \frac{1}{2} \sum_{k,l} \left(3x'^k x'^l - r'^2 \delta_k^l\right) \frac{x^k x^l}{r^5} + \ldots$$

- Dove $x^{1,2,3}=x,y,z$ e $r = \sqrt{x^2 + y^2 + z^2}$

- Il potenziale gravitazionale diventa:

$$\phi(\vec{x}) = -\frac{GM}{r} - \frac{G}{r^3} \sum_k x^k D^k - \frac{G}{2} \sum_{k,l} Q^{kl} \frac{x^k x^l}{r^5} + \ldots$$

Potenziale gravitazionale, termini di quadrupolo

- Dove: $M = \int_V \rho(x')d^3x'$ $D^k = \int_V x'^k \rho(x')d^3x'$ $Q^{kl} = \int_V \left(3x'^k x'^l - r'^2\delta_l^k\right)\rho(\vec{x}')d^3x'$
- E' sempre possibile scegliere l'origine del sistema di riferimento coincidente con il centro di massa in modo che i termini di dipolo D^k siano identicamente nulli
- Rimangono i termini di quadrupolo Q^{kl} tali da, se la distribuzione di massa consente $Q^{kl} \neq 0$, introdurre un termine $\propto r^{-3}$ nel potenziale (r^{-4} nella forza).
- La Terra presenta una differenza fra diametro polare e quello equatoriale di 3 parti su 10^3. Questo produce un termine di quadrupolo che influenza le orbite dei satelliti artificiali:
  - Precessione dell'ellisse di Keplero, cioè una piccola rotazione dell'ellisse intorno all'asse terrestre

Metric in Special Relativity

- In special relativity the space-time is flat and the distance is defined: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$
- With a change of notation: $x^0 \equiv ct, \quad x^1 \equiv x, \quad x^2 \equiv y, \quad x^3 \equiv z$

$$ds^2 = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3}\eta_{\mu\nu}dx^\mu dx^\nu \equiv \eta_{\mu\nu}dx^\mu dx^\nu$$

- Where $\eta_{\mu\nu}$ is the Minkowski metric tensor

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Equazioni di Maxwell

$$\vec{\nabla}\cdot\vec{D}=\rho$$ Legge di Gauss elettrica

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$ Legge di Faraday

$$\vec{\nabla}\cdot\vec{B}=0$$ Legge di Gauss magnetica

$$\frac{\partial\vec{D}}{\partial t}-\vec{\nabla}\times\vec{H}+\vec{J}=0$$ Legge di Ampère-Maxwell

$$\begin{cases} \nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=-\frac{\rho}{\varepsilon} \\ \nabla^2 A_x-\frac{1}{c^2}\frac{\partial^2 A_x}{\partial t^2}=-\mu\rho v_x \\ \nabla^2 A_y-\frac{1}{c^2}\frac{\partial^2 A_y}{\partial t^2}=-\mu\rho v_y \\ \nabla^2 A_z-\frac{1}{c^2}\frac{\partial^2 A_z}{\partial t^2}=-\mu\rho v_z \end{cases}$$

Velocità di propagazione dell'onda

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

In qualunque sistema di riferimento in moto relativo (uniforme)

ε_0 permettività elettrica del vuoto

μ_0 permeabilità magnetica del vuoto

What means 10^{-19}m?

ΔL expected in Virgo ~10^{-19}m

mm (10^{-3}m)

μm (10^{-6}m)

nm (10^{-9}m)

pm (10^{-12}m)

fm (10^{-15}m)

am (10^{-18}m)

Human cells

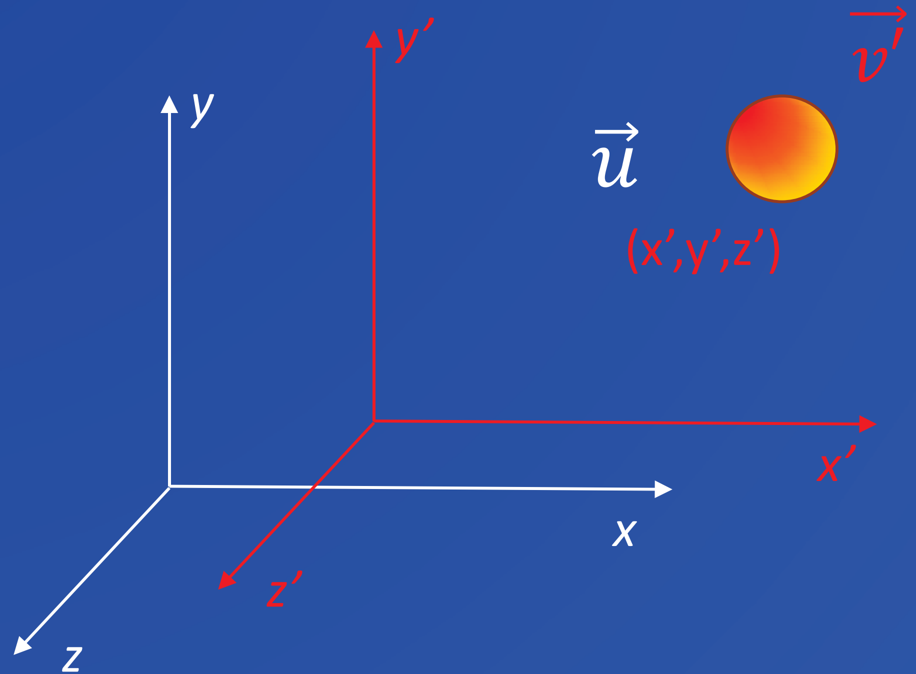
Hydrogen atom

Proton radius

Quark “size”

In 1km size detector

Trasformazioni: Galileiane e di Lorentz



$\vec{v}$?

Trasformazioni Galileiane

$$\begin{cases} x = x' + u_x t \\ y = y' \\ z = z' \\ t = t' \end{cases}$$

$$\begin{cases} v_x = v'_x + u_x \\ v_y = v'_y \\ v_z = v'_z \end{cases} \Rightarrow \vec{v} = \vec{v}' + \vec{u}$$

Trasformazioni Lorenziane

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \Rightarrow \begin{cases} x' = \dfrac{x - u_x t}{\sqrt{1 - \left(\frac{u_x}{c}\right)^2}} \\ y = y' \\ z = z' \\ t' = \dfrac{t - \frac{u_x}{c^2} x}{\sqrt{1 - \left(\frac{u_x}{c}\right)^2}} \end{cases}$$

Formalismo della Relatività Ristretta

- In RR abbiamo uno spazio-tempo quadridimensionale, in cui è definita la distanza:

$$dl^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$ (intervallo spazio-temporale)

- Per mantenere invariata la distanza, si introducono le trasformazioni di Lorentz:

$$ds'^2 = ds^2 \quad \Rightarrow \quad \begin{cases} ct' = \dfrac{ct - \dfrac{v}{c}x}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}} \\ x' = \dfrac{x - vt}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}} \\ y' = y \\ z' = z \end{cases}$$

- Principio di relatività:
  - *Tutte le leggi fisiche devono essere invarianti per trasformazioni di Lorentz*
  - Invarianza qui significa che ogni legge conserva la stessa forma matematica e che le costanti numeriche conservano lo stesso valore

...formalismo della Relatività Ristretta (2)

- Introduciamo la notazione: $x^0 \equiv ct, \quad x^1 \equiv x, \quad x^2 \equiv y, \quad x^3 \equiv z$
- La distanza diventa:

$$ds^2 = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu \equiv \eta_{\mu\nu} dx^\mu dx^\nu$$

- Dove si è usata la convenzione di somma sugli indici ripetuti (muti)
- Il termine $\eta_{\mu\nu}$ è la matrice:

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \end{pmatrix}$$

che prende il nome di *tensore metrico* o *tensore di Minkowski*.

- Definiamo:

$$x_\mu \equiv \eta_{\mu\nu} x^\nu \quad \Rightarrow \quad \begin{cases} x_0 = ct \\ x_1 = -x \\ x_2 = -y \\ x_3 = -z \end{cases} \quad \Rightarrow \quad ds^2 = dx^\mu dx_\mu$$

- Quantità con indici greci in alto sono dette controvarianti, quelle con indici in basso covarianti

…formalismo della Relatività Ristretta (3)

- Utilizzando questa notazione (e la convenzione c=1), la trasformazione di Lorentz sopracitata diventa:

$$x'^{\mu} = a^{\mu}{}_{\nu}\, x^{\nu}$$

- dove $a^{\mu}{}_{\nu}$ è la matrice ($^{\mu}$ è l'indice di riga, $_{\nu}$ di colonna):

$$a^{\mu}{}_{\nu} = \begin{pmatrix} \frac{1}{\sqrt{1-v^2}} & \frac{-v}{\sqrt{1-v^2}} & 0 & 0 \\ \frac{-v}{\sqrt{1-v^2}} & \frac{1}{\sqrt{1-v^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Grazie alla matrice $a^{\mu}{}_{\nu}$ delle trasformazioni di Lorentz, si può definire cos'è un tensore:
  - Un tensore di rango r è un oggetto $A^{\mu\nu\ldots\kappa}$ con 4^r componenti che per una trasformazione di Lorentz diventa:

$$A'^{\alpha\beta\ldots\gamma} = a^{\alpha}{}_{\mu}\, a^{\beta}{}_{\nu} \ldots a^{\gamma}{}_{\kappa}\; A^{\mu\nu\ldots\kappa}$$

- Tensore di rango 0:
  - uno scalare che rimane invariato per trasf. di Lorentz
  - La distanza ds^2
  - Il tempo proprio: $d\tau = \sqrt{dx^{\mu} dx_{\mu}} = \sqrt{dt^2 - dx^2 - dy^2 - dz^2} = dt\sqrt{1-v^2}$

…formalismo della Relatività Ristretta (4)

- Tensore di rango 1:
  - Un vettore a quattro componenti
  - La quadri-velocità $u^\mu = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v_x}{\sqrt{1-v^2}}, \frac{v_y}{\sqrt{1-v^2}}, \frac{v_z}{\sqrt{1-v^2}} \right)$
  - Il quadri-vettore energia-impulso per una particella di massa (a riposo) m:

$$p^\mu = \left(\frac{m}{\sqrt{1-v^2}}, \frac{mv_x}{\sqrt{1-v^2}}, \frac{mv_y}{\sqrt{1-v^2}}, \frac{mv_z}{\sqrt{1-v^2}} \right) = \left(\frac{E}{c}, p_x, p_y, p_z \right)$$

- Tensore di rango 2:
  - Una “matrice” a 16 componenti che segue le trasf. di Lorentz
  - Tensore energia-impulso per un sistema di particelle non interagenti. Sia n la densità di particelle e **v** la sua velocità. Le componenti $T^{\mu\nu}$ di tale tensore sono:

$$T^{00} = [\text{densità di energia}] = \frac{nm}{\sqrt{1-v^2}}$$

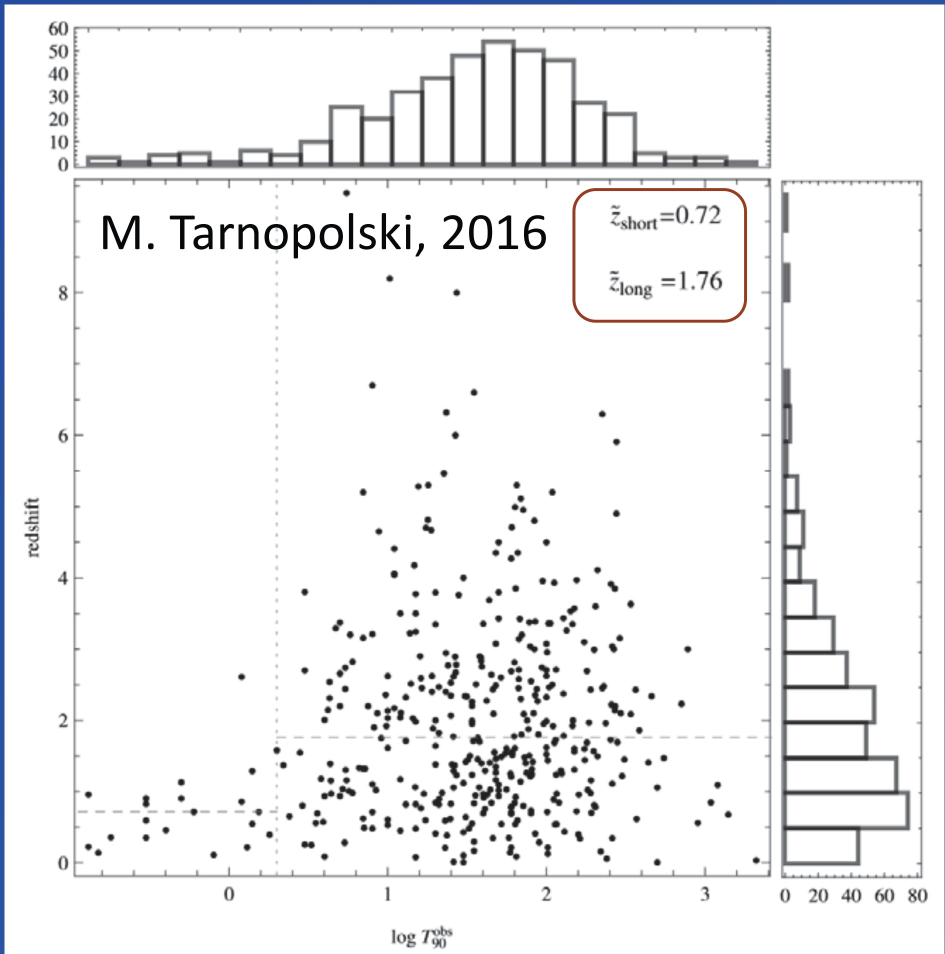
$$T^{0k} = T^{k0} = [\text{densità di impulso } k] = [\text{densità di flusso di energia}] = \frac{nmv^k}{\sqrt{1-v^2}}$$

$$T^{kl} = T^{lk} = [\text{densità di flusso di impulso } k \text{ in direzione } l] = \frac{nmv^k v^l}{\sqrt{1-v^2}}$$

- Definendo la densità di massa propria (cioè nel sistema di riposo locale delle particelle): $\rho_0 = mn\sqrt{1-v^2} \quad \Rightarrow \quad T^{\mu\nu} = \rho_0 u^\mu u^\nu$

Cosmography

- But, how far GWDs need to observe GW from BNS to have the coincidence?



- We need 10^5 of coincidences at high z and high SNR
  - Future detectors (ET)

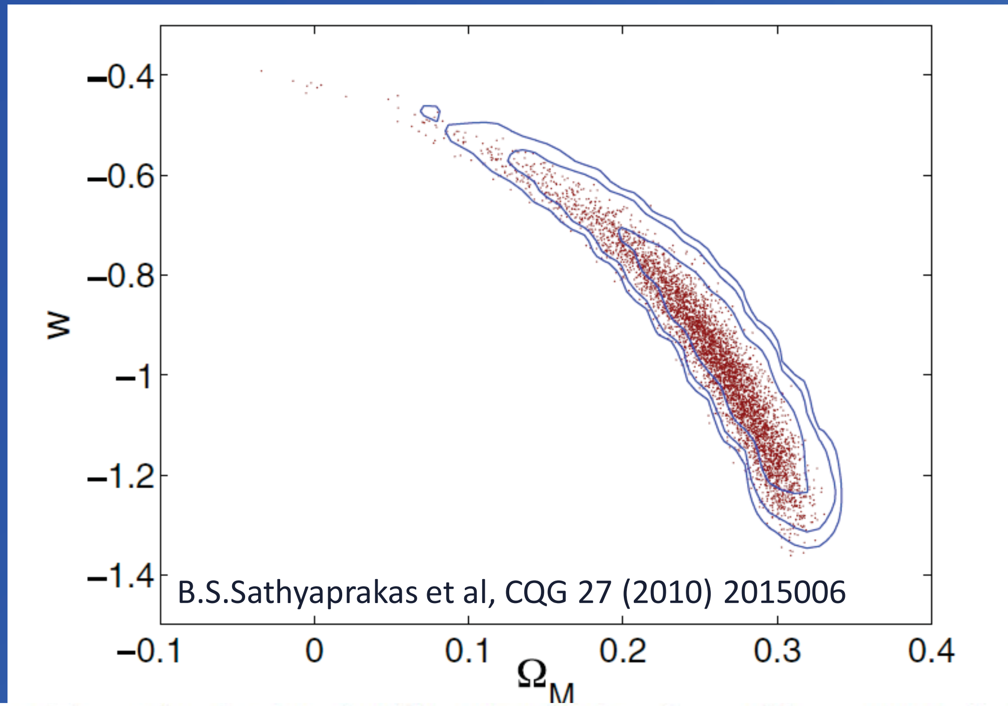


Figure 3. Scatter plot of the retrieved values for (Ω_Λ, w), with 1-σ, 2-σ and 3-σ contours, in the case where weak lensing is not corrected.

Timescale of Telescopes, Missions, Surveys