The vacuum is not empty: fluctuations, virtual particles, Casimir forces and dark energy

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Introduction

The question of the existence of the vacuum was already discussed in ancient Greece.
Well known is the opinion of Parmenides (Italy, $\sim$500 BC). He argued that the concept of something which is emptied of all existing things is a contradiction in itself.

$$\mu\eta\delta\grave{\epsilon}\nu\ \delta'\ o\upsilon\kappa\ \acute{\epsilon}\sigma\tau\iota\nu$$

"Nothing is no thing"

This was interpreted as the nonexistence of empty space.
Aristoteles argued that moving bodies would never come to rest if empty space would exist, because there would be no distinguished point. This is an interesting early form of the law of inertia which was refuted because of an apparent contradiction with experience.

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The opposite opinion was hold e.g. by Democrite ($\sim$400 BC).

According to him there must be empty space in which the indivisible smallest constituents of matter can move.

Modern physics has taken this view. In particular Newton assumes the existence of an absolute space.

The fact that matter is not continuous, but consists of elementary constituents followed originally from the laws of chemistry, later from kinetic theory of gases, the theory of Brownian motion (Einstein) and Rutherford's scattering experiments.

First doubts: fields and quanta

A first hint that atomism is not the whole truth came from Maxwell's theory of electromagnetism.

According to this theory, the space, in the absence of matter, (traditionally called the vacuum) supports the electromagnetic field. But from Einstein's equation

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(equivalence of mass and energy) we know that there is no fundamental difference between fields and matter.

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Nonrelativistic quantum mechanics describes particles by wave functions. The particles themselves are eternal and undestroyable, and the vacuum is defined by the absence of particles.

On the first sight, quantum mechanics seems to be compatible with the existence of empty space. But under closer inspection one observes where problems might originate. Let us look at the example of the harmonic oscillator.

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Quantum theory and special relativity

Combining field theory and quantum theory yields that a vacuum in the sense of an empty space does not exist. This can be nicely illustrated on the example of the electromagnetic field.

The components of the electrical field **E** and the magnetical field **B** do not commute with each other,

$$[E_j(t,\mathbf{x}), B_k(t,\mathbf{y})] = i\varepsilon_{jkl}\partial_l\delta(\mathbf{x}-\mathbf{y}) \ .$$

This implies uncertainty relations according to which there is no state in which both fields vanish.

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In relativistic quantum field theory, nevertheless the concept of a vacuum state plays an important role. But this means the ground state, i.e. the state of lowest energy. This does not at all correspond to the concept of empty space, as one clearly sees on the example of the so-called Dirac sea.

Dirac replaced the relativistic energy-momentum relation

$$E^2 = \mathbf{p}^2 + m^2$$

by a linear equation

$$E = \vec{\alpha} \cdot \mathbf{p} + \beta m$$

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But both equations have solutions with negative energy. A state corresponding to empty space would be highly unstable under decay into particle pairs with opposite energies.

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what we call the vacuum, is the state where all states with negative energy are occupied.

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The existence of the Dirac sea, however, implies that the vacuum has a rich internal structure.
In particular it implies the existence of antiparticles, the possibility of creation of particle-antiparticle pairs and the presence of fluctuations in local observables, as e.g. current density and energy momentum tensor.

In the case of the electromagnetic field the vacuum fluctuations are responsible for the decay of excited states of atoms. These states would be stable for the isolated atom and become unstable by the interaction with the (never vanishing) electromagnetic field.

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Why are fluctuations unavoidable? The reason is that there are two basic principles of relativistic quantum physics which seem, on a first sight, to contradict each other, namely locality and positivity of energy.

Locality means that operators which represent local observables commute with each other if they are mutually spacelike localized (Einstein causality).

Positivity of energy in a relativistic system means that the spectrum of energy is positive in all Lorentz systems, i.e.

$$e^{\mu}P_{\mu} \geq 0$$

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Therefore, for an arbitrary state vector Ψ, the map

$$\mathbb{C}^4 \ni z^\mu \mapsto e^{iz^\mu P_\mu}\Psi$$

is analytic if $\mathrm{Im} z^\mu$ is timelike and future directed. But then the matrix elements

$$\langle \Phi, e^{ix^\mu P_\mu}\Psi \rangle$$

are boundary values of an analytic function. Thus if Φ is orthogonal to $e^{ix^\mu P_\mu}\Psi$ for x^μ in the neighborhood of some point, it must be orthogonal for all x^μ.

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This fact has been exploited and led to the Reeh-Schlieder Theorem:

Theorem

Let Ψ be a state vector with bounded energy spectrum and let $\mathcal{O}$ be a nonempty open region in Minkowski space.
Then the set of state vectors

$$\{A\Psi, A \textit{ localized in } \mathcal{O}\}$$

is dense in the Hilbert space.

Now let $B = B^*$ be any local observable, and let $\mathcal{O}$ be spacelike to the localization region of B. Assume that Ψ has bounded energy spectrum and is an eigenstate of B,

$$B\Psi = b\Psi \ , b \in \mathbb{R} \ .$$

By local commutativity, all A localized in $\mathcal{O}$ commute with B, hence

$$BA\Psi = AB\Psi = bA\Psi$$

thus all vectors of the form $A\Psi$ are eigenvectors with eigenvalue b.

But by the Reeh-Schlieder Theorem, every vector can be approximated by a vector of this form and is therefore also an eigenvector with the same eigenvalue. Thus B is a multiple of the unit operator and hence a trivial observable.

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We conclude that, in a state with bounded energy, all nontrivial local observables have fluctuations. These fluctuations remind on thermal fluctuations. This is actually the content of a deep mathematical theorem due to Tomita and Takesaki.

Theorem

Let $\mathcal{N}$ be a weakly closed algebra of Hilbert space operators (i.e. a von Neumann algebra), and let Ψ be a vector which is cyclic (i.e. $\mathcal{N}\Psi$ is dense) and separating (i.e. $A\Psi = 0$ implies $A = 0$). Then there exists a selfadjoint operator H with $H\Psi = 0$ and

1. $$e^{itH}\mathcal{N}e^{-itH} = \mathcal{N}$$
2. *Ψ is a thermal equilibrium state (a KMS-state) if H is interpreted as the Hamiltonian.*

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Physical implications:

- Unruh effect: Accelerated systems have thermal properties (see Buchholz-Verch 2015 for a detailed discussion).
- Hawking effect: Existence of horizons induces thermal radiation (see Dappiaggi-Moretti-Pinamonti (2009) for a full construction of the radiating state).
- Intrinsic time evolution in quantum gravity? (Connes-Rovelli 1994)

But there is a disturbing fact: according to theory, the fluctuations of the field strength at a point are infinitely large. Only averages of the field over some region have finite fluctuations. According to classical electrodynamics, the energy density is

$$u = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 .$$

But this would mean that the energy density is infinitely large, a clearly meaningless statement .

We are facing the following paradox:

The theory predicts infinitely large fluctuations, but they seem to have no practical consequences .

Formal solution of the problem: the formula for the energy density is modified by subtracting the vacuum expectation value (normal ordering).

But this does not concern the (according to the theory unavoidable) fluctuations. Can they be detected?

Casimir effect

A particularly clear effect was predicted by Casimir.

He considers two parallel conducting plates.
The associated boundary conditions (the electrical field tangential to the surface has to vanish) yield changes of the fluctuations of the fields and, as a consequence, changes of the energy density u and of the pressure p,

$$u = -\frac{\pi^2}{720 d^4} \; , p = -\frac{\pi^2}{240 d^4} \; ,$$

d distance between the plates.

While the existence of the effect itself is generally accepted, there is a lot of ongoing work

for a better theoretical understanding

(see, e.g. Niekerken 2009, Dappiaggi-Nosari-Pinamonti 2015)

and for a more precise experimental confirmation

(see, e.g. M. Bordag et al., Advances in the Casimir Effect, Oxford 2014)

Quantum energy inequalities

An interesting feature is that the energy density is smaller than in the (global) ground state. The existence of states in which the energy density assumes smaller values than in the ground state is a necessary consequence of the Reeh-Schlieder theorem.

Namely, let $B \neq 0$ be any local observable with $\langle \Omega, B\Omega \rangle = 0$ where Ω is the state vector of the vacuum. Assume $\langle \Omega, B^3\Omega \rangle < \infty$. Then the minimal value of the expectation value of B in a state induced by the vectors $(1 + \lambda B)\Omega$ is given by

$$\langle B \rangle_{\min} = \sqrt{\langle \Omega, B^2\Omega \rangle} \left(\frac{a}{2} - \sqrt{\frac{a^2}{4} + 1} \right) < 0$$

where $a = \frac{\langle \Omega, B^3\Omega \rangle}{\langle \Omega, B^2\Omega \rangle^{\frac{3}{2}}}$.

Negative energy densities could lead to strange effects:

- Stability of localized physical systems questionable. Problems with the 2nd law of thermodynamics (Ford)
- Exotic spacetimes (worm holes etc.) possible.

Partial remedy: Quantum energy inequalities (Fewster et al.)
Typical example: Time average of the (normal ordered) electromagnetic energy density u with the square of a smooth real function f,

$$\int dt\, f(t)^2 u(t,\mathbf{x}) = \frac{1}{2}\int dt\, f(t)^2 :\mathbf{E}(t,\mathbf{x})^2: + :\mathbf{B}(t,\mathbf{x})^2:$$

Normal ordering is defined by

$$:\mathbf{E}(x)^2: = \lim_{y\to x} \mathbf{E}(x)\cdot\mathbf{E}(y) - \langle\Omega, \mathbf{E}(x)\cdot\mathbf{E}(y)\Omega\rangle$$

We insert

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha e^{i\alpha(t-t)} = \delta(t-t')$$

and obtain

$$\int dt\, f(t)^2 u(t,\mathbf{x}) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\alpha \int dt\, f(t) e^{i\alpha t} \int dt'\, f(t') e^{-i\alpha t'}]\times$$

$$\big(:\mathbf{E}(t,\mathbf{x})\cdot\mathbf{E}(t',\mathbf{x}): + :\mathbf{B}(t,\mathbf{x})\cdot\mathbf{B}(t',\mathbf{x}):\big)$$

Since the normal product is symmetric, the integrand is symmetric in α, and we can replace the integral by twice the integral of α from 0 to ∞. Inserting then the formula for normal ordering we get the difference of the manifestly positive operator

$$U(f) = \frac{1}{2\pi}\int_{0}^{\infty} d\alpha\, (\mathbf{E}(f_\alpha)^* \cdot \mathbf{E}(f_\alpha) + \mathbf{B}(f_\alpha)^* \cdot \mathbf{B}(f_\alpha))$$

and its vacuum expectation value.

Here

$$\mathbf{E}(f_\alpha) = \int dt\, \mathbf{E}(t,\mathbf{x}) f(t) e^{-i\alpha t}$$

and

$$\mathbf{B}(f_\alpha) = \int dt\, \mathbf{B}(t,\mathbf{x}) f(t) e^{-i\alpha t} \ .$$

The crucial observation is now that the vacuum expectation value of $U(f)$ is finite. One finds

$$\langle \Omega, U(f)\Omega \rangle = \frac{1}{8\pi^2} \int dt \ddot{f}(t)^2$$

Hence the quantum energy inequality assumes the form

$$\int dt f(t)^2 u(t,\mathbf{x}) \geq -\frac{1}{8\pi^2} \int dt \ddot{f}(t)^2$$

Dark energy

Since a few years we know that the expansion rate of the universe is growing.

This can be explained, if one adds to the known forms of energy
matter ("dust") (vanishing pressure)
and radiation (pressure = energy density/3)
a further form
(dark energy with pressure= −energy density).

Thereby matter amounts to about 30% and dark energy to about 70%. The contribution of radiation is tiny.

Also the contribution of known matter (essentially baryonic matter) is small;

the structure of our universe seems to be dominated by some unknown kind of matter (dark matter) and by dark energy.

There are presently many experiments which search for dark matter, in particular at the LHC and in cosmic ray detectors. Theoretically it could be easily explained by a new type of matter which is weakly interacting with known matter and with itself. Therefore its influence is mainly due to gravitation and becomes visible e.g. in rotation curves in the outer regions of galaxies, in gravitational lenses and in the expansion rate of the universe.

Dark energy, however, cannot be explained in this way.
A possible explanation is in terms of a nonzero vacuum expectation value of the energy momentum tensor.
If one adopts the idea that the infinities of quantum field theory can be replaced by large quantities depending on some natural ultraviolet cutoff, one obtains absurd values.
But if one renormalizes consistently one finds on Minkowski space for a free massive scalar field with mass m

$$\langle T_{\mu\nu} \rangle = -\frac{m^4}{32\pi^2} \log \frac{\mu^2}{m^2} \, g_{\mu\nu}$$

where μ is a mass scale.
Inserting $\mu^2 = m^2 - \frac{9}{4} H^2$ and the measured values for the Hubble parameter $H \approx 10^{-61}$ and $\langle T_{00} \rangle \approx 10^{-120}$ one finds $m \approx 1$ (in Planck units).

We conclude:

- There is no vacuum in the sense of empty space.
- As a consequence, matter in the sense of an undestroyable substance does not exist.
- In particular, there is no natural concept of elementary particles as fundamental constituents of matter.
- Quantum fields, however, are theoretically and experimentally well established entities. All tested predictions of quantum field theory were satisfied.
- It is not yet clear whether also gravity can be incorporated into quantum field theory.

  (See Brunetti-F-Rejzner 2016 for an attempt and Brunetti-F-Hack-Pinamonti-Rejzner 2016 for an application to cosmology.)