Knowing What You Don’t Know: Nuclear Reactions, Effective Field Theory & Uncertainty Quantification

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EMMI

Research supported by the US DOE and by EMMI
Hurricane forecasting

Data: National Hurricane Center, NOAA. Updated: Sep 14, 2018, 2:00 pm

http://www.vox.com
Hurricane forecasting

- Forces, e.g., Coriolis
- Conservation laws
- Parameterizations

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Nuclear reactions

\[ i\hbar \frac{\partial |\Psi\rangle}{\partial t} = (\hat{T} + \hat{V})|\Psi\rangle \]
Nuclear reactions

- Forces: electromagnetic, strong nuclear
- Conservation laws, e.g., probability, energy, momentum
- Some parameterizations
- Accurate knowledge of initial state (nuclear structure)
- Computing to evolve state forward in time
- Uncertainty quantification

\[ i\hbar \frac{\partial |\Psi\rangle}{\partial t} = (\hat{T} + \hat{V})|\Psi\rangle \]
Outline

- What we do and don’t know about the strong nuclear force
- EFT: organizing what we know, constraining what we don’t
- EFT truncation errors from a Bayesian analysis: NN scattering
- EFT for halo nuclei: universal formula for $\gamma + ^{AZ} \rightarrow ^{A-1}Z + n$
- Uncertainty quantification for fusion: $^7\text{Be}(p,\gamma)$ at solar energies
- Conclusion
Potentials from particle exchange
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- Longest range forces generated by lightest particles
The long and the short of hadron physics

\begin{center}
\begin{tabular}{c|c}
\textbf{M (MeV)} & \\
\hline
M_{N} & 939 \\
\hline
\omega & 770 \\
\rho & \\
\hline
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The long and the short of hadron physics

- Spectrum of QCD bound states

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# The long and the short of hadron physics

- Spectrum of QCD bound states
- Now understood as consequence of QCD’s spontaneously broken chiral symmetry: pions are approximate Goldstone bosons of QCD
- For probe energies ~a hundred MeV, simplifications of the rich QCD dynamics emerge: processes dominated by $\pi$s (and $\Delta$s)

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- For probe energies ~a hundred MeV, simplifications of the rich QCD dynamics emerge: processes dominated by $\pi$s (and $\Delta$s)
- Pion exchange generates longest-range part of NN force
- But short-distance dynamics too

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The NN potential: a cartoon

- Long-range part generated by one-pion exchange
- Intermediate ranges: multiple pion exchange
- Short ranges: “other stuff” exchange
- Needs to be parameterized, then fit to NN scattering data
Effective Field Theory

- Simpler theory that reproduces results of full theory at long distances
- Short-distance details irrelevant for long-distance (low-momentum) physics, e.g. multipole expansion
- Expansion in ratio of physical scales: \( \frac{p}{\Lambda_b} = \frac{\lambda_b}{r} \)
- Symmetries of underlying theory limit possibilities: all possible terms up to a given order present in EFT
- Short distances: unknown coefficients at a given order in the expansion need to be determined. Symmetry relates their impact on different processes
- Examples: standard model, chiral perturbation theory, Halo EFT
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Error grows as first omitted term in expansion
χEFT for nuclear forces
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- χPT→pion interactions are weak at low energy.
  
  Weinberg (1990), apply χPT to V, i.e. expand it in $P=p/\Lambda_b$
  
  $$(E - H_0)\psi = V\psi$$

  $V = V^{(0)} + V^{(2)} + V^{(3)} + \ldots$

  Ordonez, Ray, van Kolck (1996); Epelbaum, Meissner, Gloeckle (1999); Entem, Machleidt (2001)
$\chi$EFT for nuclear forces

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- Leading-order $V$:

  $V^{(0)} = \begin{array}{c}
  \begin{array}{c}
  \text{Cross}
  \end{array}
  \end{array} + \begin{array}{c}
  \begin{array}{c}
  \text{Dash}
  \end{array}
  \end{array}$

$$\langle p'|V|p\rangle = C^{3S_1}P_{3S_1} + C^{1S_0}P_{1S_0} + V_{1\pi}(p' - p)$$

Ordonez, Ray, van Kolck (1996); Epelbaum, Meissner, Gloeckle (1999); Entem, Machleidt (2001)
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- Weinberg (1990), apply \( \chi PT \) to \( V \), i.e. expand it in \( x = p/\Lambda_b \)

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![Diagram showing two-nucleon, three-nucleon, and four-nucleon forces with consistent 3NFS and 4NFS](image-url)

Figure courtesy E. Epelbaum
Potential regulated by local function, parameterized by $R$

$$\sigma_{np}(E_{lab}) = \sigma_{LO} \sum_{n=0}^{k} c_n(p_{rel}) \left( \frac{p_{rel}}{\Lambda_b} \right)^n$$

EKM state

$\Lambda_b = 600$ MeV

np observables at $E_{lab} = 96$ MeV at NLO, $N^2LO$, $N^3LO$ ($k=2, 3, 4$)
Successes in $A=3-12$

Epelbaum et al. (LENPIC), arXiv:1807.02848
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- $O(x^4)$ ($N^3LO$) potential, $\chi^2$ good. $\chi^2/\text{d.o.f.} = 1.00$ up to $E_{\text{lab}}=300$ MeV at $O(x^5^+)$

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- $\chi$EFT at $N^2\text{LO}$ reproduces binding energies of light nuclei reasonably well

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Successes in $A=3$-$12$

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- For proton electric polarizability, $\chi$PT $\Rightarrow \alpha_{E1}^{(p)} = 12.5 - 2.3 + 1.5 = 11.7$
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- Two questions:
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- One possibility: $c_3 = \max\{c_0, c_1, c_2\}$

Epelbaum, Krebs, Meissner (2014)

cf. McGovern, Griesshammer, Phillips (2013); many others.
Bayesian tools

Thomas Bayes (1701?-1761)

\[ \text{pr}(A|B, I) = \frac{\text{pr}(B|A, I)\text{pr}(A|I)}{\text{pr}(B|I)} \]

http://www.bayesian-inference.com
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Probability as degree of belief

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Likelihood \quad Prior

Posterior \quad Normalization

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Probability as degree of belief

Marginalization: \[
\text{pr}(x|\text{data}, I) = \int dy \text{pr}(x, y|\text{data}, I)
\]

Allows us to integrate out “nuisance” (e.g. higher-order) parameters
Probability for EFT coefficients

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- General EFT series for observable to order $k$: $X = X_0 \sum_{i=0}^{k} c_i x^i$

- Compute conditional probability distribution: $\text{pr}(c_{k+1}|c_0, \ldots, c_k, l)$

- $l$ = information about EFT, e.g. naturalness
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“Prior A”: \( \text{pr}(c_n|\bar{c}) = \frac{1}{2\bar{c}} \theta(\bar{c} - c_n) \); \( \text{pr}(\bar{c}) = \frac{1}{2 \ln(\epsilon) \bar{c}} \theta \left( \frac{1}{\epsilon} - \bar{c} \right) \theta(\bar{c} - \epsilon) \)

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Result: $\text{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 
1 \\
\left( \frac{c_{\text{max}}}{c_{k+1}} \right)^{k+2} 
\end{cases}$

if $c_{k+1} < c_{\text{max}}$

if $c_{k+1} > c_{\text{max}}$

$[-c_{\text{max}}X_0x^{k+1}, c_{\text{max}}X_0x^{k+1}]$ is a $\frac{k + 1}{k + 2} * 100\%$ DoB interval
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\[ x = 0.33; c_{\text{max}} = 1 \]
NN scattering cross sections

- NN cross section at $T_{lab}=50, 96, 143, 200$ MeV
- Potential regulated by local function, parameterized by $R$. Here: $R=0.9$ fm data
- Results at LO, NLO, N$^2$LO, N$^3$LO, N$^4$LO ($k=0, 2, 3, 4, 5$)

$$\sigma_{np}(E_{lab}) = \sigma_{LO} \sum_{n=0}^{k} c_n \left( \frac{p_{rel}}{\Lambda_b} \right)^n$$

$$x = \frac{p_{rel}}{\Lambda_b}$$

EKM state $\Lambda_b = 600$ MeV

Epelbaum, Krebs, Meissner, EPJA, 2015
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The well-calibrated EFTist
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Accuracy of three weather forecasting services

Source: "The Signal and the Noise" by Nate Silver | Author: Randy Olson (randololson.com / @randololson)
The well-calibrated EFTist

Furnstahl, Kleo, DP, Wesolowski, PRC, 2015
after: Bagnaschi, Cacciari, Guffanti, Jenniches, 2015

- Now we consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent
- Fix a given DOB interval, compute actual success ratio and compare
- Look at this over EKM predictions at four different orders and four different energies
- Interpret in terms of rescaling of $\Lambda_b$ by a factor $\lambda$
The well-calibrated EFTist

Now we consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent.

- Fix a given DOB interval, compute actual success ratio and compare.
- Look at this over EKM predictions at four different orders and four different energies.
- Interpret in terms of rescaling of $\Lambda_b$ by a factor $\lambda$.

No evidence for significant rescaling of $\Lambda_b$.
Physics from consistency plots

- Allows assessment of order-by-order convergence
- Can look at differential cross section and spin observables too
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Outline

- What we do and don’t know about the strong nuclear force
- EFT: organizing what we know, constraining what we don’t
- EFT truncation errors from a Bayesian analysis: NN scattering
- EFT for halo nuclei: universal formula for $\gamma + ^{A}Z \rightarrow ^{A-1}Z + n$
- Uncertainty quantification for fusion: $^7\text{Be}(p,\gamma)$ at solar energies
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Ordinary vs. halo nuclei
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- In nuclei, each nucleon moves in the potential generated by the others.

- The nuclear size grows as $A^{1/3}$; cross sections like $A^{2/3}$.

- Nuclear binding energies are on the order of 8 MeV/nucleon.

http://alternativephysics.org
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http://www.uni-mainz.de
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- Halo nuclei: the last few nucleons “orbit” far from the nuclear “core”

- Characterized by small nucleon binding energies, large radii, large interaction cross sections, large E1 transition strengths.

http://www.uni-mainz.de
Halo nuclei: history & examples
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- “Open quantum systems”: physics beyond mean field

- Universality: common features of weakly-bound quantum systems
Halo nuclei: history & examples

http://nupecc.org
Halo EFT

\[ \lambda \gg R_{\text{core}}; \lambda \lesssim R_{\text{halo}} \]

Bertulani, Hammer, van Kolck, NPA (2003);
Bedaque, Hammer, van Kolck, PLB (2003);
Halo EFT

- Define $R_{\text{halo}} = \langle r^2 \rangle^{1/2}$. Seek EFT expansion in $R_{\text{core}}/R_{\text{halo}}$. Valid for $\lambda \leq R_{\text{halo}}$

- Typically $R \equiv R_{\text{core}} \sim 2$ fm. And since $\langle r^2 \rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies of order 1 MeV or less

- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of Halo EFT

Predicting dissociation

\[ M = \frac{e Z g_0 2 m_R}{\gamma_0^2 + (p - \frac{k}{A})^2} \]

\[ \gamma_0 = \sqrt{2 m_R S_{1n}} \]

\[ p = \sqrt{2 m_R E} \]

\[ E_1 \propto \int_0^\infty dr \, j_1(pr) ru_0(r); \quad u_0(r) = A_0 e^{-\gamma_0 r} \]

Chen, Savage (1999)
Predicting dissociation

- Leading order: no FSI $\Rightarrow \gamma_0$ is only free parameter = 0.16 fm$^{-1}$ for $^{19}$C

\[ M = \frac{eZg_02m_R}{\gamma_0^2 + (p - \frac{k}{A})^2} \]
\[ \gamma_0 = \sqrt{2m_RS_{1n}} \]
\[ p = \sqrt{2m_RE} \]

\[ E1 \propto \int_0^\infty dr j_1(pr)ru_0(r); \quad u_0(r) = A_0e^{-\gamma_0r} \]

Chen, Savage (1999)
Predicting dissociation

- Leading order: no FSI ⇒ $\gamma_0$ is only free parameter = 0.16 fm$^{-1}$ for $^{19}$C

$$\mathcal{M} = \frac{eZg_02m_R}{\gamma_0^2 + (p - \frac{k}{A})^2} \quad \gamma_0 = \sqrt{2m_RS_{1n}}$$

$$p = \sqrt{2m_RE}$$

$$\text{E1} \propto \int_0^{\infty} dr \, j_1(pr)ru_0(r); \quad u_0(r) = A_0e^{-\gamma_0r}$$

$$\frac{dB(E1)}{e^2dE} = \frac{6m_RZ^2}{\pi^2} \frac{A^2}{A_0^2} \frac{p^3}{(\gamma_0^2 + p^2)^2}$$

Universal E1 strength formula for S-wave halos
Predicting dissociation

- Leading order: no FSI $\Rightarrow \gamma_0$ is only free parameter $= 0.16 \text{ fm}^{-1}$ for $^{19}\text{C}$

$$\mathcal{M} = \frac{eZg_02m_R}{\gamma_0^2 + (p - \frac{k}{A})^2} \quad \gamma_0 = \sqrt{2m_RS_{1n}}$$

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Universal E1 strength formula for S-wave halos

- Final-state interactions suppressed by $(R_{\text{core}}/R_{\text{halo}})^3$

- Short-distance piece of E1 m.e.: $L_{E1}\sigma^\dagger{\mathbf{E}} \cdot (n \vec{\nabla} c) + \text{h.c.} \sim \left(\frac{R_{\text{core}}}{R_{\text{halo}}}ight)^4$
Results
Results

Data: Nakamura et al., 1999, 2003; Fukuda et al., 2004
Results

Data: Nakamura et al., 1999, 2003; Fukuda et al., 2004

\[ \gamma_{0\rightarrow a_0} \]
Results

Data: Nakamura et al., 1999, 2003; Fukuda et al., 2004
Determine S-wave $^{18}\text{C}-\text{n}$ scattering parameters $\leftrightarrow^{19}\text{C}$ ANC from dissociation data.
\( a = (7.75 \pm 0.35\text{(stat.)} \pm 0.3\text{(EFT)}) \text{ fm}; \)
\( r_0 = (2.6^{+0.6}_{-0.9}\text{(stat.)} \pm 0.1\text{(EFT)}) \text{ fm}. \)
Ab initio $\rightarrow$ Halo EFT $\rightarrow$ Reaction theory

- $^{11}$Be is a halo nucleus: last neutron only bound to $^{10}$Be by 503 keV. Has a p-wave halo state with $S_{1n}=184$ keV.

- Model Coulomb dissociation of $^{11}$Be via sophisticated “Dynamical Eikonal Approximation”: includes nuclear and Coulomb $^{208}$Pb-$^{10}$Be-n potentials

- Use Halo EFT to identify important $^{10}$Be-n inputs for reaction-theory calculation: s- and p-wave phase shifts

- Take those from ab initio calculation of Calci et al. based on modern nuclear forces and NCSMC (PRL 117, 242501)


No dependence on interior of $^{10}$Be-n potential used
Why is $^7\text{Be}(p,\gamma)$ important?
Why is $^7\text{Be}(p,\gamma)$ important?

- Part of pp chain (pplI)
- Key for predictions flux of solar neutrinos, especially high-energy ($^8\text{B}$) neutrinos
- Accurate knowledge of $^7\text{Be}(p,\gamma)$ needed for inferences from solar-neutrino flux regarding chemical composition of Sun→solar-system formation history
- $S(0)=20.8 \pm 0.7 \pm 1.4$ eV b

"SFII": Adelberger et al. (2010)
This is an extrapolation problem

Thermonuclear reaction rate

\( \propto \langle \nu \sigma \rangle \propto \int_0^\infty dE \exp \left(-\frac{E}{k_B T}\right) E \sigma(E) \)
This is an extrapolation problem

Thermonuclear reaction rate \( \propto \langle v\sigma \rangle \propto \int_0^\infty dE \exp \left( -\frac{E}{k_B T} \right) E \sigma(E) \)
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\[ \sigma(E) = \frac{S(E)}{E} \exp \left( -\pi Z_1 Z_2 \alpha_\text{em} \sqrt{\frac{m_R}{2E}} \right) \]
This is an extrapolation problem

Thermonuclear reaction rate

\[ \propto \langle \nu \sigma \rangle \propto \int_0^\infty dE \exp \left( -\frac{E}{k_BT} \right) E \sigma(E) \]

\[ \sigma(E) = \frac{S(E)}{E} \exp \left( -\pi Z_1 Z_2 \alpha_{em} \sqrt{m_R \over 2E} \right) \]

“Gamow peak”
This is an extrapolation problem

Thermonuclear reaction rate
\[ \propto \left\langle \nu \sigma \right\rangle \propto \int_{0}^{\infty} dE \exp \left( -\frac{E}{k_BT} \right) E \sigma(E) \]

\[ \sigma(E) = \frac{S(E)}{E} \exp \left( -\pi Z_1 Z_2 \alpha_{em} \sqrt{\frac{m_R}{2E}} \right) \]

- E1 capture: \(^{7}\text{Be} + p \rightarrow ^{8}\text{B} + \gamma\)
- Energies of relevance 20 keV

“Gamow peak”
Status as in “Solar Fusion II”

- Energies of relevance ≈ 20 keV
- There dominated by $^7$Be-p separations ~ 10s of fm
- Below narrow $1^+$ resonance proceeds via s- and d-wave direct capture
- Energy dependence due to interplay of bound-state properties, Coulomb, strong ISI
- SF II central value used energy-dependence from Descouvemont’s ab initio eight-body calculation. Errors from consideration of energy-dependence in a variety of “reasonable models”
Capture to \( p \)-wave halo in EFT

- At LO: \( p \)-wave 1\( n \) halo described solely by its ANC and binding energy

\[
u_1(r) = A_1 \exp(-\gamma_1 r) \left( 1 + \frac{1}{\gamma_1 r} \right)
\]

- Capture to the \( p \)-wave state proceeds via the one-body \( E1 \) operator: “external direct capture”

\[
E1 \propto \int_0^\infty dr \: u_0(r) r u_1(r); \quad u_0(r) = 1 - \frac{r}{a}
\]

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator ⇒ there is an LEC that must be fit
NLO for $^7\text{Be}(p,\gamma)$

- LO calculation: ISI in $S=2$ & $S=1$ into p-wave bound state. Scattering wave functions are linear combinations of Coulomb wave functions $F_0$ and $G_0$. Bound state wave function = the appropriate Whittaker function.

- We also incorporate a low-lying excited state ($1/2^-$) in $^7\text{Be}$.

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator $\mathcal{D}$; there is an LEC that must be fit.

\[
S(E) = f(E) \sum_s C_s^2 \left[ |S_{EC}(E; \delta_s(E)) + \tilde{L}_s S_{SD}(E; \delta_s(E)) + \epsilon_s S_{CX}(E; \delta_s(E))|^2 + |\mathcal{D}(E)|^2 \right]
\]

- ANC's in $^5\text{P}_2$ and $^3\text{P}_2$: $A_{5P2}$ and $A_{3P2}$

- Scattering lengths and effective ranges in both $^5\text{S}_2$ and $^3\text{S}_1$: $a_2, r_2$ and $a_1, r_1$

- Core excitation: determined by ratio of $^8\text{B}$ couplings of $^7\text{Be}^*p$ and $^7\text{Be}-p$ states: $\epsilon_1$

- LECs associated with contact interaction, one each for $S=1$ and $S=2$: $L_1$ and $L_2$
Extrapolation to zero energy

\[
\text{pr} \left( \bar{F} \mid D; T; I \right) = \int \text{pr} \left( \bar{g}, \{ \xi_i \} \mid D; T; I \right) \delta(\bar{F} - F(\bar{g}))d\xi_1 \ldots d\xi_5 d\bar{g}
\]
Extrapolation to zero energy

\[ \text{pr} \left( \bar{F} \mid D; T; I \right) = \int \text{pr} \left( \bar{g}, \{ \xi_i \} \mid D; T; I \right) \delta(\bar{F} - F(\bar{g})) d\xi_1 \ldots d\xi_5 d\bar{g} \]
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\[ S(0) = 21.33^{+0.66}_{-0.69} \text{ eV b} \]

No N^2LO corrections

Also assessed impact of N^3LO contact operator
Extrapolation to zero energy

\[ \text{pr} \left( \bar{F} \middle| D; T; I \right) = \int \text{pr} \left( \vec{g}, \{ \xi_i \} \middle| D; T; I \right) \delta(\bar{F} - F(\vec{g})) d\xi_1 \ldots d\xi_5 d\vec{g} \]

\[ S(0) = 21.33^{+0.66}_{-0.69} \text{ eV b} \]

No N^2LO corrections

Also assessed impact of N^3LO contact operator

Some remaining uncertainty due to $^8\text{B} S_{1p}$

Uncertainty reduced by factor of two: model selection
Ongoing work along these lines

- Simultaneous fit to $^7\text{Be}+p$ scattering data: requires inclusion of resonances (TRIUMF experiment)
  
- Same techniques applied to $^3\text{He}(^4\text{He},\gamma)$
  
- Coulomb dissociation: better reaction theory and connection to \textit{ab initio} structure
  
- Rotational states as explicit degrees of freedom
  
- Gaussian process models for EFT truncation errors
  
- $\chi\text{EFT}$ truncation errors in nuclear & neutron matter
  
- Parameter estimation for 3NFs in $\chi\text{EFT}$
The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication in Physical Review A without a detailed discussion of the uncertainties involved in the measurements.

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations.....There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation.....However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made.

Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

Physical Review A Editorial, 29 April 2011
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Bayesian Uncertainty Quantification:
Errors for Your EFT
Theorists Anonymous
Theorists Anonymous

- Admit that you have a problem: your theory has uncertainties
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- Acknowledge the existence of a higher power
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- Acknowledge your mistakes
- Make amends for those mistakes
- Help others who must deal with the same issues


Backup Slides
A Generic EFT

\[ g(x) = \sum_{i=0}^{k} A_i(x) x^i \]

\[ x = \frac{p}{\Lambda_b} \]
A Generic EFT

- Suppose we are interested in a quantity as a function of a momentum, $p$, that is small compared to some high scale, $\Lambda_b$.

- EFT expansion for quantity is

$$g(x) = \sum_{i=0}^{k} A_i(x)x^i$$

$$x = \frac{p}{\Lambda_b}$$
A Generic EFT

- Suppose we are interested in a quantity as a function of a momentum, p, that is small compared to some high scale, \( \Lambda_b \).

- EFT expansion for quantity is

\[
g(x) = \sum_{i=0}^{k} A_i(x) x^i
\]

\[
A_i(x) = a_i(\mu) + f_i(x, \mu)
\]

\[a_i, f_i = O(1) \text{ for } \mu \sim \Lambda_b, x \sim 1\]
A Generic EFT

- Suppose we are interested in a quantity as a function of a momentum, $p$, that is small compared to some high scale, $\Lambda_b$.

- EFT expansion for quantity is

$$ g(x) = \sum_{i=0}^{k} A_i(x) x^i $$

$$ x = \frac{p}{\Lambda_b} $$

$$ A_i(x) = a_i(\mu) + f_i(x, \mu) \quad a_i, f_i = O(1) \text{ for } \mu \sim \Lambda_b, x \sim 1 $$

- $f_i(x, \mu)$ is a calculable function, that encodes IR physics at order $i$

- $a_i$ is a low-energy constant (LEC): encodes UV physics at order $i$. Must be fit to data

- Complications: multiple light scales, multiple functions at a given order, skipped orders, ….
Bayes $\rightarrow$ Result
Bayes → Result

- Bayes theorem: \( \text{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \frac{\text{pr}(c_0, c_1, \ldots, c_k|\bar{c}) \text{pr}(\bar{c})}{\text{pr}(c_0, c_1, \ldots, c_k)} = \mathcal{N} \text{pr}(\bar{c}) \prod_{n=0}^{k} \text{pr}(c_n|\bar{c}) \)
Bayes → Result

- **Bayes theorem:**
  \[
  \text{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \frac{\text{pr}(c_0, c_1, \ldots, c_k|\bar{c})\text{pr}(\bar{c})}{\text{pr}(c_0, c_1, \ldots, c_k)}
  = \mathcal{N}\text{pr}(\bar{c})\prod_{n=0}^{k}\text{pr}(c_n|\bar{c})
  \]

- **Marginalization:**
  \[
  \text{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c} \text{pr}(c_{k+1}|\bar{c})\text{pr}(\bar{c}|c_0, c_1, \ldots, c_k)
  \]
Bayes → Result

- **Bayes theorem:**\[ \Pr(\bar{c}|c_0, c_1, \ldots, c_k) = \frac{\Pr(c_0, c_1, \ldots, c_k|\bar{c})\Pr(\bar{c})}{\Pr(c_0, c_1, \ldots, c_k)} = \mathcal{N}\Pr(\bar{c})\prod_{n=0}^{k}\Pr(c_n|\bar{c}) \]

- **Marginalization:**\[ \Pr(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_{0}^{\infty} d\bar{c} \Pr(c_{k+1}|\bar{c})\Pr(\bar{c}|c_0, c_1, \ldots c_k) \]

- This is generic, but the integrals are simple in the case of “Prior A”\[ \Pr(\bar{c}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 0 & \text{if } \bar{c} < \max\{c_0, \ldots, c_k\} \\ \frac{1}{\bar{c}^{k+2}} & \text{if } \bar{c} > \max\{c_0, \ldots, c_k\} \end{cases} \]
\[ \Pr(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\text{max}} \\ \left(\frac{c_{\text{max}}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\text{max}} \end{cases} \]
I don’t like THAT prior!

- Modify Set A to restrict cbar to a finite range, e.g. $A_{[0.25,4]}$

- Set B: give cbar a log-normal prior: 
  $$
  \Pr(\bar{c}) = \frac{1}{\sqrt{2\pi \bar{c}\sigma}} e^{-\frac{(\log \bar{c})^2}{2\sigma^2}}
  $$

- Set C:  
  $$
  \Pr(c_n | \bar{c}) = \frac{1}{\sqrt{2\pi \bar{c}}} e^{-\frac{c_n^2}{2\bar{c}^2}} ; \Pr(\bar{c}) \propto \frac{1}{\bar{c}} \theta(\bar{c} - \bar{c}_{<}) \theta(\bar{c}_{>} - \bar{c})
  $$

- Same formulas as before can be invoked. Now numerical.

  $$
  \Pr(c_{k+1} | c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c} \Pr(c_{k+1} | \bar{c}) \Pr(\bar{c} | c_0, c_1, \ldots c_k)
  $$

  $$
  \Pr(\bar{c} | c_0, c_1, \ldots, c_k) = \mathcal{N} \Pr(\bar{c}) \prod_{n=0}^{k} \Pr(c_n | \bar{c})
  $$

- You don’t like these? Pick your own and follow the rules…

- First omitted term approximation
Λ_b determines the size of the c_n’s. Choose it too big, and they’ll be too big. Choose it too small, they’ll be too small. And progressively so as one moves to higher and higher order.

We have a theory for \( \text{pr}(c_n|c_0, c_1, \ldots, c_k) \): now use Bayes’ theorem to see how (im)probable are the c_n’s that dimensionful EFT coefficients (b_n’s) produce for a given Λ_b.
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At one energy:

\[
\text{pr}(\Lambda_b|b_2, \ldots, b_k) \propto \frac{1}{\Lambda_b} \left( \frac{\Lambda_b^{k+2}}{(k+1)\langle b^2 \rangle} \right)^{\frac{k-1}{2}}
\]

(NLO: k=2, NNLO: k=3, N^3LO: k=4, etc.)
Λ_b determines the size of the c_n’s. Choose it too big, and they’ll be too big. Choose it too small, they’ll be too small. And progressively so as one moves to higher and higher order.

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\]

(NLO: k=2, NNLO: k=3, N^3LO: k=4, etc.)

Using 5 energies (and 2 angles):
BREAKDOWN-SCALE INFERENCES

- \( \Lambda_b \) determines the size of the \( c_n \)'s. Choose it too big, and they'll be too big. Choose it too small, they'll be too small. And progressively so as one moves to higher and higher order.

- We have a theory for \( \text{pr}(c_n|c_0, c_1, \ldots, c_k) \): now use Bayes’ theorem to see how (im)probable are the \( c_n \)'s that dimensionful EFT coefficients (\( b_n \)'s) produce for a given \( \Lambda_b \).

At one energy:

\[
\text{pr}(\Lambda_b|b_2, \ldots, b_k) \propto \frac{1}{\Lambda_b} \left( \frac{\Lambda_b^{k+2}}{(k+1)(\langle b^2 \rangle)} \right)^{\frac{k-1}{2}}
\]

(NLO: \( k=2 \), NNLO: \( k=3 \), \( N^3\text{LO} \): \( k=4 \), etc.)

Using 17 energies (and 7 angles):
\( \Lambda_b \) determines the size of the \( c_n \)'s. Choose it too big, and they’ll be too big. Choose it too small, they’ll be too small. And progressively so as one moves to higher and higher order.

We have a theory for \( \text{pr}(c_n|c_0, c_1, \ldots, c_k) \): now use Bayes’ theorem to see how (im)probable are the \( c_n \)'s that dimensionful EFT coefficients (\( b_n \)'s) produce for a given \( \Lambda_b \).

At one energy:

\[
\text{pr}(\Lambda_b|b_2, \ldots, b_k) \propto \frac{1}{\Lambda_b} \left( \frac{\Lambda_b^{k+2}}{(k+1)
\langle b^2 \rangle} \right)^{\frac{k-1}{2}}
\]

(NLO: \( k=2 \), NNLO: \( k=3 \), \( N^3 \text{LO} \): \( k=4 \), etc.)

Using 17 energies (and 7 angles):

\( R=1.2 \text{ fm} \)
But we don’t have 119 independent data points.

We have a function for each observable at each order.

Can we understand the properties of these functions, so we can do $\Lambda_b$ inference and compute success ratios rigorously?

$$\sigma(E) = \sigma_0(E) \left[ 1 + c_2(E)x^2 + c_3(E)x^3 + c_4(E)x^4 + c_5(E)x^5 \right]$$
OBSERVATIONS AND QUESTIONS

- $c_n$’s do not grow or shrink with $n$: good $\Lambda_b$ choice
- Bounded functions, mostly between -2 and 2
- Each “takes a turn” at being largest
- Not oscillating quickly in this energy range

$\Lambda_b = 600$ MeV
OBSERVATIONS AND QUESTIONS

- $c_n$’s do not grow or shrink with $n$: good $\Lambda_b$ choice
- Bounded functions, mostly between -2 and 2
- Each “takes a turn” at being largest
- Not oscillating quickly in this energy range

$\Lambda_b = 400$ MeV
OBSERVATIONS AND QUESTIONS

- $c_n$'s do not grow or shrink with $n$: good $\Lambda_b$ choice
- Bounded functions, mostly between -2 and 2
- Each “takes a turn” at being largest
- Not oscillating quickly in this energy range

$\Lambda_b = 600$ MeV
OBSERVATIONS AND QUESTIONS

- $c_n$’s do not grow or shrink with $n$: good $\Lambda_b$ choice
- Bounded functions, mostly between -2 and 2
- Each “takes a turn” at being largest
- Not oscillating quickly in this energy range

Physics questions:
- Do curves all fluctuate around zero with some common variance?
- What is the correlation length? Is it different at each order?

$\Lambda_b=600$ MeV
Coulomb dissociation of halo nuclei

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus

- Do with different Z, different nuclear sizes, different energies to test systematics
Coulomb dissociation of halo nuclei

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus

- Do with different Z, different nuclear sizes, different energies to test systematics

- Coulomb excitation dissociation cross section (p.v. $b \gg R_{\text{target}}$)

\[
\frac{d\sigma_C}{2\pi bdb} = \sum \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_{\pi L}^{\gamma L}(E_\gamma)
\]

- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus.
Coulomb dissociation of halo nuclei

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus

- Do with different Z, different nuclear sizes, different energies to test systematics

- Coulomb excitation dissociation cross section (p.v. $b \gg R_{\text{target}}$)

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma^\pi_L(E_\gamma)$$

- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus.

- $\sigma^\pi_L(E_\gamma)$ can then be extracted: it’s the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity $\pi L$. 
The multi-dimensional Halo EFT space

$N_n$  $N_p$  $L$

**Single-neutron halos**

- (s-wave) $d, ^{19}C$
- (p-wave) $^8Li$
The multi-dimensional Halo EFT space

- Two-neutron halos (s-wave)
  - $^{11}\text{Li}$, $^{14}\text{Be}$, $^{22}\text{C}$

- Two-neutron halos (p-wave)
  - $^6\text{He}$

- Single-neutron halos (s-wave)
  - $^d$, $^{19}\text{C}$

- Single-neutron halos (p-wave)
  - $^8\text{Li}$
The multi-dimensional Halo EFT space

- Single-neutron halos (p-wave): $^{8}\text{Li}$
- Single-neutron halos (s-wave): $^{17}\text{F}^*$, $^{11}\text{Li}$, $^{14}\text{Be}$, $^{22}\text{C}$
- Two-neutron halos (p-wave): $^6\text{He}$
- Two-neutron halos (s-wave): $^{10}\text{Li}$, $^{14}\text{Be}$, $^{22}\text{C}$
- Single-proton halos (p-wave): $^8\text{B}$
- Single-proton halos (s-wave): $^{19}\text{C}$
The multi-dimensional Halo EFT space

- Single-neutron halos
  - (p-wave) $^8$Li
  - (s-wave) $^8$B

- Two-neutron halos
  - (s-wave) $^{11}$Li, $^{14}$Be, $^{22}$C
  - (p-wave) $^6$He

- Single-proton halos
  - (s-wave) $^{17}$F*
  - (p-wave) $^3$Li, $^{19}$C

Plus complementary direction: $N_\alpha$
E.g. $^9$Be, $^{12}$C*, etc.
Lagrangian for s- and p-wave states

\[ \mathcal{L} = c^\dagger \left( i \partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left( i \partial_t + \frac{\nabla^2}{2m} \right) n + \sigma^\dagger \left[ \eta_0 \left( i \partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi^\dagger_j \left[ \eta_1 \left( i \partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j - g_0 \left[ \sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[ \pi^\dagger_j (n \overset{\rightharpoonup}{\nabla}_j c) + (c^\dagger \overset{\rightharpoonup}{\nabla}_j n^\dagger) \pi_j \right] - \frac{g_1}{2} \frac{M - m}{M_{nc}} \left[ \pi^\dagger_j \overset{\rightharpoonup}{\nabla}_j (n c) - \overset{\rightharpoonup}{\nabla}_j (n^\dagger c^\dagger) \pi_j \right] + \ldots, \]

- c, n: “core”, “neutron” fields. c: boson, n: fermion
- \( \sigma, \pi_j \): S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

s-wave: Kaplan, Savage, Wise (1998); van Kolck (1999); Birse, Richardosn, McGovern 1999
Dressing the s-wave state

- $\sigma_{nc}$ coupling $g_0$ of order $R_{\text{halo}}$, nc loop of order $1/R_{\text{halo}}$. Therefore need to sum all bubbles:

$$D_{\sigma}(p) = \frac{1}{\Delta_0 + \eta_0 [p_0 - \mathbf{p}^2/(2M_{nc})]} - \Sigma_{\sigma}(p)$$

$$\Sigma_{\sigma}(p) = -\frac{g_0^2 m_R}{2\pi} \left[ \mu + i\sqrt{2m_R \left( p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + i\eta \right)} \right]$$

$$t = \frac{2\pi}{m_R} \left( \frac{1}{a_0} - \frac{1}{2} r_0 k^2 + i k \right)$$

$$D_{\sigma}(p) = \frac{2\pi \gamma_0}{m_R^2 g_0^2} \frac{1}{1 - r_0 \gamma_0} \frac{1}{p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + B_0} + \text{regular}$$

Counting in S waves:
- $a_0 \sim R_{\text{halo}} \sim 1/\gamma_0$;
- $r_0 \sim R_{\text{core}}$.
- $r_0 = 0$ at LO.
One-slide p-wave review

\[ \langle k| t_1 | k' \rangle = - \frac{6\pi}{m_R} - \frac{\mathbf{k} \cdot \mathbf{k}'}{\frac{1}{a_1}} + \frac{1}{2} r_1 k^2 - ik^3 \]

Bethe (1949)
One-slide p-wave review

- For a short-ranged potential, if $kR \ll 1$:

\[
\langle \mathbf{k}|t_1|\mathbf{k}' \rangle = -\frac{6\pi}{m_R} \left( \frac{k \cdot k'}{a_1} + \frac{1}{2} r_1 k^2 - ik^3 \right)
\]

Bethe (1949)
One-slide p-wave review

- For a short-ranged potential, if $kR \lesssim 1$:

$$\langle k|t_1|k' \rangle = -\frac{6\pi}{m_R} \left( \frac{k \cdot k'}{a_1} + \frac{1}{2} r_1 k^2 - i k^3 \right)$$

Bethe (1949)

- “Natural case” $a_1 \sim R^3; r_1 \sim l/R. \Rightarrow t_1 \sim R^3 k^2$, so small cf. $t_0 \sim l/k$ ($N^3LO$)
One-slide p-wave review

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- But what if there is a low-energy p-wave resonance?
One-slide p-wave review

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- Causality says $r_1 \preceq -1/R$
One-slide p-wave review

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- But what if there is a low-energy p-wave resonance?

- Causality says \( r_1 \preceq -1/R \)

- So low-energy resonance/bound state would seem to have to arise due to cancellation between \( -1/a_1 \) and \( 1/2 r_1 k^2 \) terms.

- \( a_1 \sim R/M_{\text{lo}}^2 \) gives \( k_R \sim M_{\text{lo}} \)

Wigner (1955); Hammer & Lee (2009); Nishida (2012)

Dressing the p-wave state

Proceed similarly for p-wave state as for s-wave state

\[
D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - p^2/(2M_{nc})] - \Sigma_\pi(p)}
\]

Here both \(\Delta_1\) and \(g_1\) are mandatory for renormalization at LO

\[
\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi} \left[ \frac{3}{2\mu + i k} \right]
\]

Reproduces ERE. But here (cf. s waves) cannot take \(r_1=0\) at LO.
Dressing the p-wave state


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- If \( a_1 > 0 \) then pole is at \( k = i\gamma_1 \) with \( B_1 = \gamma_1^2/(2m_R) \):

\[ D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2} \frac{2}{r_1 + 3\gamma_1} \frac{i}{p_0 - p^2/(2M_{nc}) + B_1} + \text{regular} \]
A narrow p-wave resonance/bound state


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- First EFT paper to do this assigned $a_1 \sim 1/M_{lo}^3; r_1 \sim M_{lo}$  

- Here we adopt $r_1 \sim 1/R, a_1 \sim M_{lo}^2/R$  
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- So, off resonance, $\text{Re}[t^{-1}] > \text{Im}[t^{-1}]$: phase shifts are $O(M_{lo}R)$ and scattering is perturbative away from resonance

\[
\langle k|t_1|k' \rangle = -\frac{12\pi}{m_R r_1} \frac{k \cdot k'}{k^2 - k_R^2} \quad k_R^2 = \frac{2}{a_1 r_1}
\]
A narrow p-wave resonance/bound state

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\]

- Resonance width is $\sim E_R k_R/r_1$, so it is parametrically narrow. Need to resum width if $k^2 - k_R^2$ gets small
P-wave FSI in $\gamma_{E1} + ^{11}\text{Be} \rightarrow ^{10}\text{Be} + n$


- $^{11}\text{Be}: 1/2^- (P\text{-wave})$ state bound by 0.18 MeV
P-wave FSI in $\gamma_{E1} + ^{11}\text{Be} \rightarrow ^{10}\text{Be} + n$


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- FSI in spin-1/2 channel: stronger, but “kinematic” nature of P-wave bound state means P-wave scattering is perturbative away from it. EFT analysis in terms of scales:
  \[ k^3 \cot \delta_1 = -1/2 \, r_1 \, (k^2 + \gamma_1^2) \Rightarrow \delta_1 \sim R_{\text{core}}/R_{\text{halo}} \text{ if } k \sim 1/R_{\text{halo}} \sim \gamma_1. \]

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- Need both $\gamma_1$ and $r_1 \equiv A_1$ at NLO in this observable. $A_0$ also becomes a free parameter at NLO; fit it to Coulomb dissociation data
Coulomb dissociation of $^{11}$Be: result

Data: Palit et al., 2003

- Reasonable convergence
- Information on value of $r_0$ through fitting of $A_0$:
  $r_0=2.7$ fm
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- Here value of $r_1$ used to fit $B(E1; 1/2^+ \rightarrow 1/2^-)$ works. $r_1 = -0.66$ fm$^{-1}$

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NLO: $(<r_c^2>+<r_{Be}^2>)^{1/2}=2.44$ fm
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NLO: $(<r_c^2> + <r_{Be}^2>)^{1/2} = 2.44 \text{ fm}$

Use of ab initio input, e.g. for ANC?

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$^7\text{Li} + n \rightarrow ^8\text{Li} + \gamma_{E1}$

- $^7\text{Li}$ ground state is $3/2^-$: S-wave n scattering in $^5S_2$ and $^3S_1$

\[ a_{S=2} \sim R_{\text{halo}}; \ a_{S=1} \sim R_{\text{core}} \]

![Energy level diagram with transitions and energy levels labeled]

\[ ^3\text{H} + ^4\text{He} \]
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- $a_{S=2} = -3.63(5)$ fm, $a_{S=1} = 0.87(7)$ fm

- $a_{S=2} \sim R_{\text{halo}}$; $a_{S=1} \sim R_{\text{core}}$

![Energy level diagram](attachment:image.png)
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\[ 3^+ \quad 0.22 \quad n + ^7\text{Li} \]
\[ 1^+ \quad -1.05 \]
\[ 2^+ \quad -2.03 \]

$^7\text{Li} + n \rightarrow ^8\text{Li} + \gamma_{E1}$
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- LO calculation: $S=2$ (with ISI) and $S=1$ into P-wave bound state

$$E1 \propto \int_0^\infty dr \: u_0(r)ru_1(r);$$

$$u_0(r) = 1 - \frac{r}{a}; \quad u_1(r) = A_1 \exp(-\gamma_1r) \left(1 + \frac{1}{\gamma_1r}\right)$$
Fixing $^8\text{Li}$ parameters

- $^8\text{Li}$ ground state is $2^+$: both $^5\text{P}_2$ and $^3\text{P}_2$ components
- $^8\text{Li}$ first excited state: $1^+$, bound by 1.05 MeV

Zhang, Nollett, Phillips, PRC (2014)
c.f. Rupak, Higa, PRL 106, 222501 (2011);
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\[
\begin{array}{c}
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\end{array}
\]

\[
\begin{array}{c}
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1/2^- & 0.478 \\
3/2^- & 0 \\
\end{array}
\]

\[
\begin{array}{c}
\text{n} + ^7\text{Li} \\
^8\text{Li} \\
\end{array}
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- VMC calculation with AV18 + UIX gives all ANCs: infer $r_1 = -1.43$ fm$^{-1}$

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$R_{\text{core}}$

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\[ \gamma_1 \sim 1/R_{\text{halo}} \]

\[ r_1 \sim 1/R_{\text{core}} \]

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<td>Trache</td>
<td>-0.284(23)</td>
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<td>0.187(16)</td>
<td>0.217(13)</td>
</tr>
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</table>
LO results for $^{7}\text{Li} + n \rightarrow ^{8}\text{Li} + \gamma_{E1}$

LO results for $^7\text{Li} + n \rightarrow ^8\text{Li} + \gamma_{\text{EI}}$


$$\frac{\sigma( {^5S_2} \rightarrow 2^+) }{\sigma( \rightarrow 2^+ )} = 0.95$$

Experiment $> 0.86$

Barker, 1996
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$$\frac{\sigma(\rightarrow 2^+) }{\sigma(\rightarrow 2^+) + \sigma(\rightarrow 1^+)} = 0.89$$

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Experiment $= 0.88$
Lynn et al., 1991

Dynamics **predicted** through *ab initio* input
Data situation

- 42 data points for $100 \text{ keV} < E_{\text{c.m.}} < 500 \text{ keV}$
  - Junghans (BE1 and BE3)
  - Fillipone
  - Baby
  - Hammache (1998 and 2001)

- Subtract M1 resonance: negligible impact at 500 keV and below

- Deal with CMEs by introducing five additional parameters, $\xi_j$

- CMEs
  - 2.7% and 2.3%
  - 11.25%
  - 5%
  - 2.2% (1998)
Building the pdf

- Bayes:

\[
\text{pr} \left( \vec{g}, \{\xi_i\} | D; T; I \right) = \text{pr} \left( D | \vec{g}, \{\xi_i\}; T; I \right) \text{pr} \left( \vec{g}, \{\xi_i\} | I \right),
\]

- First factor: likelihood

\[
\ln \text{pr} \left( D | \vec{g}, \{\xi_i\}; T; I \right) = c - \sum_{j=1}^{N} \frac{\left( (1 - \xi_j) S(\vec{g}; E_j) - D_j \right)^2}{2\sigma_j^2},
\]

- Second factor: priors
  - Independent gaussian priors for $\xi_j$, centered at zero and with width=CME
  - Gaussian priors for $a_{S=1}$ and $a_{S=2}$, based on Angulo et al. measurement
  - All other EFT parameters assigned flat priors, corresponding to natural ranges
  - No s-wave resonance below 600 keV
Marginalizing $\rightarrow$ pdfs

$$\text{pr} \left( g_1, g_2 | D; T; I \right) = \int \text{pr} \left( \vec{g}, \{ \xi_i \} | D; T; I \right) \, d\xi_1 \ldots d\xi_5 \, dg_3 \ldots dg_9$$

- ANCs are highly correlated but sum of squares strongly constrained
- One spin-1 short-distance parameter: $0.33 \, \bar{L}_1 / (\text{fm}^{-1}) - \epsilon_1$
More questions we can answer

42 data points, 7 parameters “fit” to these data, 5 $\xi_i$’s fixed to their mean values
More questions we can answer

- Is it a “good fit”?

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More questions we can answer

- Is it a “good fit”?
- Did the experimentalists understand their systematic errors?
More questions we can answer

- Is it a “good fit”?
- Did the experimentalists understand their systematic errors?
- Are there parameters that are not well constrained by these data?
Truncation error

- N2LO correction=0 (technically only in absence of excited state)
- EFT s-wave scattering corrections (shape parameter)~0.8%
- E2, M1 contributions < 0.01%, Radiative corrections: ~0.1%
- So first correction is at N3LO, i.e., $\vec{L}_i \rightarrow \vec{L}_i + k^2 \vec{L}'_i$
Planning improvements

Use extrapolant to simulate impact of hypothetical future data that could inform posterior pdf for $S(0)$

Left-to-right: 42 data points all of similar quality to Junghans et al.

A: ANC

$S$: $a_{S=1}$ and $a_{S=2}$

L: short-distance

Note that 1 keV uncertainty in $S_{1p}$ of $^8$B may not be negligible effect
A sneak peek at $^3\text{He}(^4\text{He},\gamma)$

Preliminary results from Zhang, Nollett, DP, in preparation.
A sneak peek at $^3$He($^4$He,γ)

Zhang, Nollett, DP, in preparation
Halo EFT as a “super model”

- Halo EFT is also the EFT of all the models used to extrapolate the cross section in “Solar Fusion II”
- Differences are sub-% level between 0 and 0.5 MeV
- Parameters generally obey $a \sim 1/R_{\text{halo}}$, $r \sim R_{\text{core}}$, $L \sim R_{\text{core}}$, as expected
- Absolute size of $S(0)$ over-predicted in all models, but curves rescaled in fits for Solar Fusion II