# Testing the Standard Model with the lepton g-2

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# Outline

- $\mu$: The muon g-2: recent theory progress
- e: Testing the Standard Model with the electron g-2
- $\tau$: The tau g-2: opportunities or fantasies? Surprises?

# Lepton magnetic moments: the basics

# The beginning: g = 2

- **Uhlenbeck and Goudsmit in 1925 proposed:**

$$\boxed{\begin{aligned} \vec{\mu} &= g\frac{e}{2mc}\vec{s} \\ g &= \underline{2 \quad (\text{not } 1!)} \end{aligned}}$$

- **Dirac 1928:**

$$(i\partial_\mu - eA_\mu)\gamma^\mu\psi = m\psi$$

- **A Pauli term in Dirac's eq would give a deviation…**

$$a\frac{e}{2m}\sigma_{\mu\nu}F^{\mu\nu}\psi \quad \rightarrow \quad g = 2(1+a)$$

**…but there was no need for it! g=2 stood for ~20 yrs.**

# Theory of the g-2: Quantum Field Theory

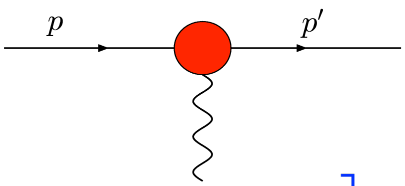
- **Kusch and Foley 1948:**

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc}\,(1.00119 \pm 0.00005)$$

- **Schwinger 1948 (triumph of QED!):**

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc}\left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- **Keep studying the lepton–γ vertex:**




$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \dots\right]u(p)$$

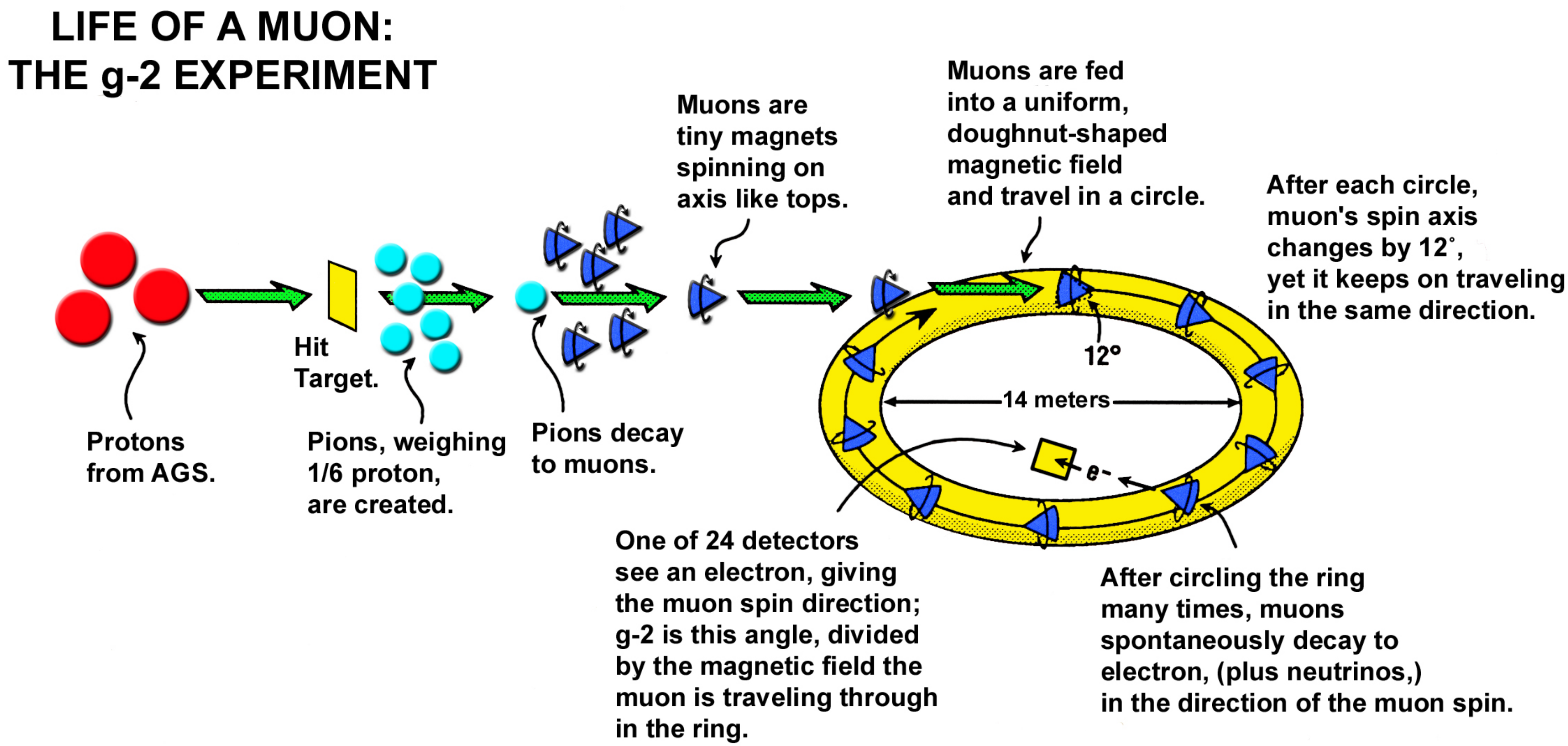
$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

A pure “quantum correction” effect!

$\mu$

## The muon g-2 experiment

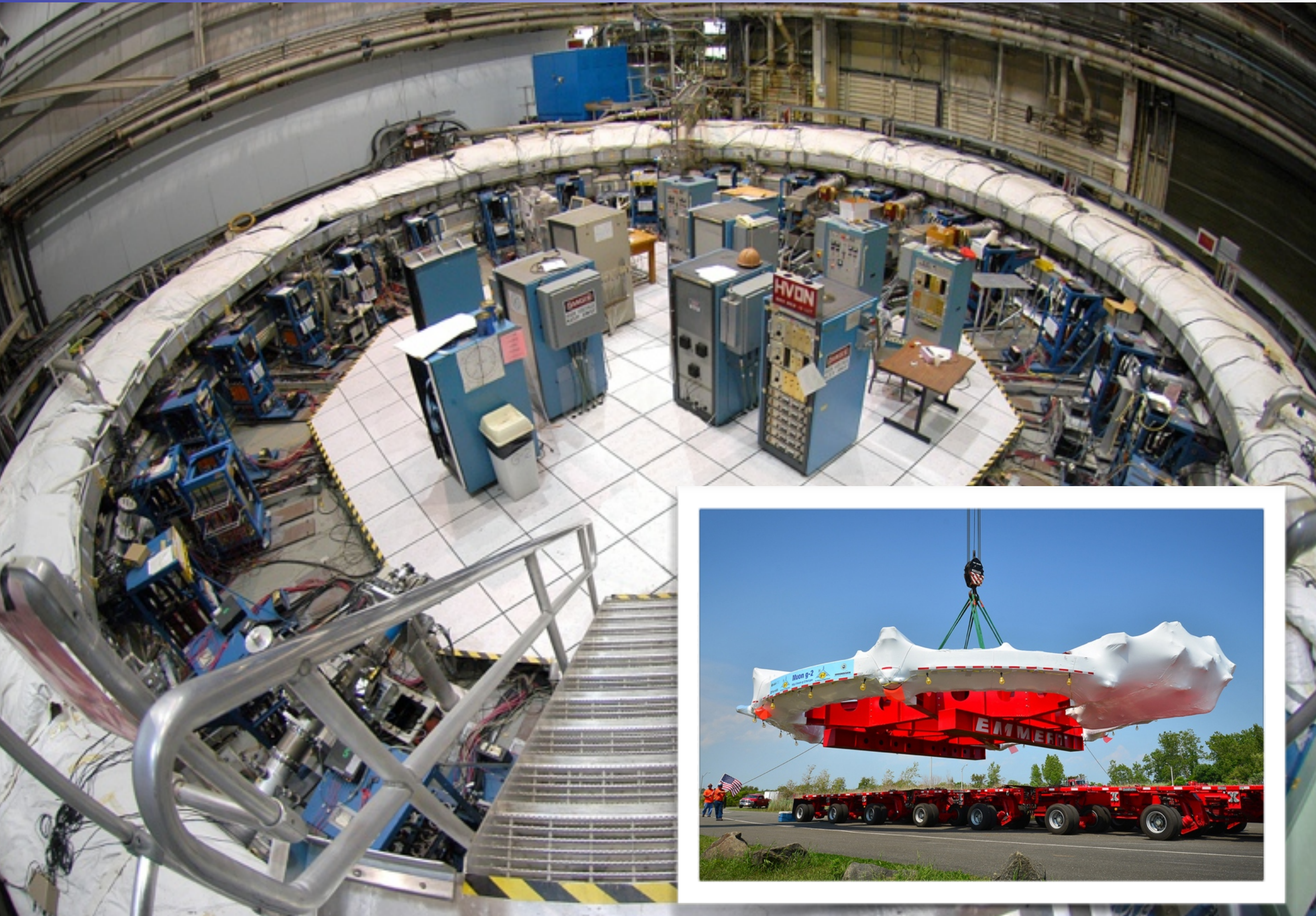
#### The old experiment E821

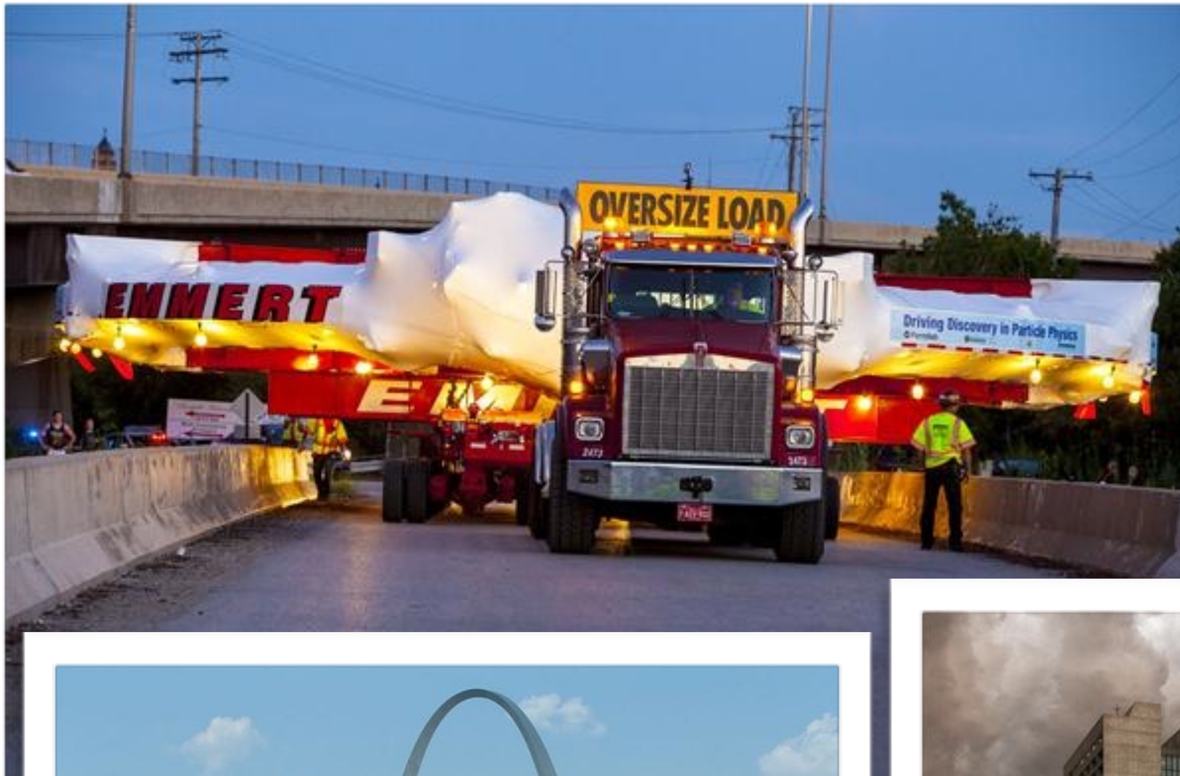



**E821 @ BNL**

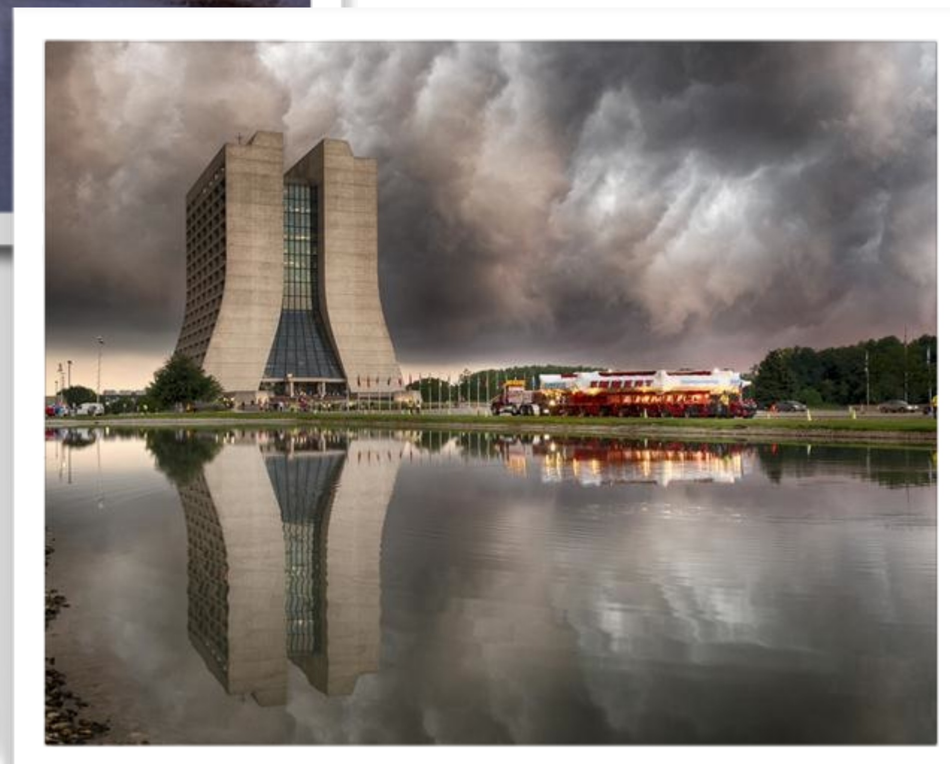
# The old experiment E821 (2)

μ



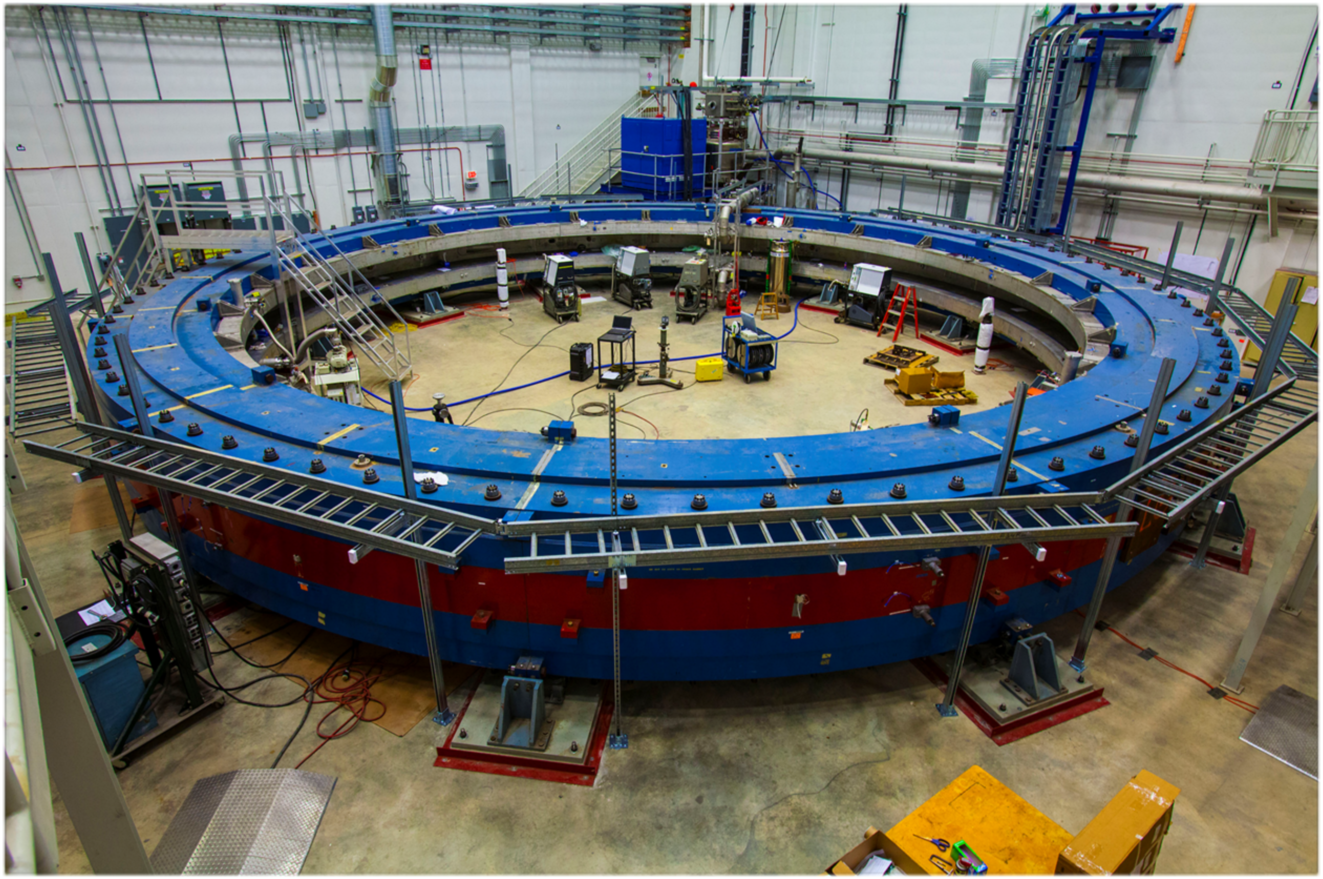



**The Big Move:**
**BNL to Fermilab**
**Summer 2013**



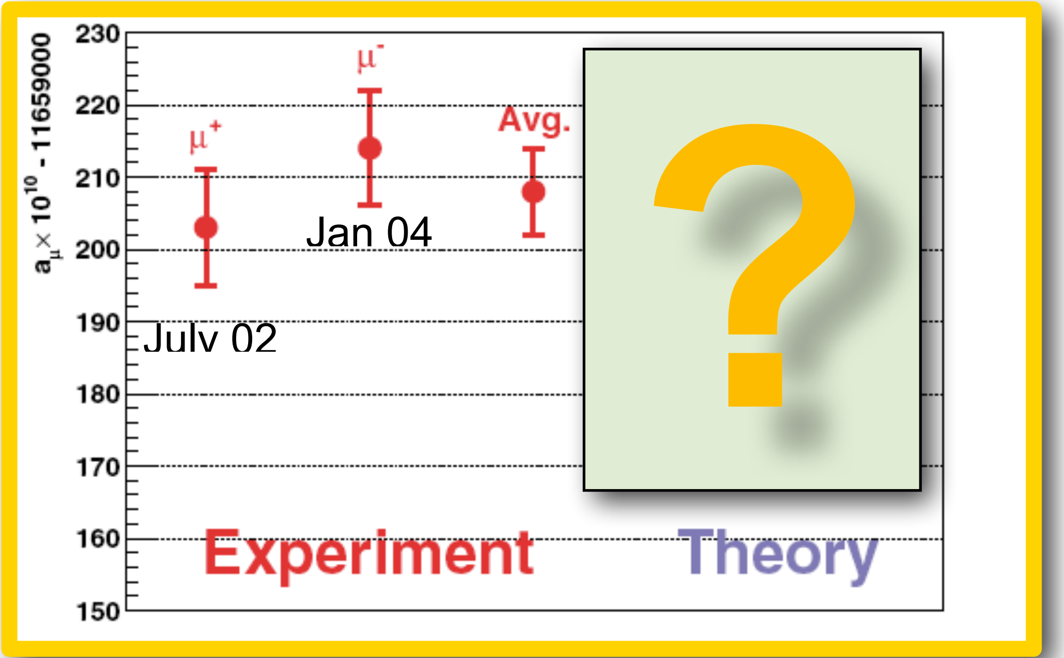
μ

# Fermilab: 2015



**Ring fully assembled and connected to cryogenic system; magnet powered**

# The muon g-2: experimental status




- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments at:
  - **Fermilab E989:** aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm. Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.
  - **J-PARC proposal:** aiming at 2019 Phase 1 start with 0.4ppm.
- Are theorists ready for this (amazing) precision? **Not yet**

# The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \ 24.05050988\ (28)\ (\alpha/\pi)^3$

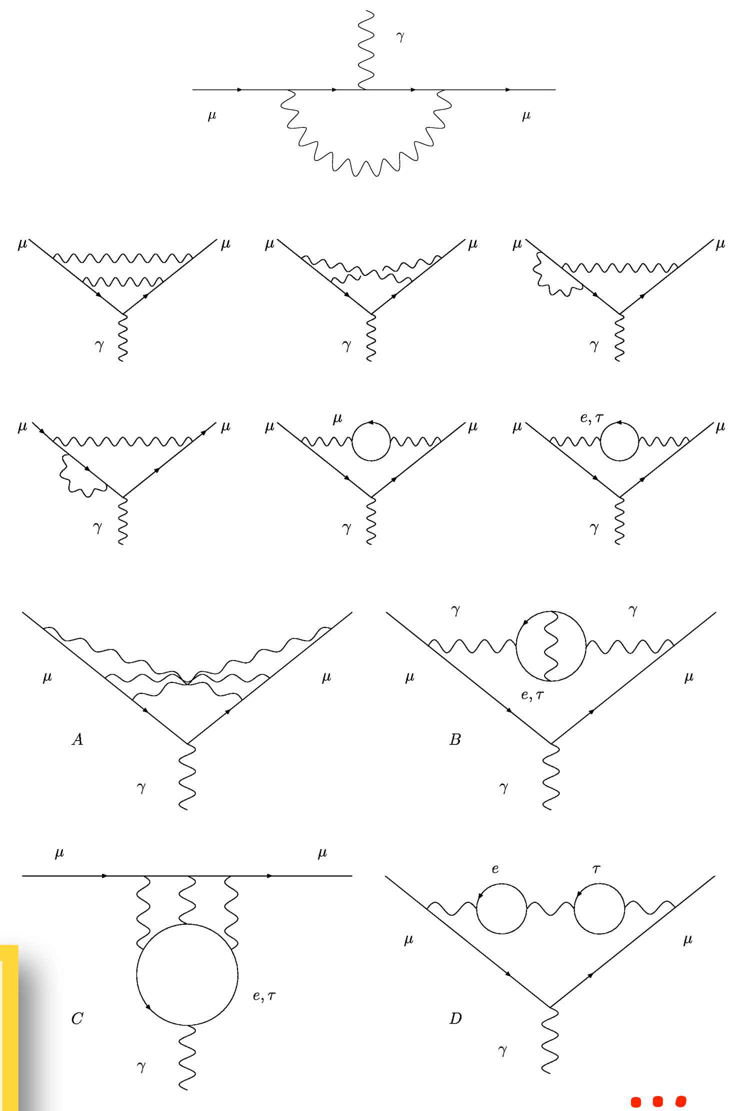
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \ 130.8773\ (61)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Lee, Marquard, Smirnov$^2$, Steinhauser 2013 (electron loops, analytic),
Kurz, Liu, Marquard, Steinhauser 2013 ($\tau$ loops, analytic);
Steinhauser et al. 2015 & 2016 (all electron & $\tau$ loops, analytic).

$+ \ 752.85\ (93)\ (\alpha/\pi)^5$ **COMPLETED!**

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,…, Kataev, Kinoshita & Nio '06; Kinoshita et al. 2012 & 2015



## Adding up, we get:

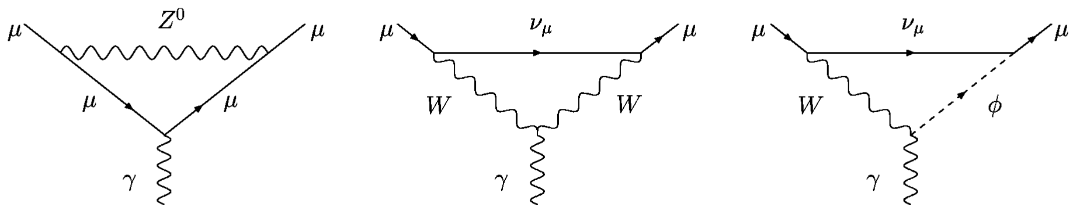
$a_\mu^{QED} = 116584718.941\ (21)(77) \times 10^{-11}$

from coeffs, mainly from 4-loop unc ↲ ↳ from $\delta\alpha$(Rb)

with $\alpha = 1/137.035999049(90)$ [0.66 ppb]

# The muon g-2: the electroweak contribution

- **One-loop term:**




$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

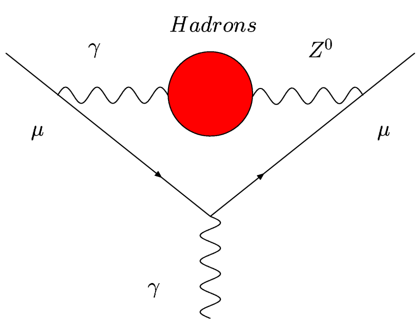
- **One-loop plus higher-order terms:**

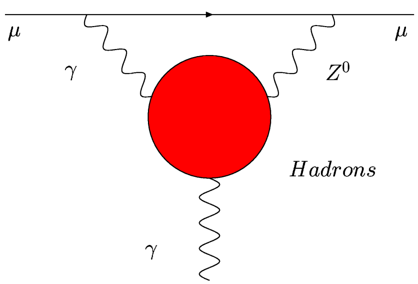
$$\boxed{a_\mu^{\mathrm{EW}} = 153.6\ (1) \times 10^{-11}}$$

with $M_{\mathrm{Higgs}}$ = 125.6 (1.5) GeV

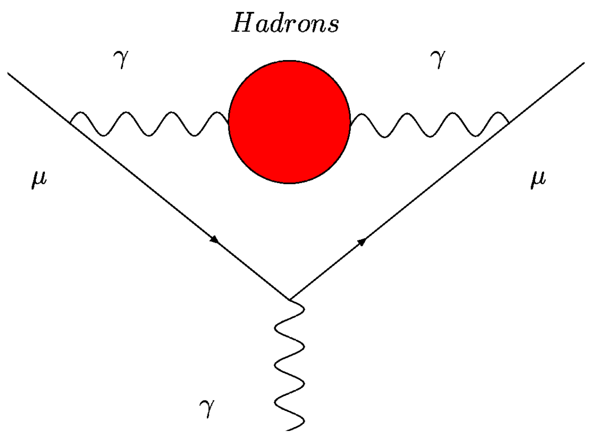
Hadronic loop uncertainties
and 3-loop nonleading logs.

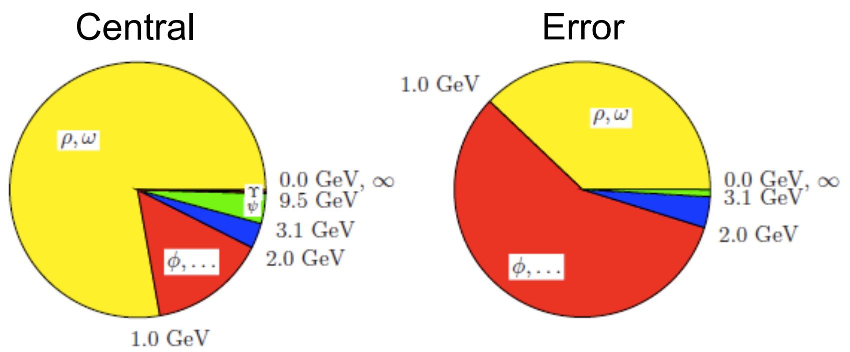
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



# The muon g-2: the hadronic LO contribution (HLO)




F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\,\sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s} K(s)R(s)$$

$a_\mu^{\mathrm{HLO}} = 6870\ (42)_{\mathrm{tot}} \times 10^{-11}$ — F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)

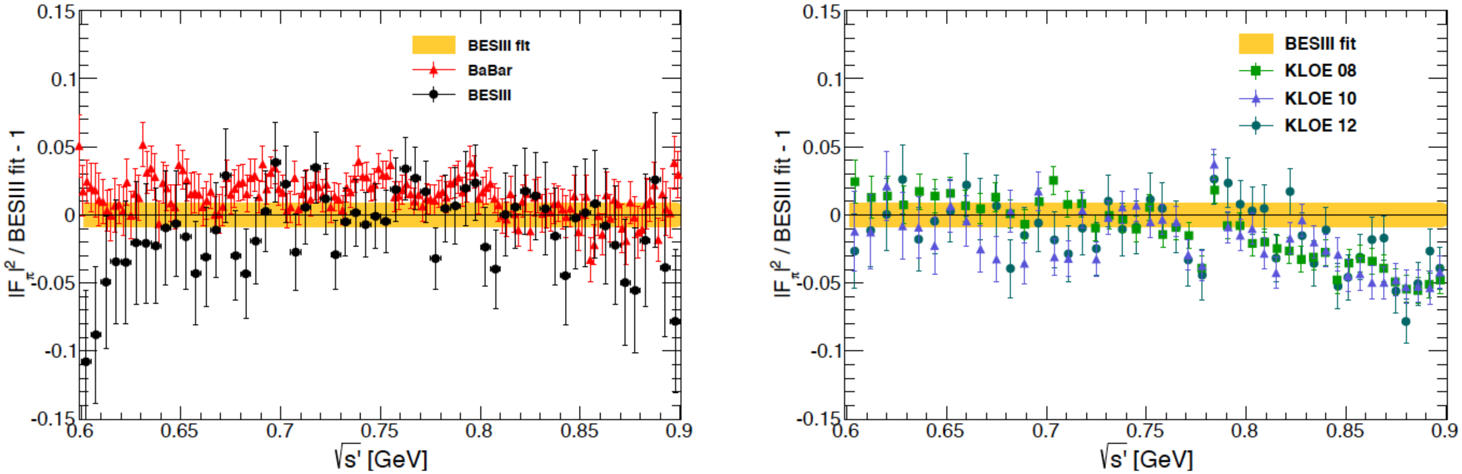
$= 6923\ (42)_{\mathrm{tot}} \times 10^{-11}$ — Davier et al, EPJ C71 (2011) 1515

$= 6949\ (37)_{\mathrm{exp}}\ (21)_{\mathrm{rad}} \times 10^{-11}$ — Hagiwara et al, JPG 38 (2011) 085003

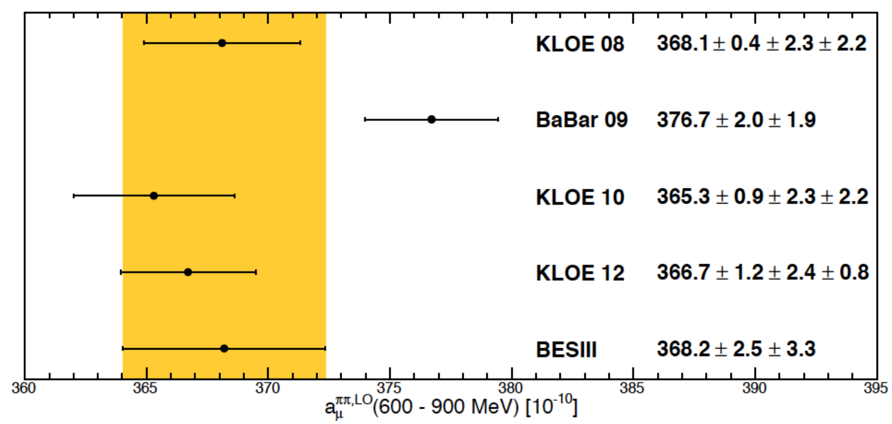
- **Radiative Corrections are crucial!**
- **BabaYaga MC event generator developed in Pavia**

Balossini, Barzè, Bignamini, Carloni Calame, Montagna, Nicrosini, Piccinini

# New from BESIII: measurement of the $e^+e^- \rightarrow \pi^+ \pi^-$ cross section between 600 & 900 MeV using initial state radiation




BESIII Collaboration, arXiv:1507.08188 (PLB 2016)



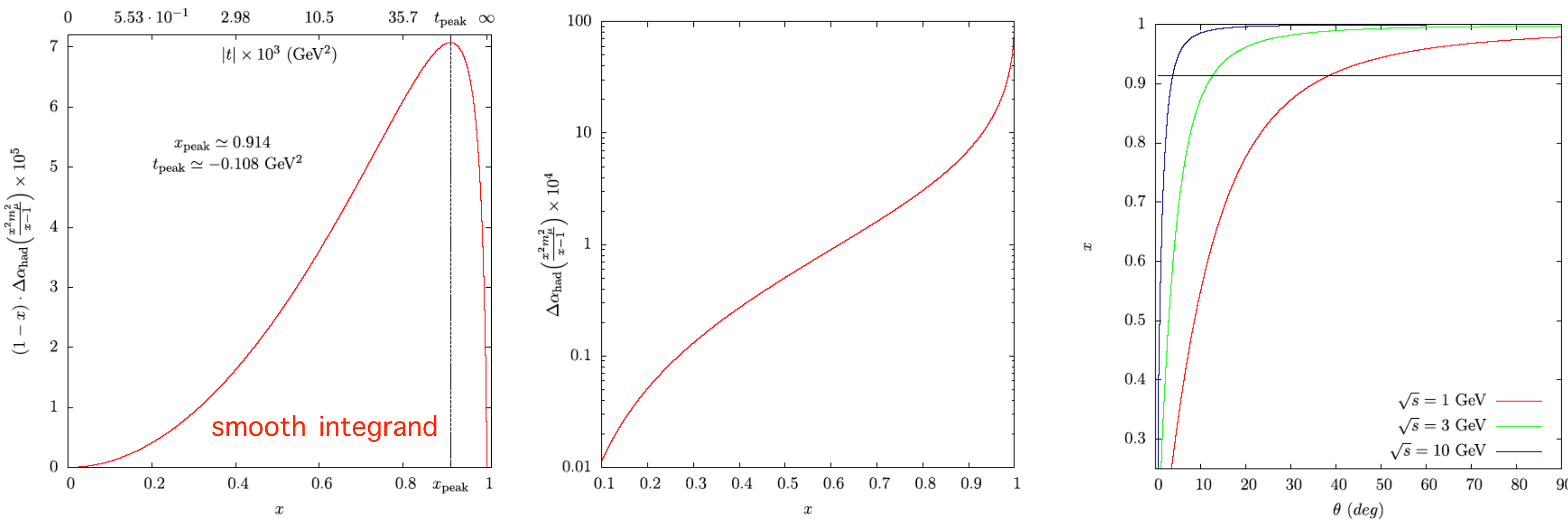

## Upcoming $e^+e^- \rightarrow \pi^+ \pi^-$ cross section data from VEPP 2000

# New independent space-like approach for HLO

- **Alternatively, exchanging the x and s integrations in $a_\mu^{HLO}$,**

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)] \qquad\qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

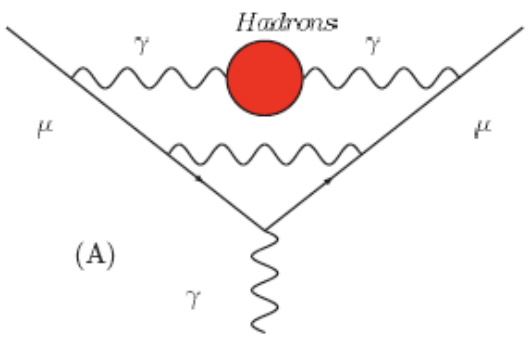
**involving the hadronic contrib. to the running of $\alpha$ in the space-like region, which can be extracted from Bhabha scattering data!**

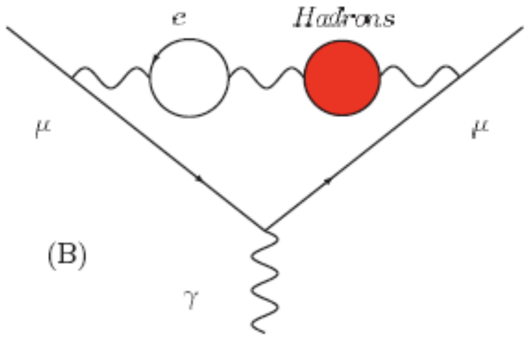



Carloni Calame, MP, Trentadue, Venanzoni, PLB 746 (2015)

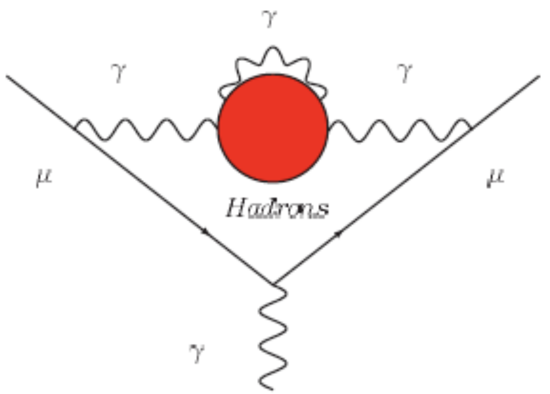
- **Requires Bhabha cross section at small angles at better than $10^{-4}$. Challenging: must improve by at least 1 order of magnitude.**
- **Dedicated feasibility study in progress.** Carloni Calame, Montagna, Nicrosini, MP, Piccinini, Trentadue, Venanzoni.

- ## HNLO: Vacuum Polarization




Already included in $a_\mu^{HLO}$

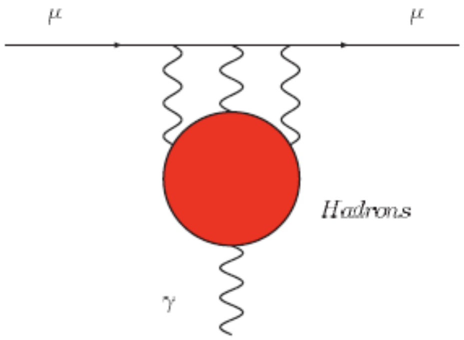



**O($\alpha^3$) contributions of diagrams containing hadronic vacuum polarization insertions:**

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause ’96, Alemany et al. ’98, Hagiwara et al. 2011

# HNLO: Light-by-light contribution




- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.
- This term had a troubled life! Latest values:

$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$ Knecht & Nyffeler '02

$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$ Melnikov & Vainshtein '03

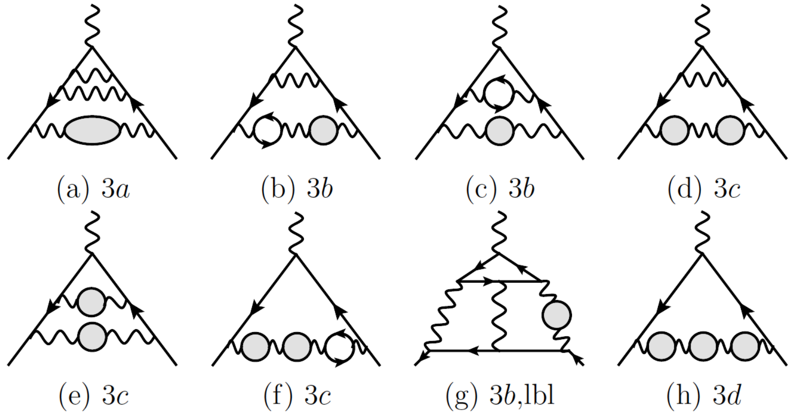
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$ Prades, de Rafael, Vainshtein '09

$a_\mu^{HNLO}(lbl) = +102\ (39) \times 10^{-11}$ Jegerlehner, arXiv:1511.04473

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- Improvements expected in the $\pi^0$ transition form factor A. Nyffeler 1602.0339
- Dispersive approach proposed Colangelo, Hoferichter, Procura, Stoffer, 2014 & 2015 Pauk and Vanderhaeghen 2014.
- Lattice? Very hard but promising Tom Blum et al. 2015

## HNNLO: Vacuum Polarization




**O($\alpha^4$) contributions of diagrams containing hadronic vacuum polarization insertions:**

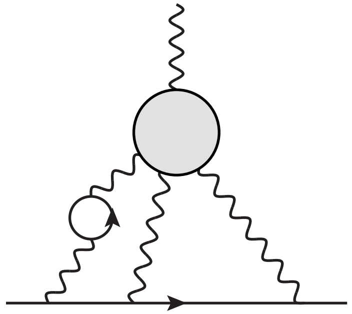
$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

## HNNLO: Light-by-light

$$a_\mu^{HNNLO}(lbl) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



# The muon g-2: SM vs. Experiment

## Comparisons of the SM predictions with the measured g-2 value:

$a_\mu^{EXP} = 116592091\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda = \mu_\mu/\mu_p$ from CODATA'10

| $a_\mu^{SM} \times 10^{11}$ | $\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$ | $\sigma$ |
|---|---|---|
| $116\,591\,795\ (56)$ | $296\ (86) \times 10^{-11}$ | $3.5$ [1] |
| $116\,591\,815\ (57)$ | $276\ (85) \times 10^{-11}$ | $3.2$ [2] |
| $116\,591\,841\ (58)$ | $250\ (86) \times 10^{-11}$ | $2.9$ [3] |

with the very recent "conservative" hadronic light-by-light $a_\mu^{HNLO}(lbl) = 102\ (39) \times 10^{-11}$ of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

[1] Jegerlehner, arXiv:1511.04473 (includes BaBar, KLOE10-12 & BESIII $2\pi$)
[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 $2\pi$)
[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 $2\pi$)

- Can $\Delta a_\mu$ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$\boxed{\begin{aligned} a &= \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), & f(s) &= \frac{K(s)}{4\pi^3},\ s_u < M_Z^2, \\ b &= \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), & g(s) &= \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}}$$

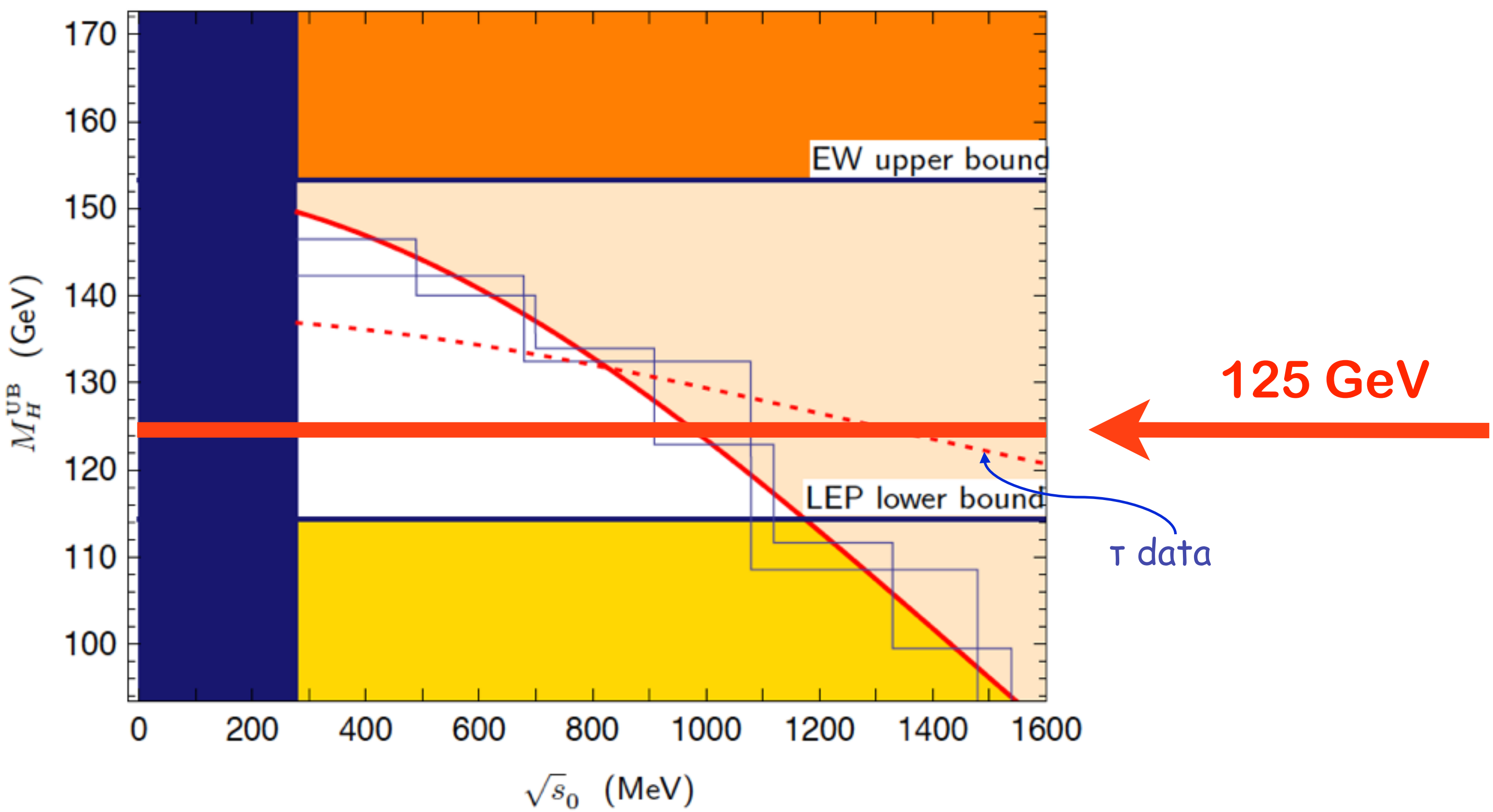
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\epsilon>0$), in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

- **How much does the $M_H$ upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate $\Delta a_\mu$ ?**




**W.J. Marciano, A. Sirlin, MP, 2008 & 2010**

# The muon g-2: connection with the SM Higgs mass (2)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

- Vice versa, assuming we now have a SM Higgs with $M_H$ = 125 GeV, if we bridge the $M_H$ discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010

# Testing the SM with the electron g-2

# The QED prediction of the electron g-2

$$a_e^{QED} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\ldots$$

O($10^{-18}$) in $a_e$

$$A_2^{(4)}\,(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$$

$$A_2^{(4)}\,(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$$

$$+\ 1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$$

O($10^{-19}$) in $a_e$

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\ldots$$

$$A_2^{(6)}\,(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$$

$$A_2^{(6)}\,(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$$

$$A_3^{(6)}\,(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$$

$$-\ 1.91206\,(84)\,(\alpha/\pi)^4$$

0.2 $10^{-13}$ in $a_e$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012 & 2015; Kurz, Liu, Marquard & Steinhauser 2014: analytic heavy virtual lepton part.

$$+\ 7.79\,(34)\,(\alpha/\pi)^5$$

**Complete Result! (12672 mass indep. diagrams!)**

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; PRD 91 (2015) 3, 033006

0.2 $10^{-13}$ in $a_e$

**NB: $(\alpha/\pi)^6 \sim O(10^{-16})$**

# The SM prediction of the electron g-2

e

- The SM prediction is:

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from Codata10]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{HAD} = 17.10\,(17) \times 10^{-13}$$

$$a_e^{HLO} = +\,18.66\,(11) \times 10^{-13}$$

$$a_e^{HNLO} = [-2.234(14)_{vac} + 0.39(13)_{lbl}] \times 10^{-13}$$

$$a_e^{HNNLO} = +\,0.28\,(1) \times 10^{-13}$$

- Which value of $\alpha$ should we use to compute $a_e^{SM}$?

# The electron g-2 gives the best determination of alpha



- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8$\sigma$ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $\boxed{a_e^{SM}(\alpha) = a_e^{EXP}}$ → best determination of alpha (2015):

$$\alpha^{-1} = 137.035\,999\,157\ (33) \qquad [0.24 \text{ ppb}]$$

- Compare it with other determinations (independent of $a_e$):

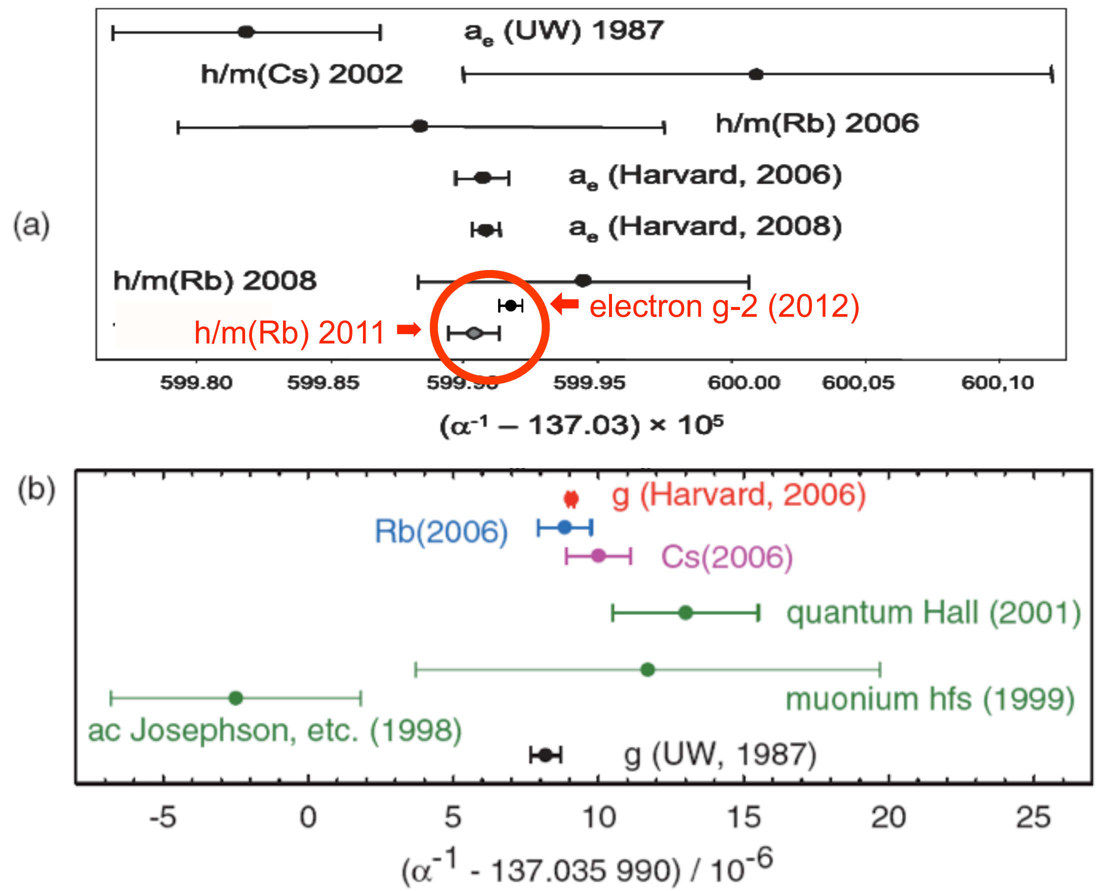
$\alpha^{-1} = 137.036\,000\,0\ (11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

$\alpha^{-1} = 137.035\,999\,049\ (90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

# Old and new determinations of alpha

e




Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801
Bouchendira et al, PRL106 (2011) 080801

- Using $\alpha$ = 1/137.035 999 049 (90) [$^{87}$Rb, 2011], the SM prediction for the electron g-2 is

$$a_e^{SM} = 115\,965\,218\,16.5\ \underbrace{(0.2)}_{\delta C_4^{qed}}\ \underbrace{(0.2)}_{\delta C_5^{qed}}\ \underbrace{(0.2)}_{\delta a_e^{had}}\ \underbrace{(7.6)}_{\text{from } \delta\alpha} \times 10^{-13}$$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -9.2\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1$\sigma$).
NB: The 4-loop contrib. to $a_e^{QED}$ is $-556 \times 10^{-13} \sim 70\ \delta\Delta a_e$!
(the 5-loop one is $6.2 \times 10^{-13}$)

# The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.2)_{\text{QED4}}, \quad (0.2)_{\text{QED5}}, \quad (0.2)_{\text{HAD}},}_{(0.4)_{\text{TH}}} \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

← may drop to 0.2

- The $(g\text{-}2)_e$ exp. error may soon drop below $10^{-13}$ and work is in progress for a significant reduction of that induced by $\delta\alpha$.

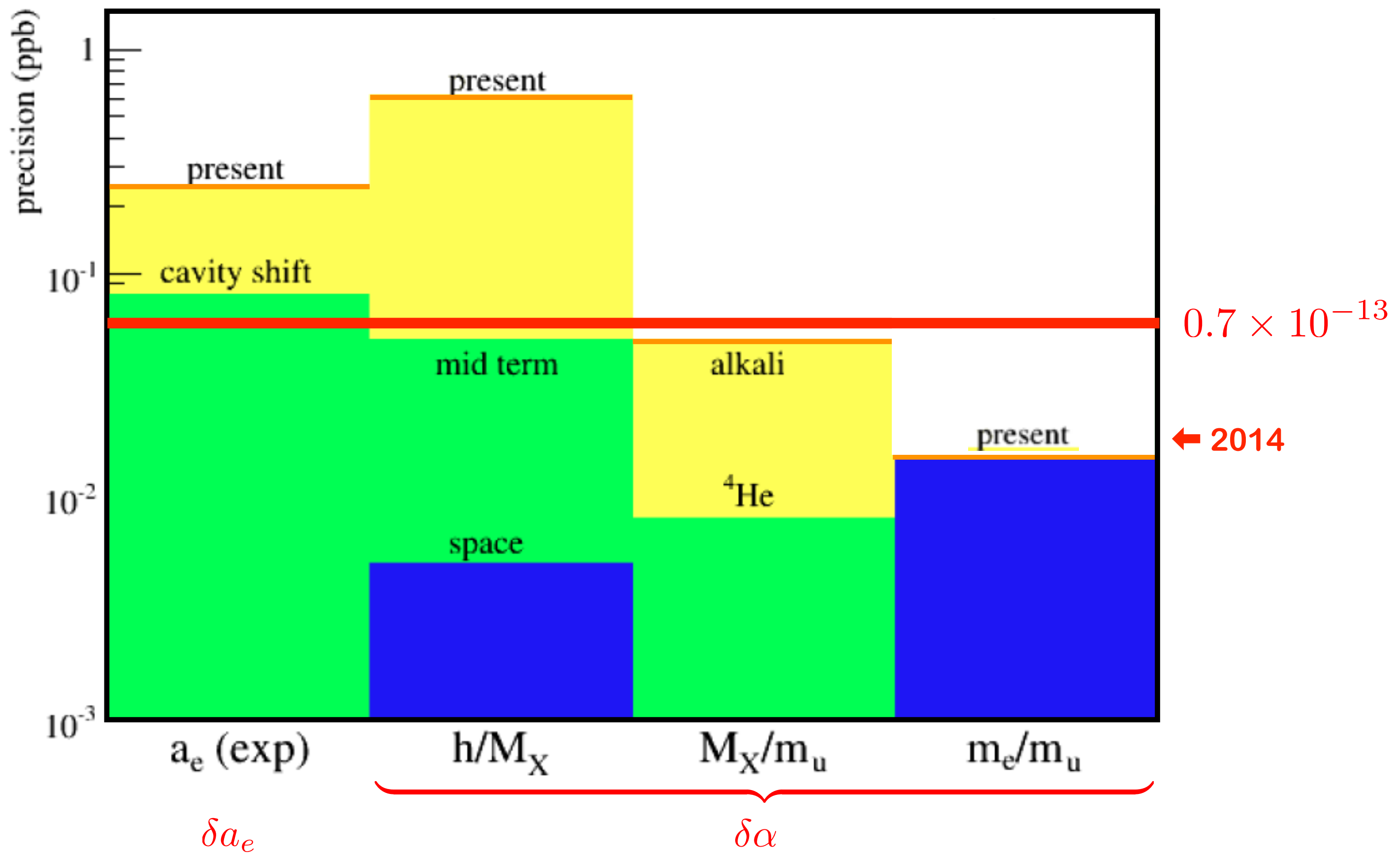
→ sensitivity of $10^{-13}$ may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to $a_l$ scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\boxed{\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}}$$




**Summary of the exp. contributions to the relative uncertainty of δα and δaₑ (in ppb).**

F. Terranova & G.M. Tino, PRA89 (2014) 052118

# The electron g-2 sensitivity and NP tests (3)

- The experimental sensitivity in $\Delta a_e$ is not very far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in $\Delta a_e$ and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), $\Delta a_e$ can reach $10^{-12}$ (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

Giudice, Paradisi & MP, arXiv:1208.6583

# The tau g-2: opportunities or fantasies?

# The SM prediction of the tau g-2



- The Standard Model prediction of the tau g-2 is:

| | | | | |
|---|---|---|---|---|
| $a_\tau^{SM}$ = | 117324 (2) | x $10^{-8}$ | QED |
| + | 47.4 (0.5) | x $10^{-8}$ | EW |
| + | 337.5 (3.7) | x $10^{-8}$ | HLO |
| + | 7.6 (0.2) | x $10^{-8}$ | HHO (vac) |
| + | 5 (3) | x $10^{-8}$ | HHO (lbl) |

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

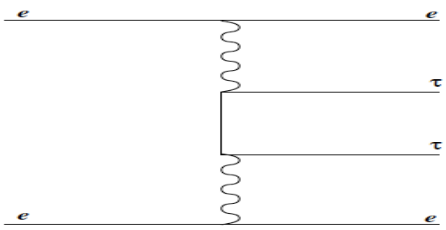
| | Muon | Tau |
|---|---|---|
| aEW/aH | 1/45 | 1/7 |
| aEW/ δaH | 3 | 10 |

… if only we could measure it!!

# The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine $a_\tau$ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):




$$\boxed{a_\tau = -0.018\ (17)}$$

PDG 2014

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$\boxed{-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ CL)}$$

Gonzáles-Sprinberg et al 2000

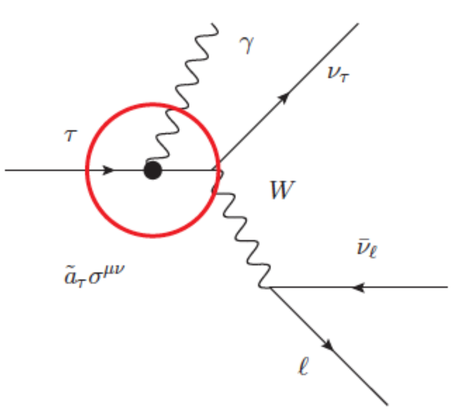
- Bernabéu et al, propose the measurement of $F_2(q^2=M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

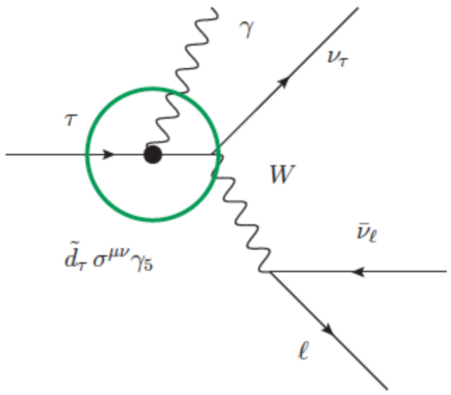
# A new proposal: the τ g-2 via τ radiative leptonic decays

$\tau$

- $a_\tau$ via the radiative leptonic decays $\tau \to e\bar{\nu}\nu\gamma, \ \tau \to \mu\bar{\nu}\nu\gamma$ comparing the theoretical prediction for the differential decay rates with precise data from high-luminosity B factories:

$$\boxed{d\Gamma = d\Gamma_0 + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\rm NLO} + \tilde{a}_\tau \, d\Gamma_{\rm a} + \tilde{d}_\tau \, d\Gamma_{\rm d}}$$




- Detailed feasibility study performed in Belle-II conditions: we expect a (modest) improvement of the present PDG bound.

Eidelman, Epifanov, Fael, Mercolli, MP, arXiv:1601.07987 (JHEP 2016)

# Radiative leptonic tau decays: branching ratios

$\tau$

B.R. of radiative $\tau$ leptonic decays ($\omega_0 = 10$ MeV)

| | $\tau \to e\bar{\nu}\nu\gamma$ | $\tau \to \mu\bar{\nu}\nu\gamma$ |
|---|---|---|
| $\mathcal{B}_{\rm LO}$ | $1.834 \times 10^{-2}$ | $3.663 \times 10^{-3}$ |
| $\mathcal{B}^{\rm Inc}_{\rm NLO}$ | $-1.06\,(1)_n(10)_N \times 10^{-3}$ | $-5.8\,(1)_n(2)_N \times 10^{-5}$ |
| $\mathcal{B}^{\rm Exc}_{\rm NLO}$ | $-1.89\,(1)_n(19)_N \times 10^{-3}$ | $-9.1\,(1)_n(3)_N \times 10^{-5}$ |
| $\mathcal{B}^{\rm Inc}$ | $1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$ | $3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$ |
| $\mathcal{B}^{\rm Exc}$ | $1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$ | $3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$ |
| $\mathcal{B}^{\dagger}_{\rm EXP}$ | $1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$ | $3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$ |

$(n)$: numerical errors

$(N)$: uncomputed NNLO corr.

$\sim (\alpha/\pi)\ln r \ln(\omega_0/M) \times \mathcal{B}^{\rm Exc/Inc}_{\rm NLO}$

†BABAR - PRD 91 (2015) 051103

(th): combined $(n) \oplus (N)$

$(\tau)$: experimental error of $\tau$ lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$ s

| | $\tau \to e\bar{\nu}\nu\gamma$ | $\tau \to \mu\bar{\nu}\nu\gamma$ |
|---|---|---|
| $\Delta^{\rm Exc}$ | $2.02\,(57) \times 10^{-3} \to 3.5\sigma$ | $1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$ |

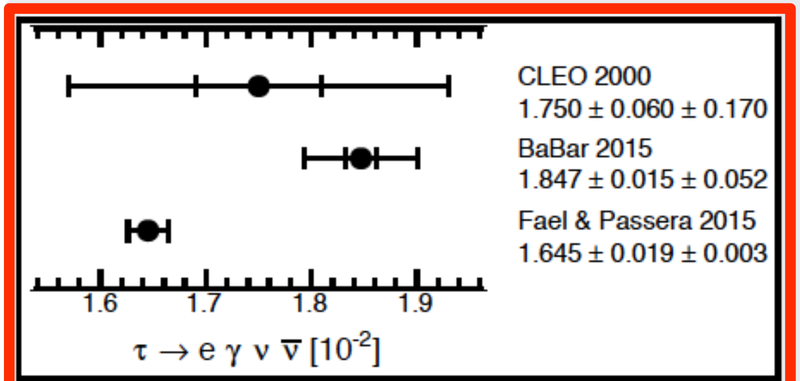
- **Agreement with MEG's recent μ→eννγ measurement [EPJ C76 (2016) 3, 108]**

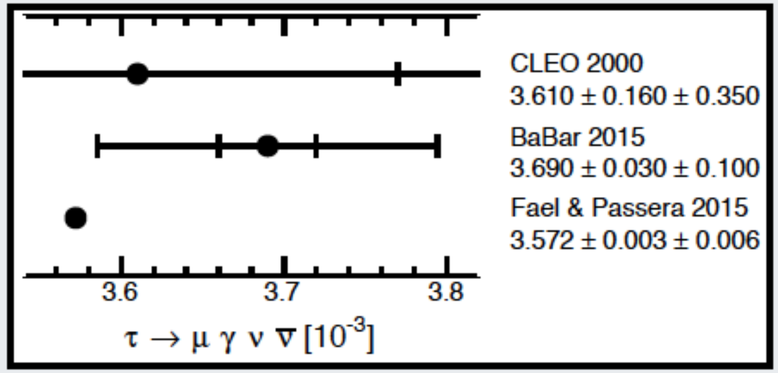
**Fael, Mercolli and MP, 1506.03416 (JHEP 2015)**
**Fael and MP, 1602.00457**

# Radiative leptonic tau decays: branching ratios (II)



## Tau radiative leptonic decays ($E_\gamma > 10\,\mathrm{MeV}$)




- (see also M.Passera presentation in this workshop)
- CLEO 2000: T. Bergfeld et al., PRL 84 (2000) 830
- *BABAR* 2015: PRD 91, 051103 (2015)
- Fael & Passera 2015: NLO calculation, JHEP 07 (2015) 153, arXiv:1602.00457 [hep-ph]
- $3.5\sigma$ discrepancy between *BABAR* 2015 and NLO calculation, to be investigated

# Conclusions

- The lepton g-2 provide beautiful examples of interplay between theory and experiment, and different areas of Physics.
- Muon g-2: $\Delta a_\mu \sim 3.5\ \sigma$. $\Delta a_\mu$ can be explained by errors in the theory predictions, in the g-2 experiment, or, we hope, by New Physics! New upcoming experiment: lots of progress in the theory prediction, but the hadronic sector is not yet ready.
- Electron g-2: the present sensitivity to New Physics is limited by experimental uncertainties, but a strong exp. program is under way to improve both $\alpha$ & $a_e$. It could allow to test if the discrepancy in the muon g-2 manifests also in the electron g-2!
- Tau g-2: essentially unknown. New proposal to measure it via radiative leptonic tau decays. Modest improvement expected. BaBar's recent precise measurement of $\mathcal{B}(\tau \to e\bar{\nu}\nu\gamma)$ differs from the SM prediction by $3.5\ \sigma$!

**The End**