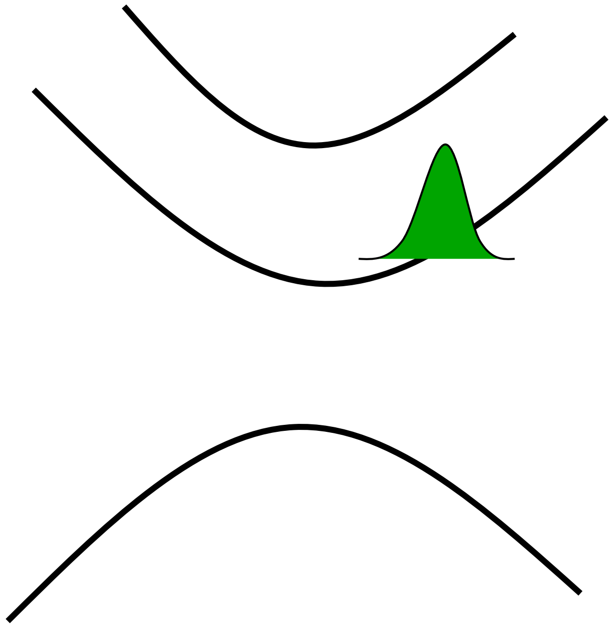
Dynamics of the effective mass and the anomalous velocity

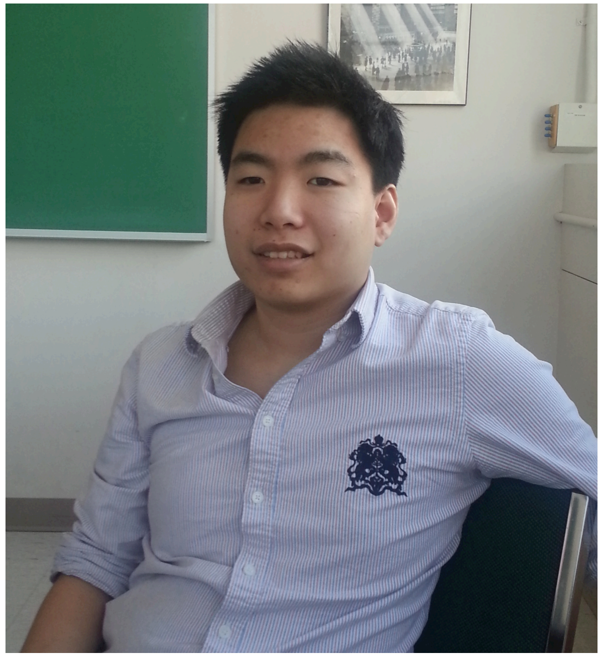


J.E. Sipe
Department of Physics
University of Toronto

Federico Duque-Gomez

Aephraim Steinberg and his research group

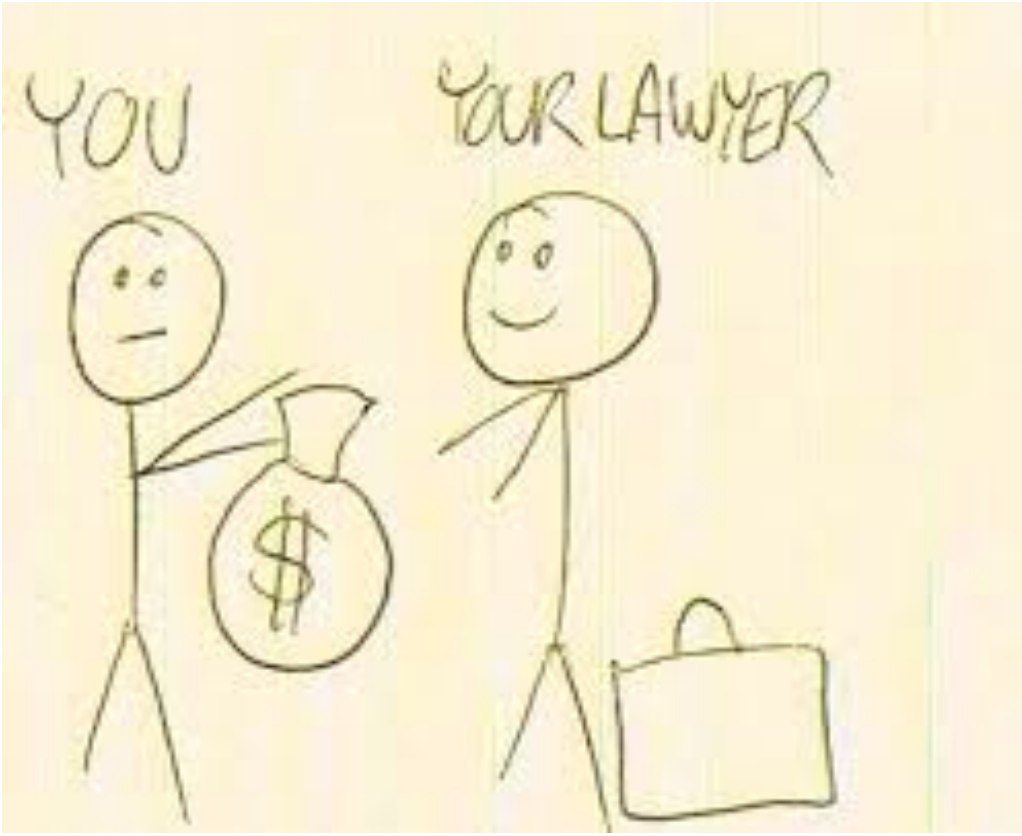
Mike Fang




It's Here
CANADA
U. S. A.
MEXICO
Hudson Bay
ATLANTIC OCEAN
Gulf of Mexico
Tropic of Cancer
Alberta
Saskatchewan
Manitoba
Ontario
Quebec
Newfoundland
British Columbia
Washington
Oregon
California
Nevada
Idaho
Montana
Wyoming
Utah
Arizona
Colorado
New Mexico
Texas
Oklahoma
Kansas
Nebraska
South Dakota
Minnesota
Iowa
Missouri
Arkansas
Louisiana
Mississippi
Alabama
Georgia
Florida
Tennessee
Kentucky
Illinois
Indiana
Ohio
Michigan
Wisconsin
Pennsylvania
West Virginia
Virginia
North Carolina
South Carolina
New York
Massachusetts
Maine
New Brunswick
Prince Edward Island
Lake Superior
Lake Michigan
Lake Huron
Lake Erie
Lake Ontario
L. Winnipeg
Missouri
Mississippi
Ohio
Colorado
Rio Grande
Bear Lake
Fort Chipewyan
Churchill
Inukjuak
Dawson Creek
Prince Rupert
Fort McMurray
Grande Prairie
Thompson
Labrador City
Prince George
Edmonton
Flin Flon
Red Deer
Prince Albert
Kamloops
Saskatoon
Calgary
Vancouver
Victoria
Moosonee
Swift Current
Regina
Seattle
Spokane
Brandon
Winnipeg
Timmins
Amos
Quebec
Fredericton
Portland
Williston
Grand Forks
Helena
Butte
Duluth
Ottawa
Montreal
Bismarck
Concord
Augusta
Boise
St. Paul
Minneapolis
Toronto
Rochester
Boston
Providence
Hartford
Twin Falls
Milwaukee
Detroit
Buffalo
Reno
Salt Lake City
Cheyenne
Scottsbluff
Chicago
Cleveland
New York
Philadelphia
Pittsburgh
San Francisco
Oakland
Provo
Boulder
Omaha
Indianapolis
Dayton
Baltimore
Denver
Washington D. C.
Kansas City
Cincinnati
Las Vegas
St. Louis
Louisville
Richmond
Norfolk
Nashville
Charlotte
Los Angeles
Santa Fe
Oklahoma City
Memphis
San Diego
Phoenix
Albuquerque
Columbia
Mexicali
Little Rock
Birmingham
Atlanta
Tucson
Dallas
Jackson
Tallahassee
Jacksonville
Hermosillo
Austin
Baton Rouge
New Orleans
Orlando
Chihuahua
San Antonio
Houston
Tampa
West Palm Beach
Nassau
Miami
THE BAHAMAS
Culiacan
Monterrey
Durango
La Paz
Ciudad
Havana


SUE THE BASTARDS


It's
Here
CANADA
U. S. A.
MEXICO
Hudson Bay
ATLANTIC
OCEAN
Gulf
of
Alberta
Manitoba
Saskatchewan
Ontario
Quebec
Newfoundland
British Columbia
Ottawa
Toronto
Rochester
Buffalo
Detroit
Cleveland
Pittsburgh
Montreal
Quebec
New York
Boston
Washington D. C.
Chicago
Minneapolis
St. Paul
Winnipeg
Regina
Calgary
Edmonton
Vancouver
Seattle
Portland
Salt Lake City
Denver
Kansas City
St. Louis
Dallas
Houston
New Orleans
Atlanta
Miami
Los Angeles
San Diego
Phoenix
Oakland
Nassau
THE BAHAMAS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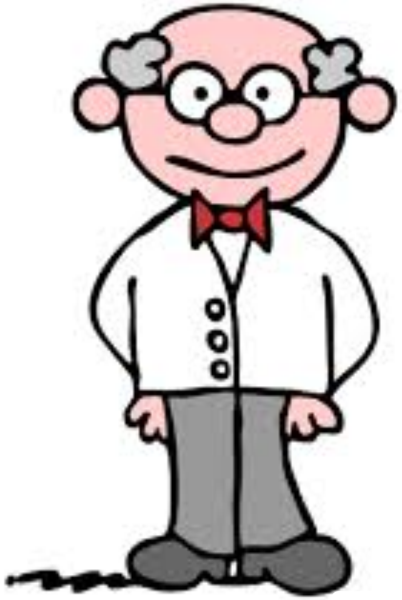
YOUR LAWYER


SUE THE BASTARDS


SUE THE BASTARDS


Outline

The conventional wisdom

The conventional wisdom is wrong!

When the conventional wisdom is not so wrong

Beyond the conventional wisdom

Experiments and possible experiments

The moral(s) of the story

Outline

The conventional wisdom

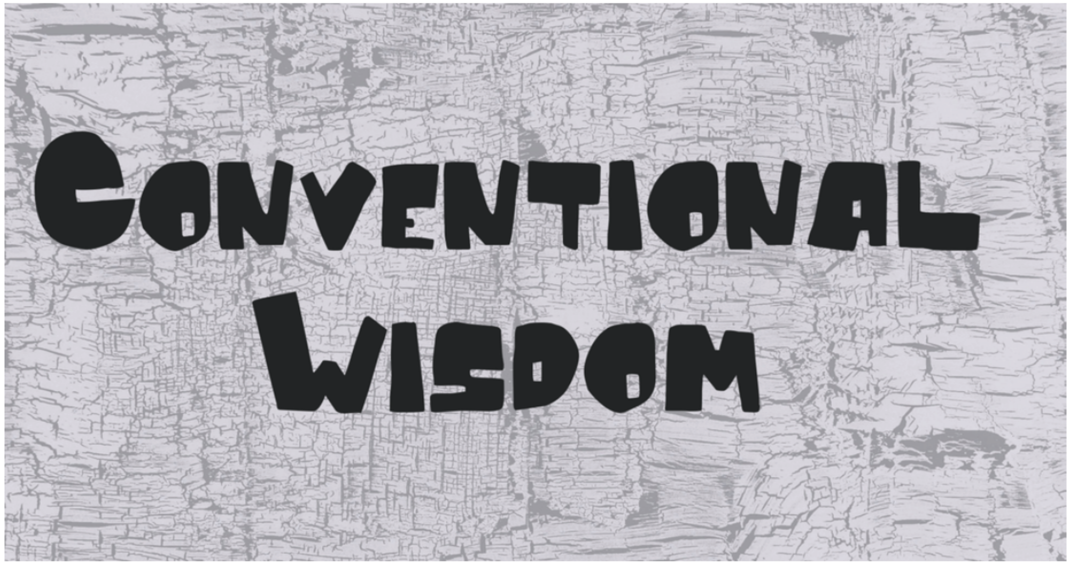
The conventional wisdom is wrong!

When the conventional wisdom is not so wrong

Beyond the conventional wisdom

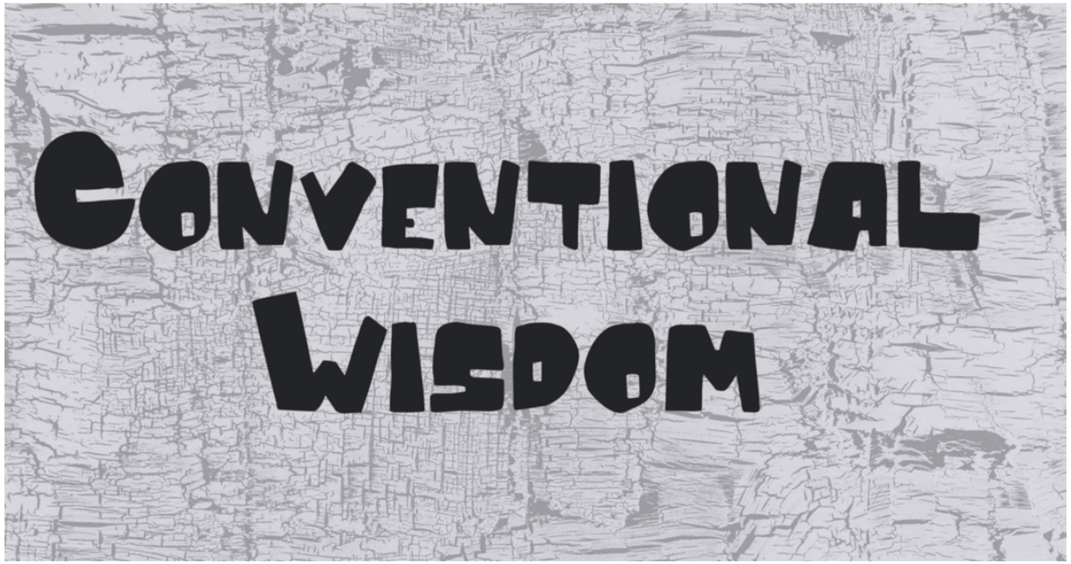
Experiments and possible experiments

The moral(s) of the story



Effective mass and Bloch oscillations

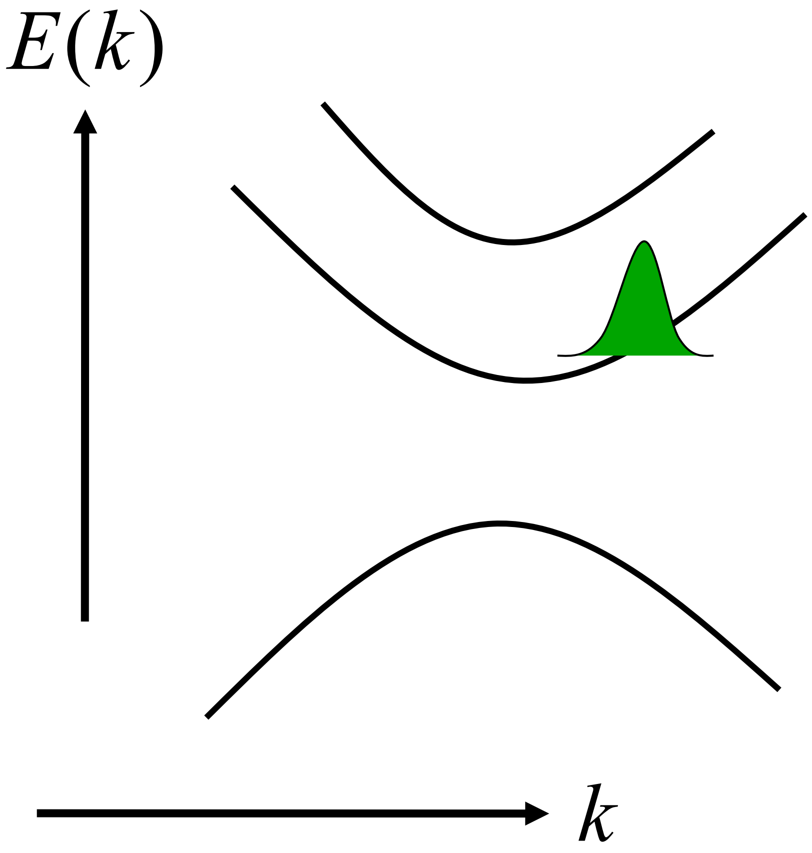
The anomalous velocity



Effective mass and Bloch oscillations

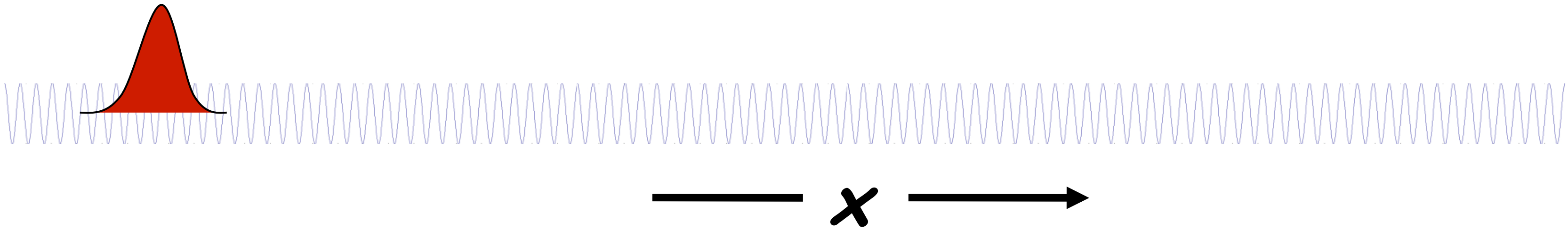
The anomalous velocity

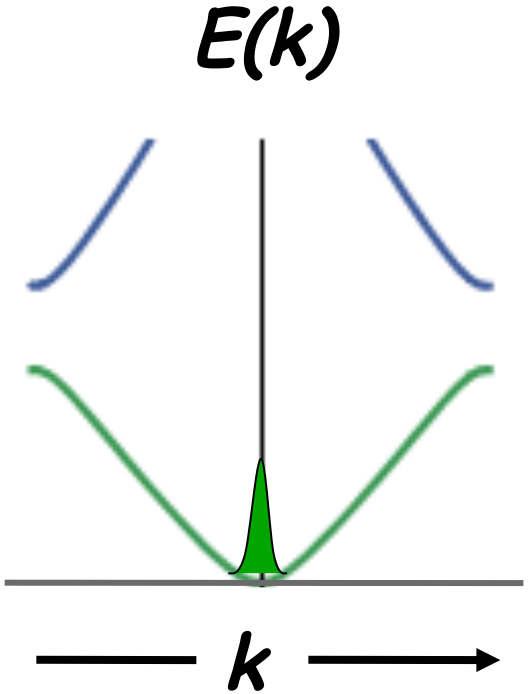
Effective mass "theorem"

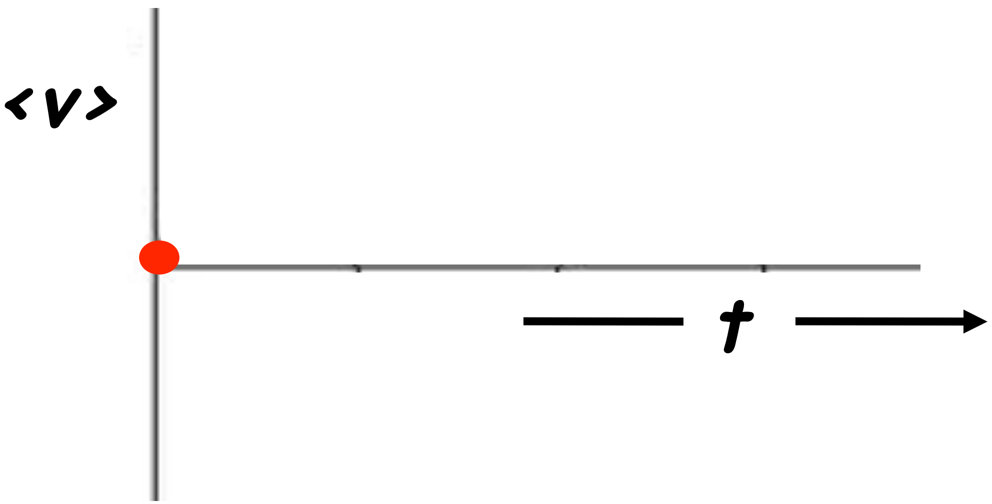


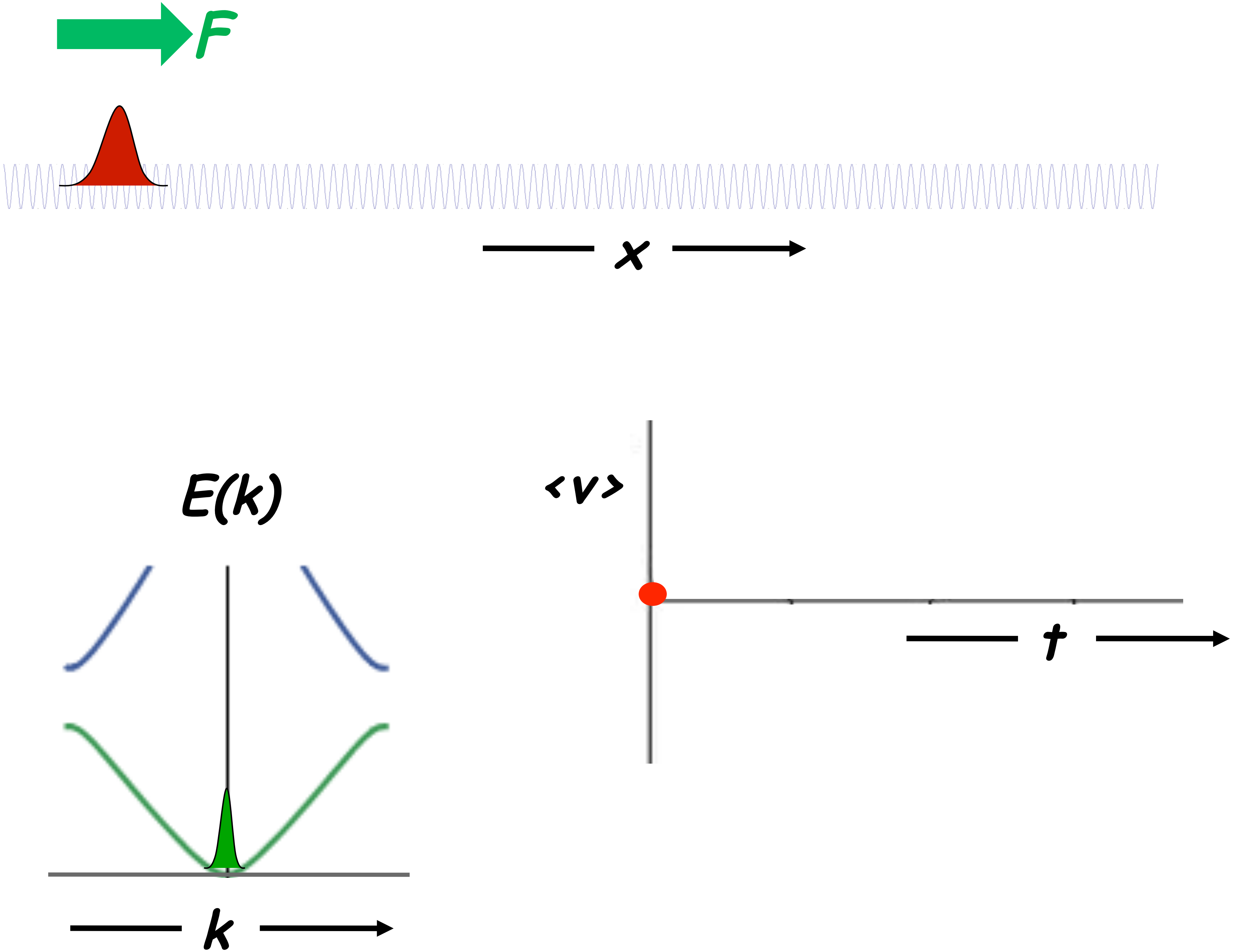
$$\langle a \rangle = \frac{F}{m^*(k)}$$

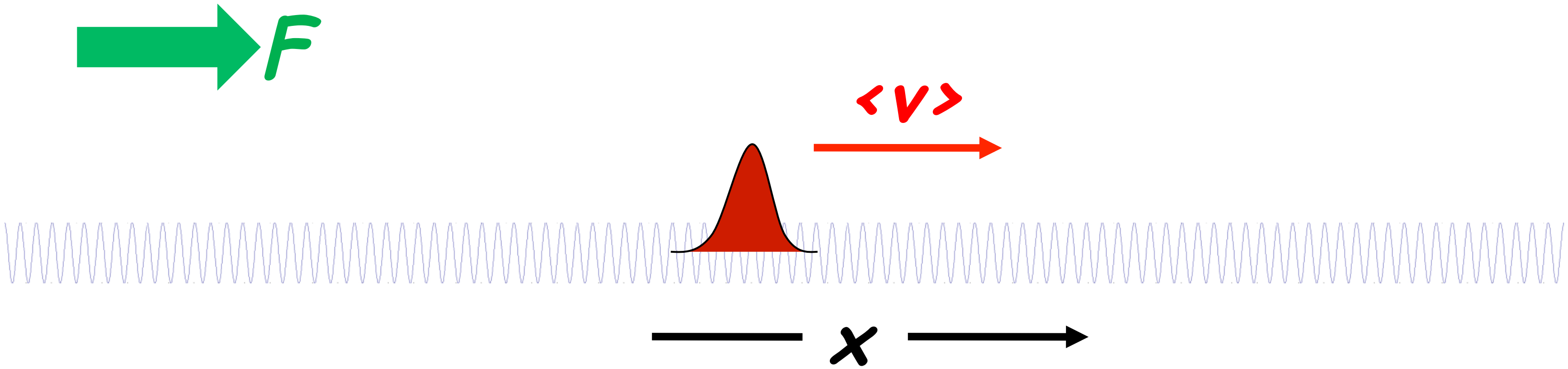
$$\frac{1}{m^*(k)} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$

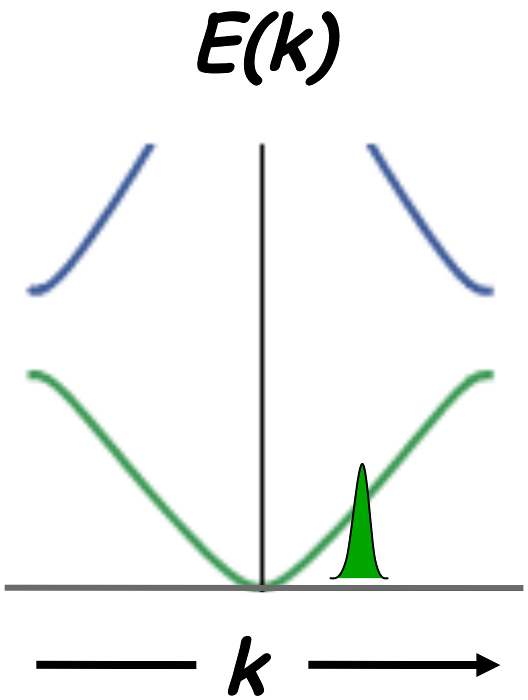

x

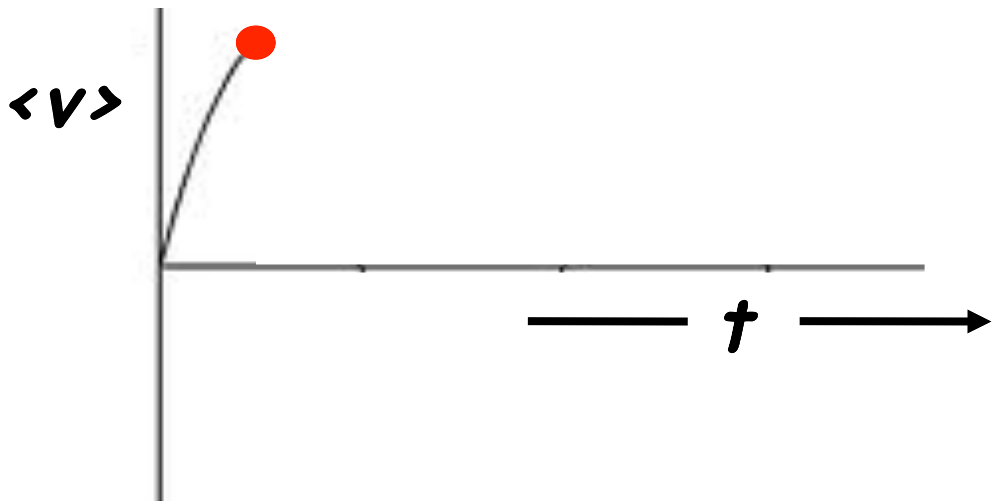

E(k)
k


<v>
t

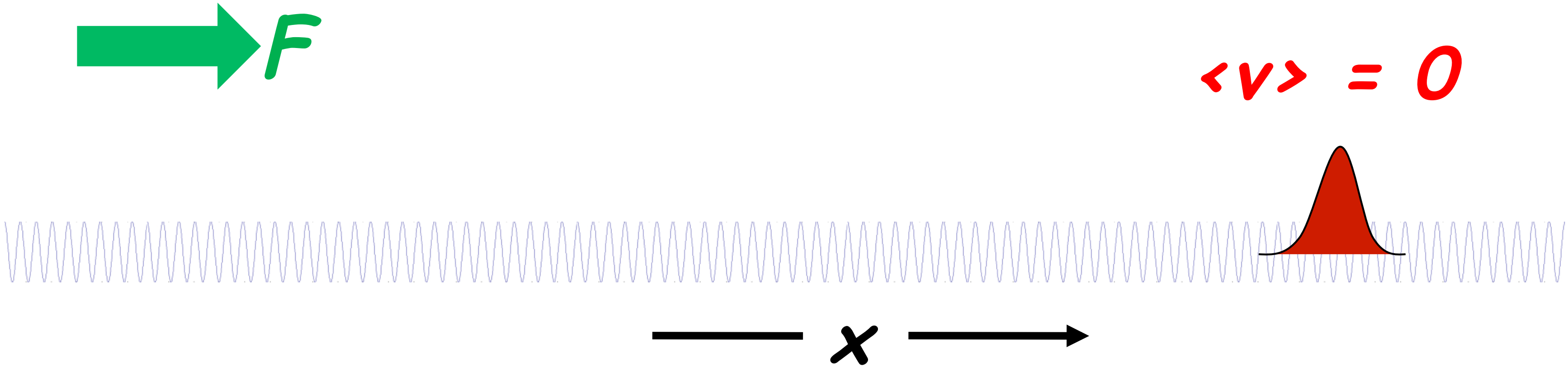

F
x
E(k)
k
<v>
t

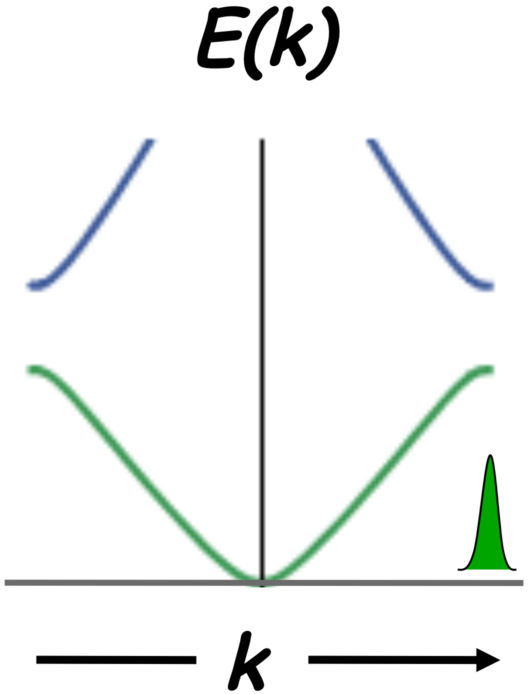


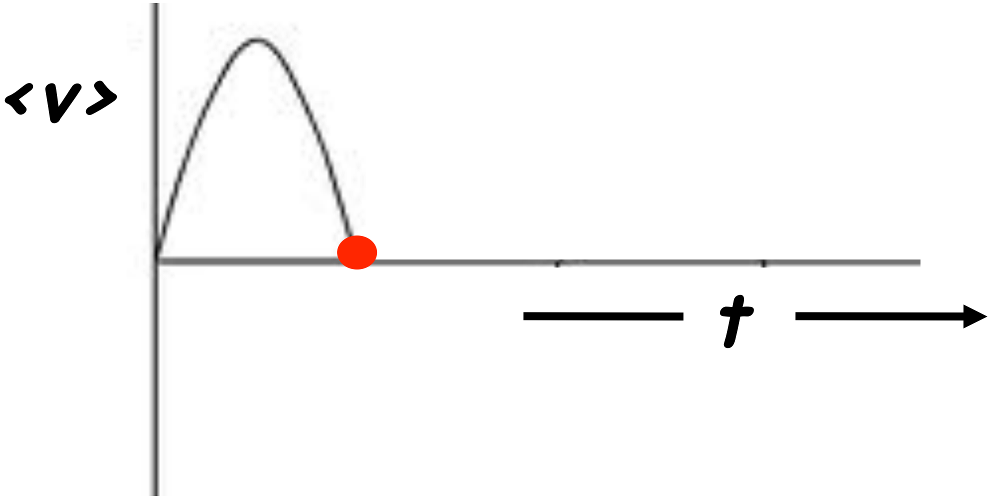


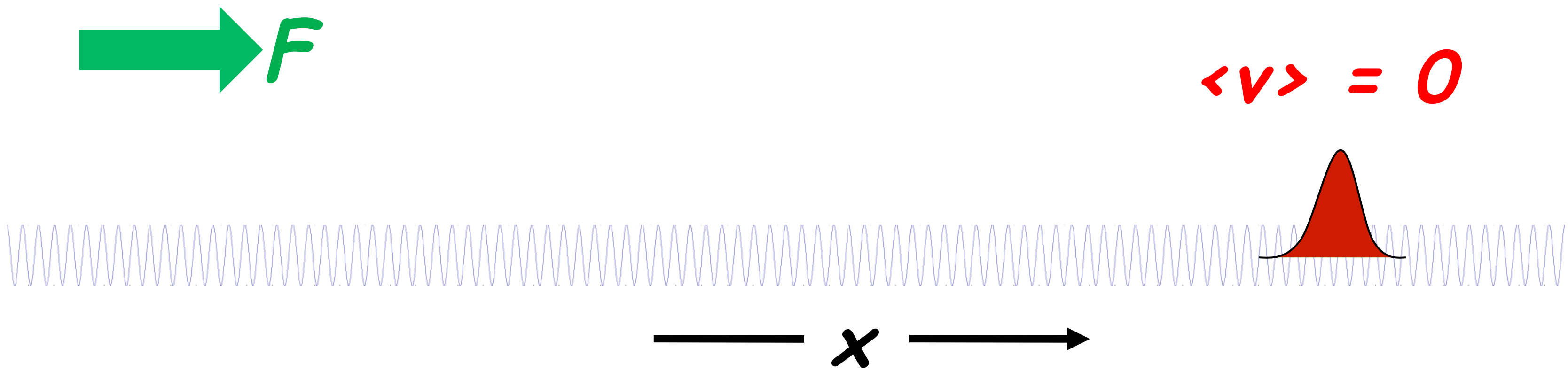


$$\langle v \rangle = \frac{d\omega(k)}{dk}$$


F
<v> = 0
x


E(k)
k


<v>
t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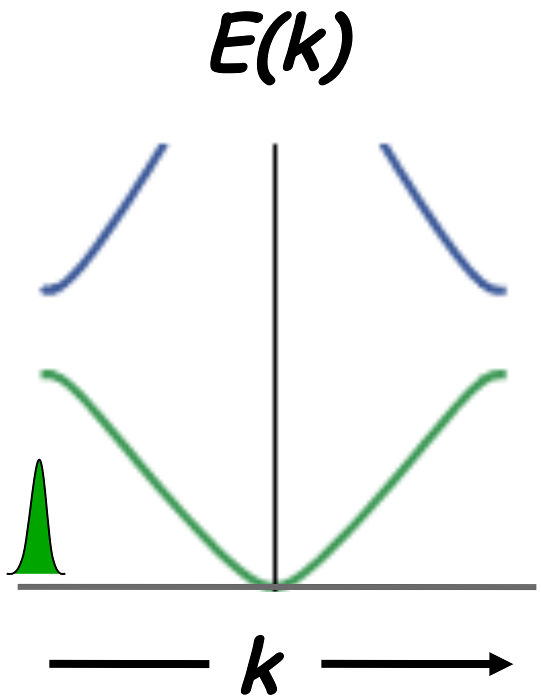
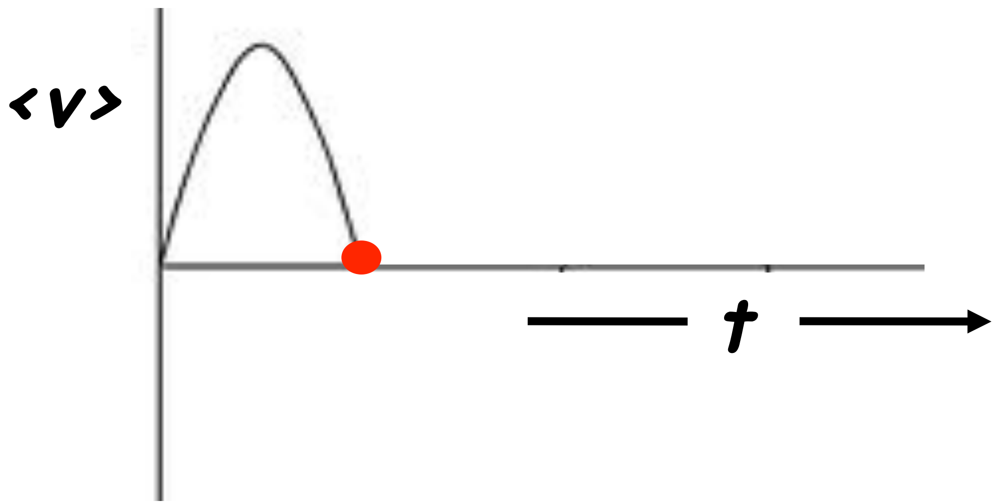
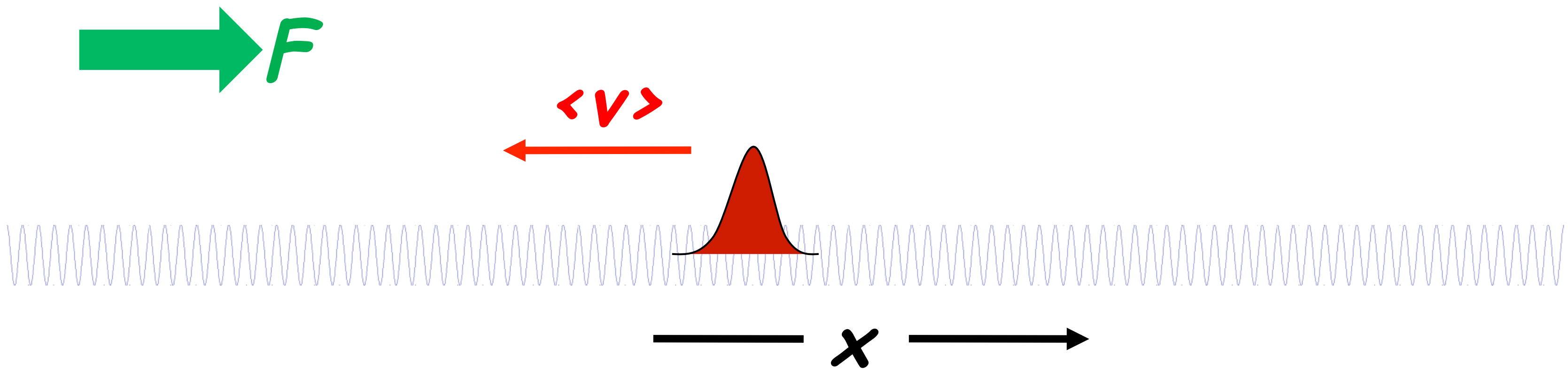
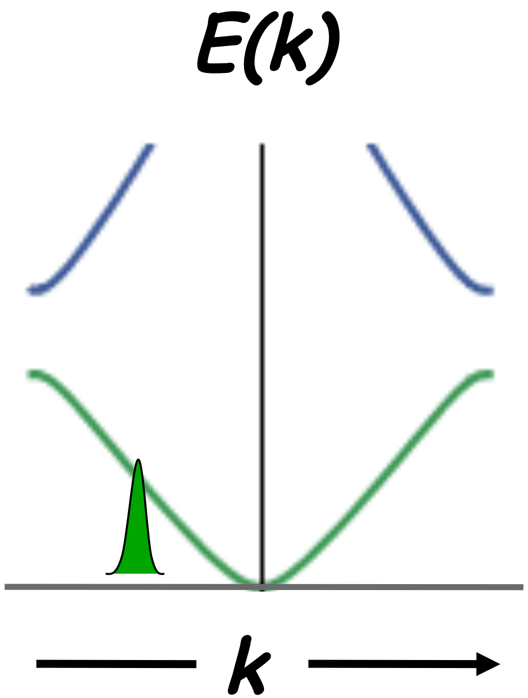
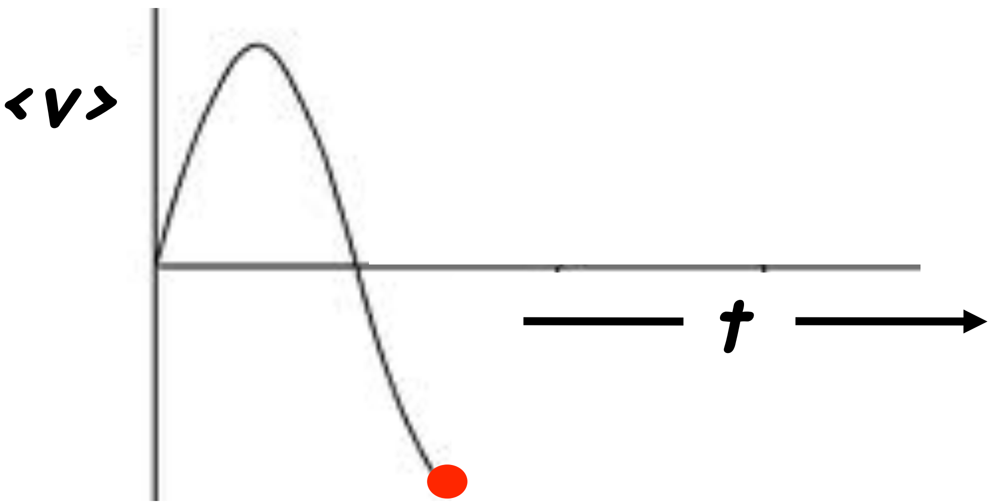
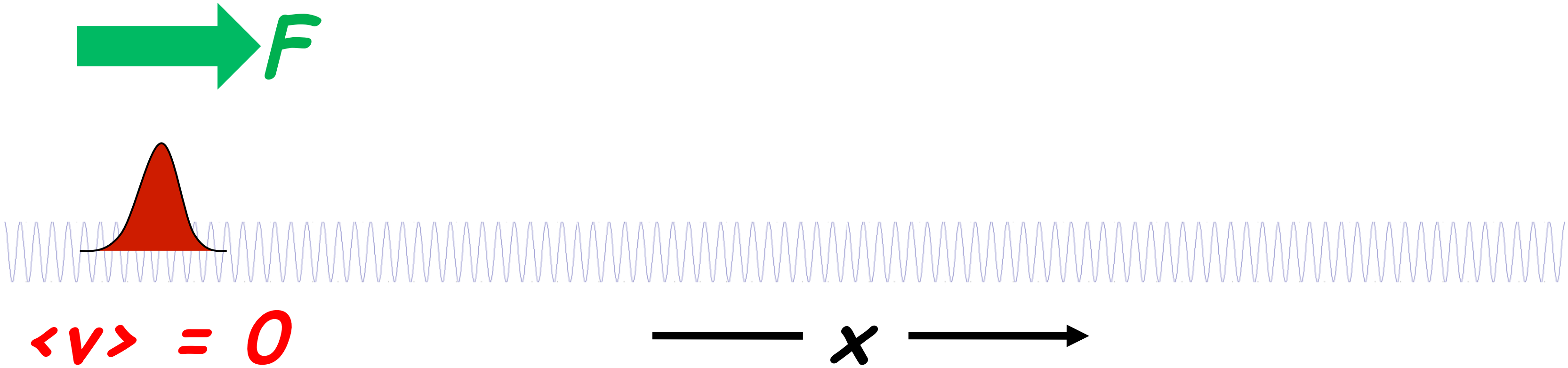

E(k)
k

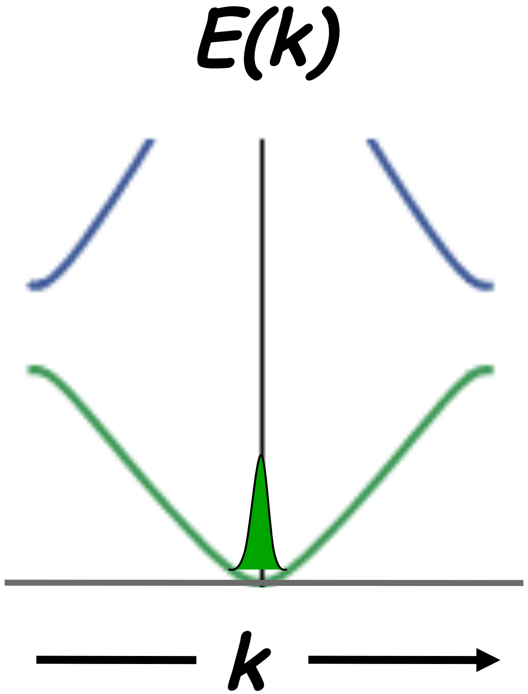

<v>
t

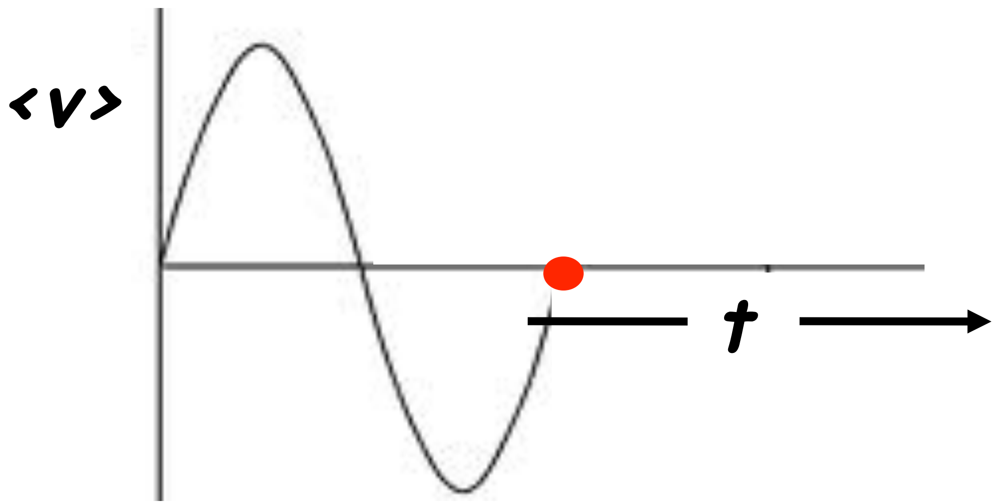

F
<v>
x

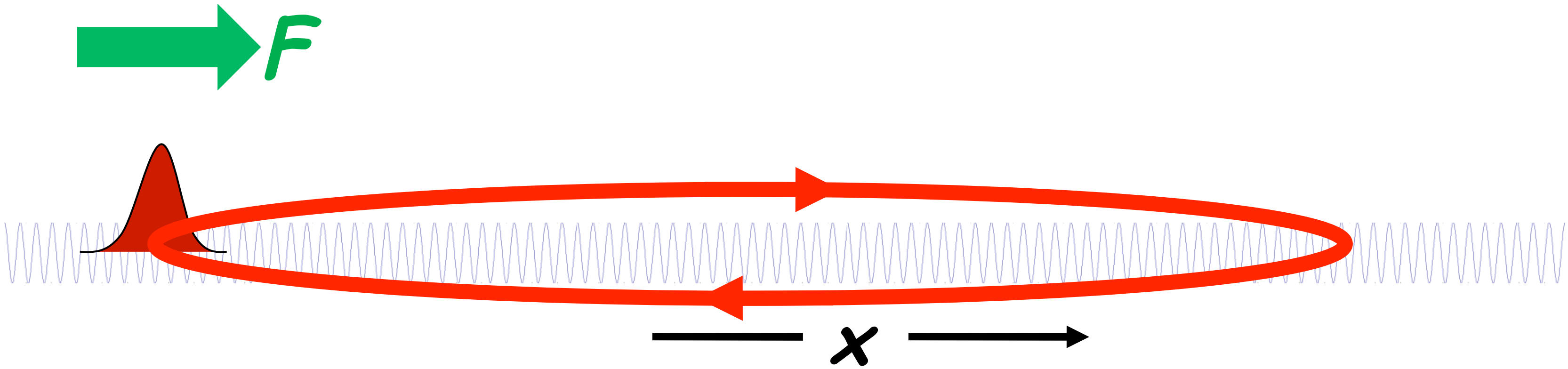

E(k)
k

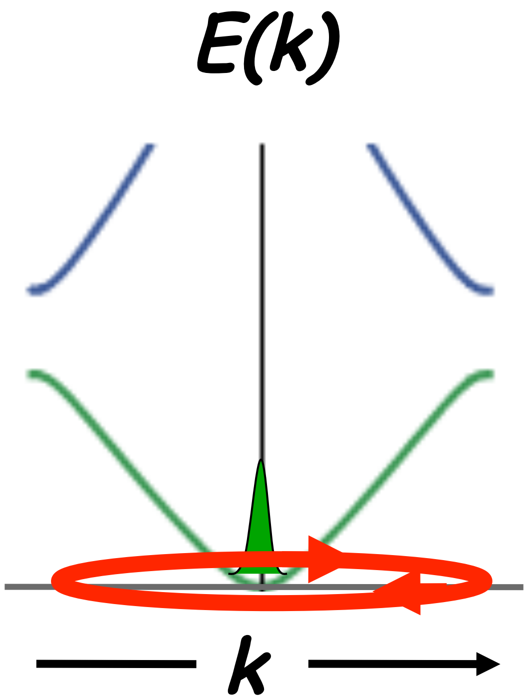

<v>
t

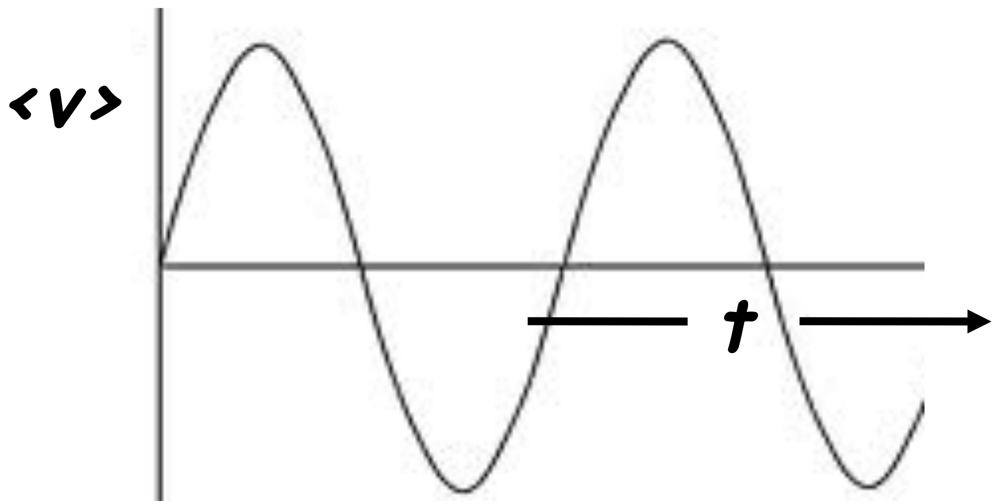

F
<v> = 0
x


E(k)
k


<v>
t



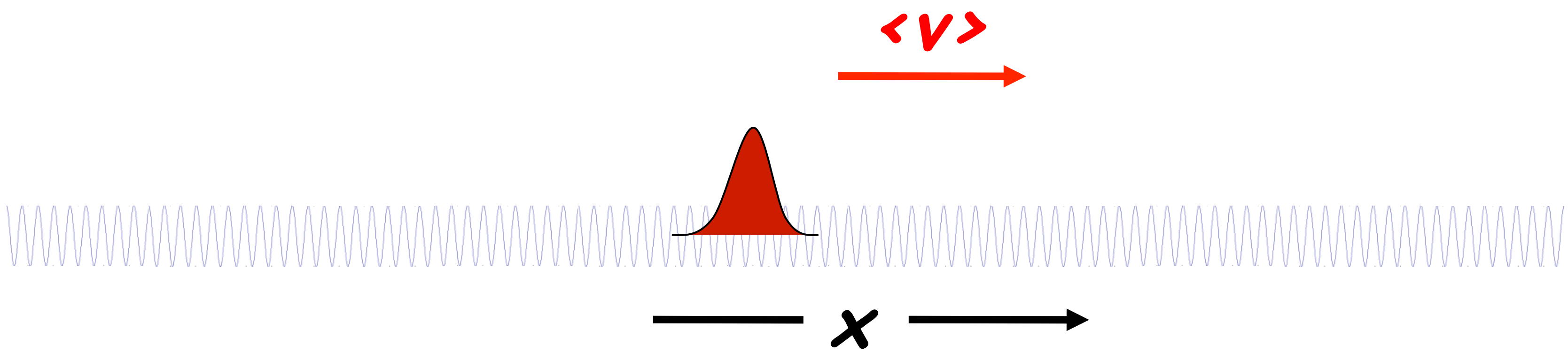




Bloch oscillations

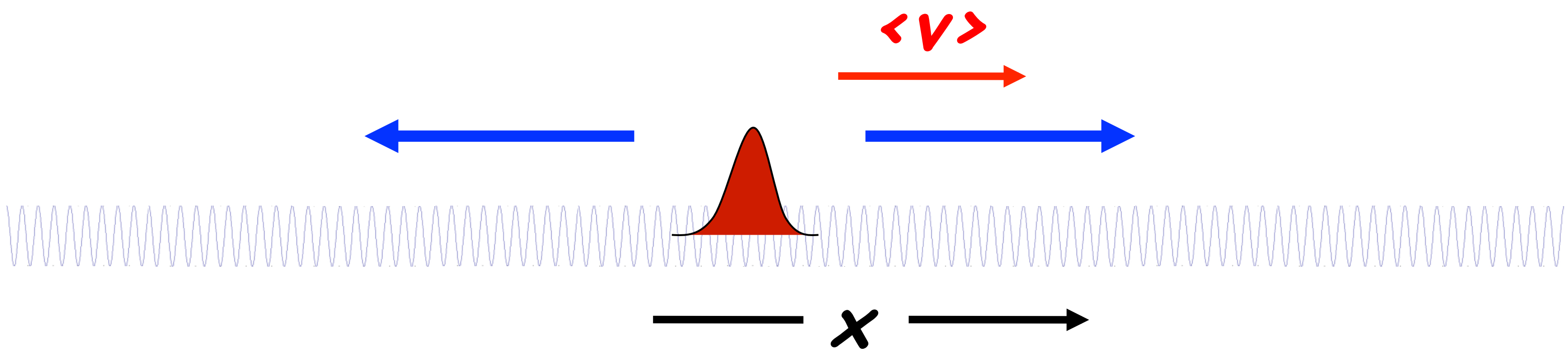
Difficulties in observing Bloch oscillations

Wavepacket spreading



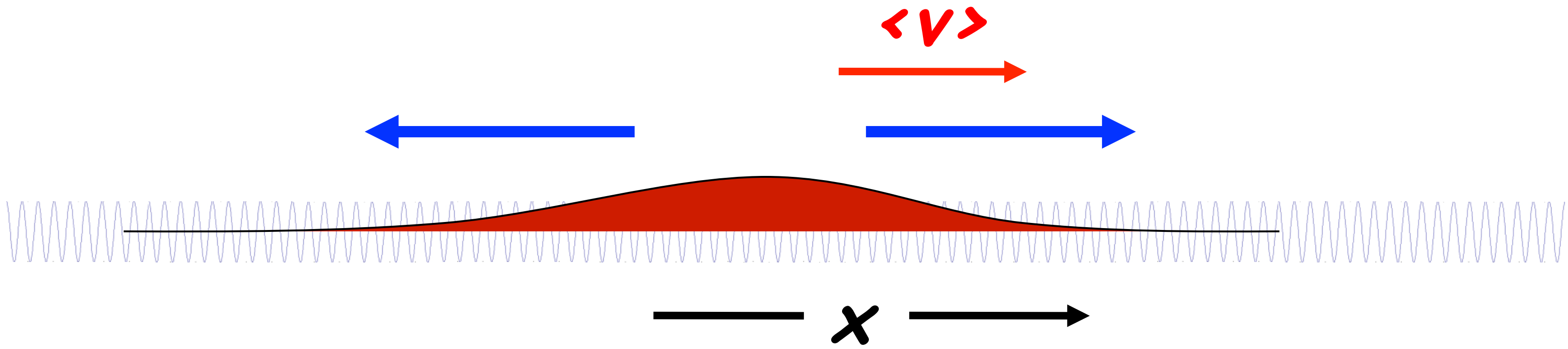
Difficulties in observing Bloch oscillations

Wavepacket spreading



Difficulties in observing Bloch oscillations

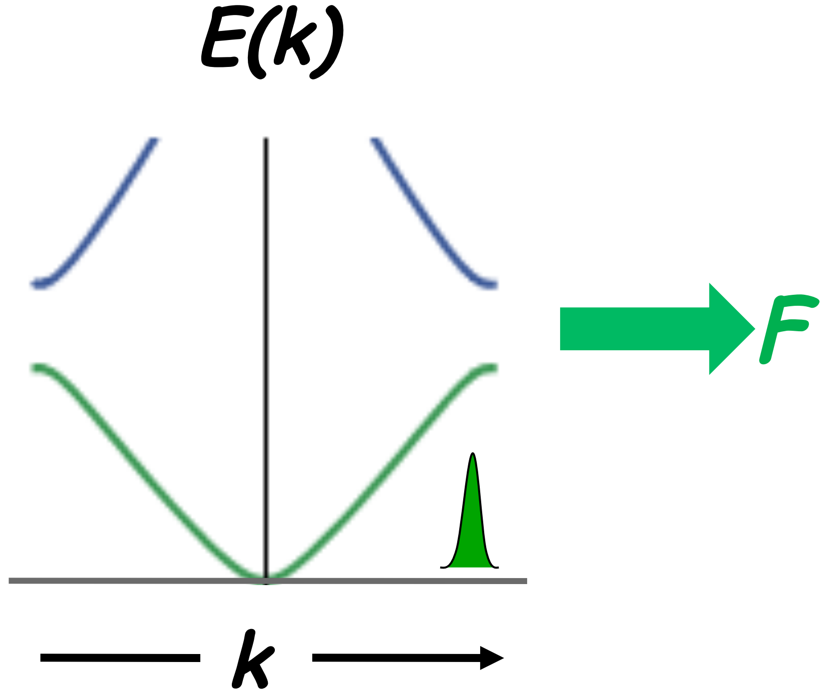
Wavepacket spreading



Difficulties in observing Bloch oscillations

Wavepacket spreading

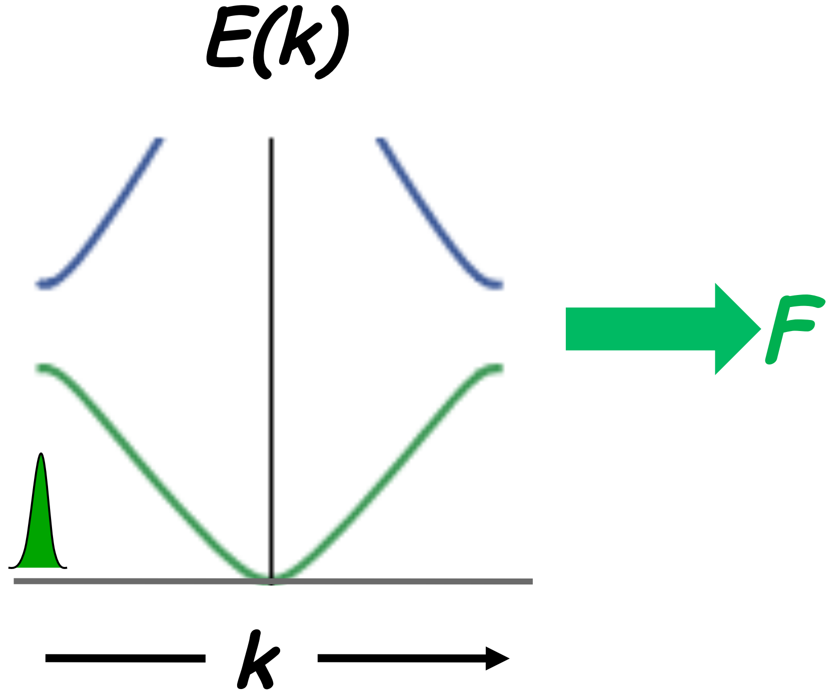
Zener tunneling



Difficulties in observing Bloch oscillations

Wavepacket spreading

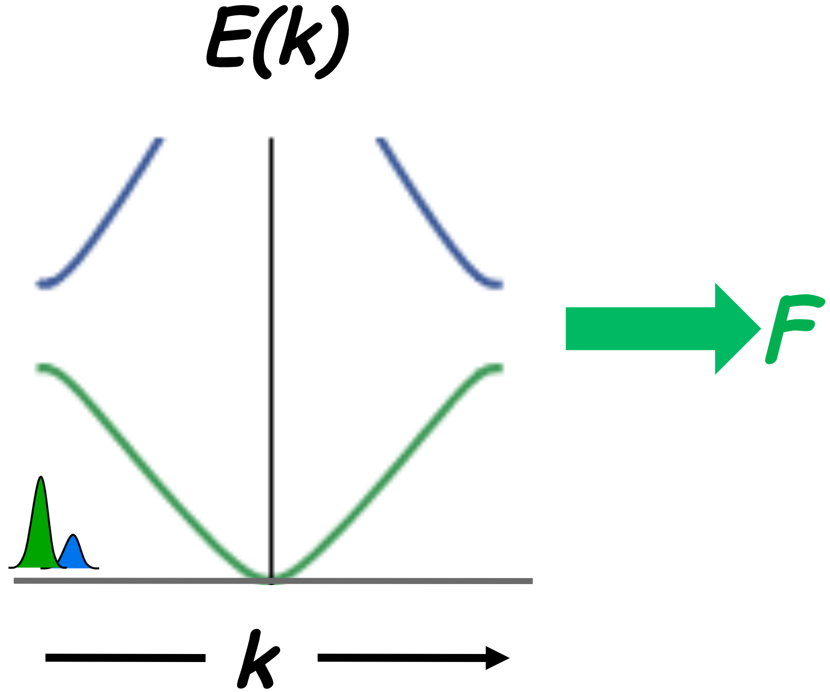
Zener tunneling



Difficulties in observing Bloch oscillations

Wavepacket spreading

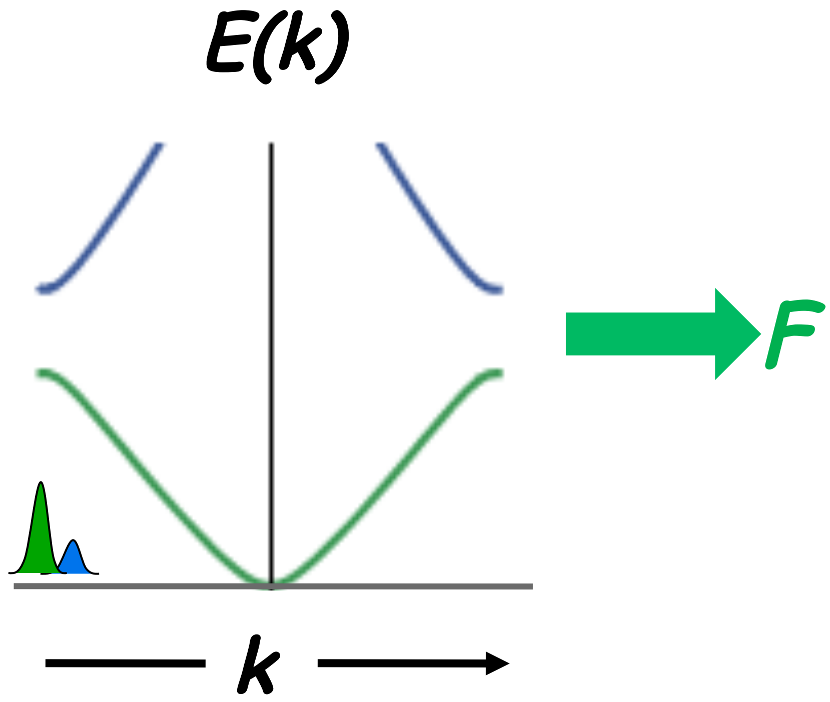
Zener tunneling

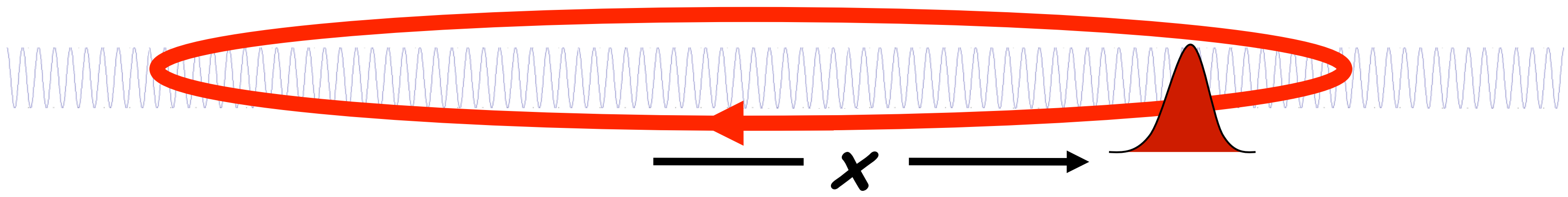


Difficulties in observing Bloch oscillations

Wavepacket spreading

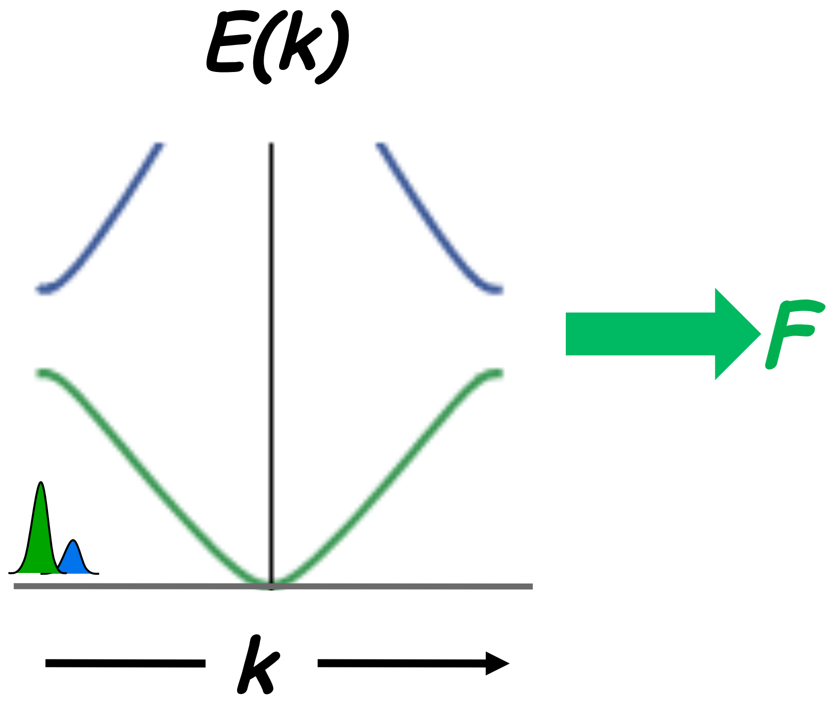
Zener tunneling

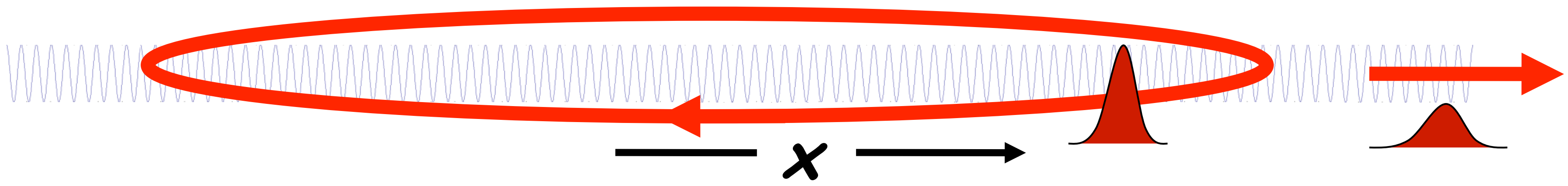




Difficulties in observing Bloch oscillations

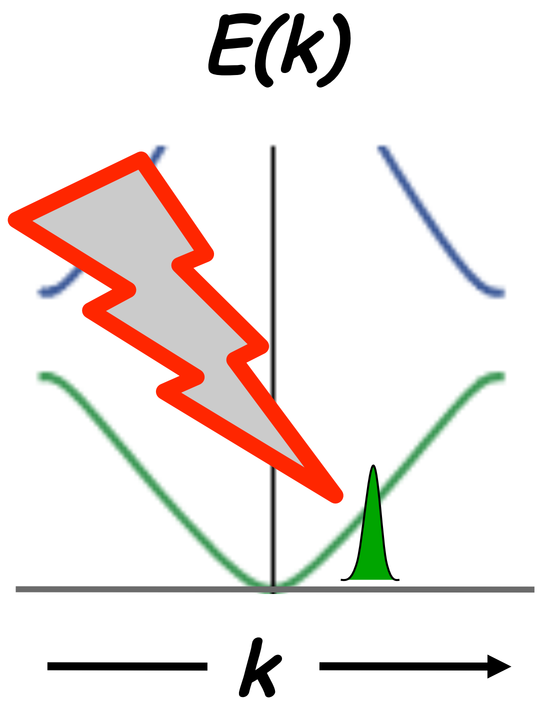
Wavepacket spreading
Zener tunneling

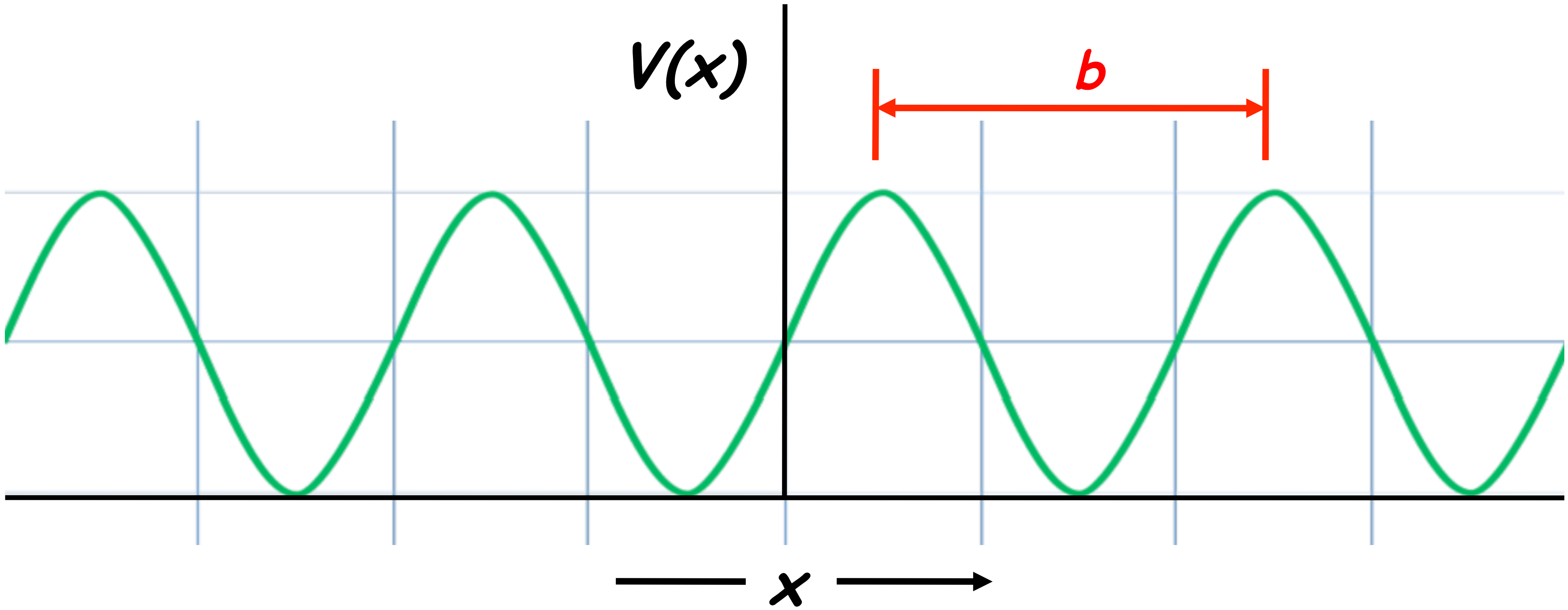




Difficulties in observing Bloch oscillations

- Wavepacket spreading
- Zener tunneling
- Scattering processes





$$t_{Bloch} = \frac{h}{bF} = \frac{h}{beE}$$

$$b = 0.5nm$$

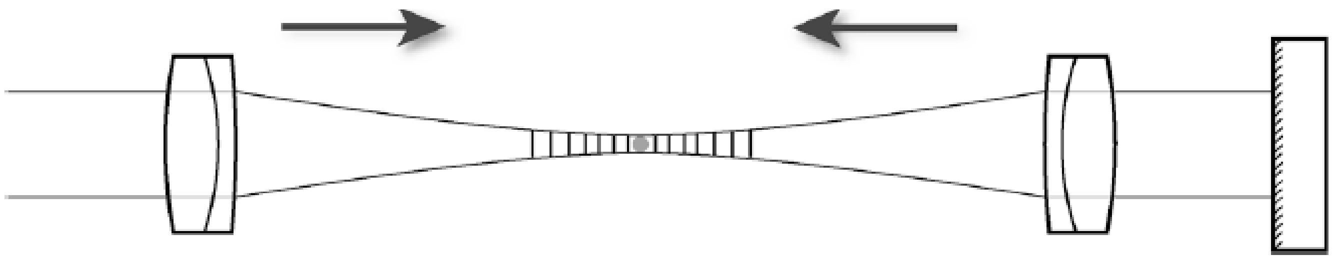
$$E = \frac{170\ kV}{cm} = \frac{17\ V}{\mu m}$$

$$t_{Bloch} \approx 500\ fs$$

Semiconductor superlattices

Waschke et al.,
Phys. Rev. Lett. 70, 3319 (1993)

Optical lattices



I. Bloch, M. Grenier (2005)

Ultracold atoms: small spread in momentum

Adiabatic loading: wavepacket in one band

Acceleration of the lattice: inertial force in the lattice frame

Turn off the lattice: measure the velocity at any desired time

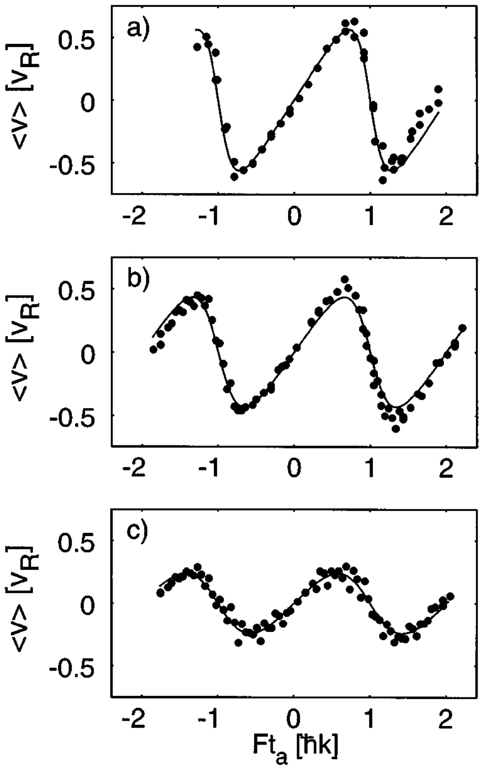
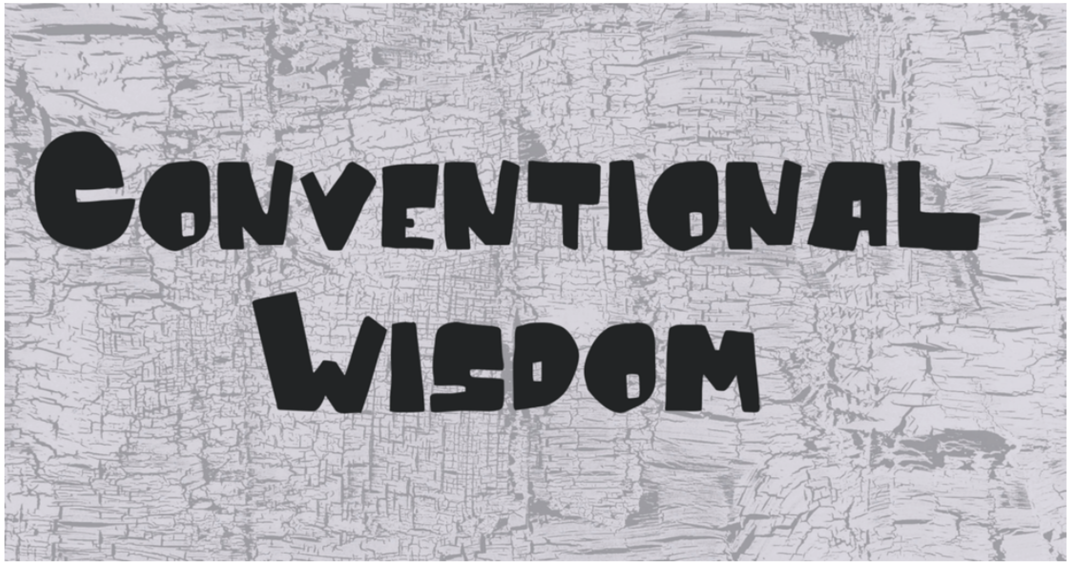


FIG. 3. Mean atomic velocity $\langle v \rangle$ as a function of the acceleration time t_a for three values of the potential depth: (a) $U_0 = 1.4E_R$, (b) $U_0 = 2.3E_R$, (c) $U_0 = 4.4E_R$. The negative values of Ft_a were measured by changing the sign of F. Solid lines: theoretical prediction.

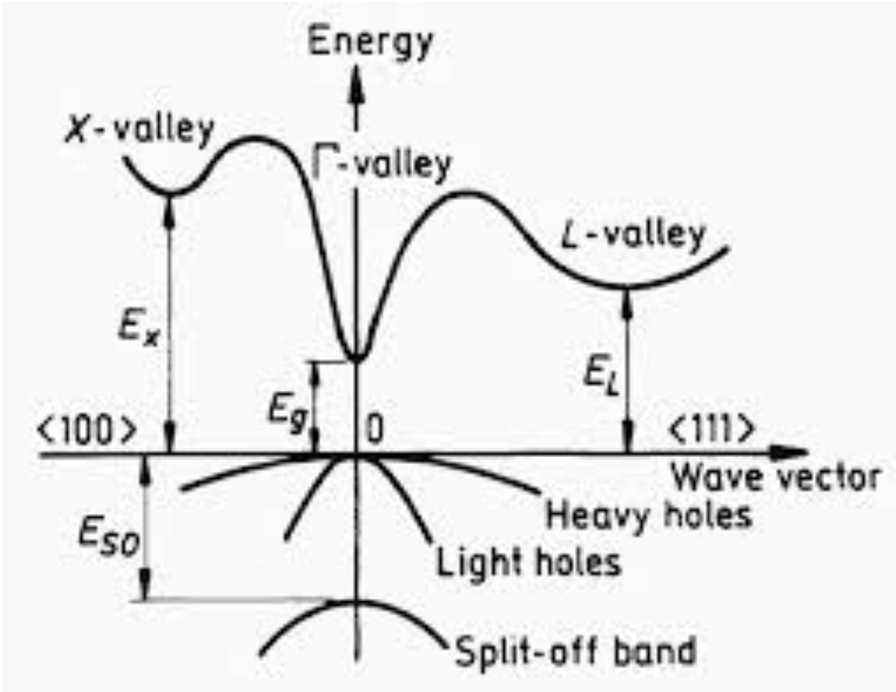
Dehan et al., Phys. Rev. Lett. 76, 4508 (1996)
Peik et al., Phys. Rev. A55, 2989 (1997)

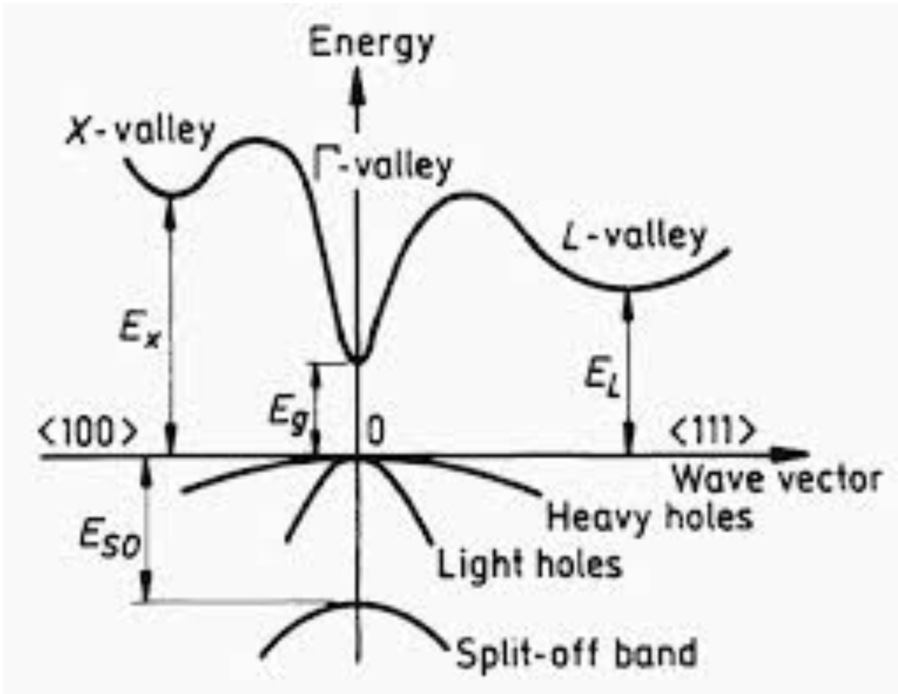
Acceleration of a BEC:
Denschlag et al, J. Phys. B35, 3095 (2002)



Effective mass and Bloch oscillations

The anomalous velocity


Energy
X - valley
Γ-valley
L-valley
E_x
E_g
E_L
<100>
0
<111>
Wave vector
Heavy holes
Light holes
E_{SO}
Split-off band



Energy
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E_x
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0
⟨100⟩
⟨111⟩
Wave vector
Heavy holes
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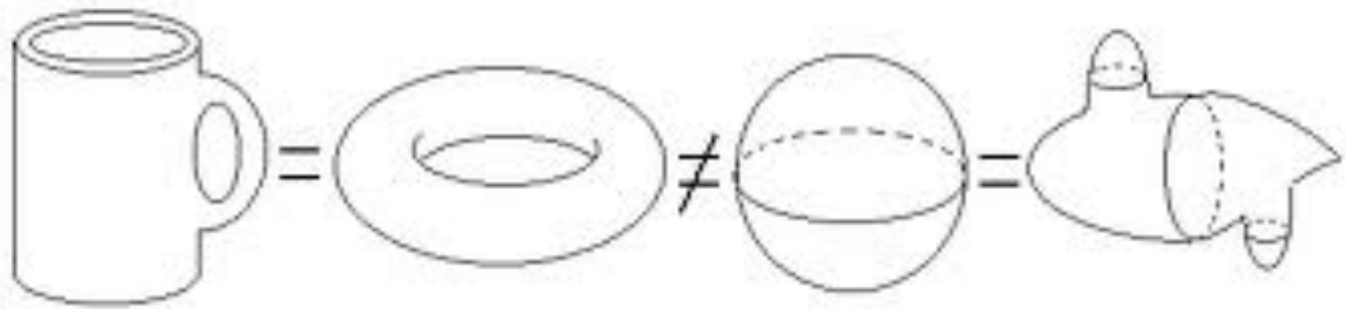

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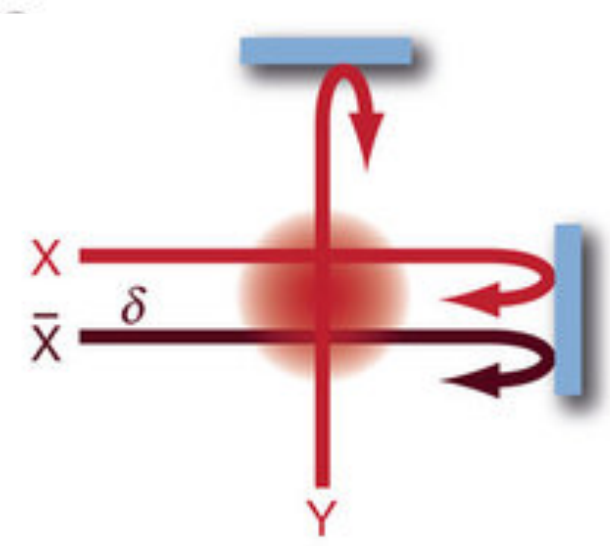
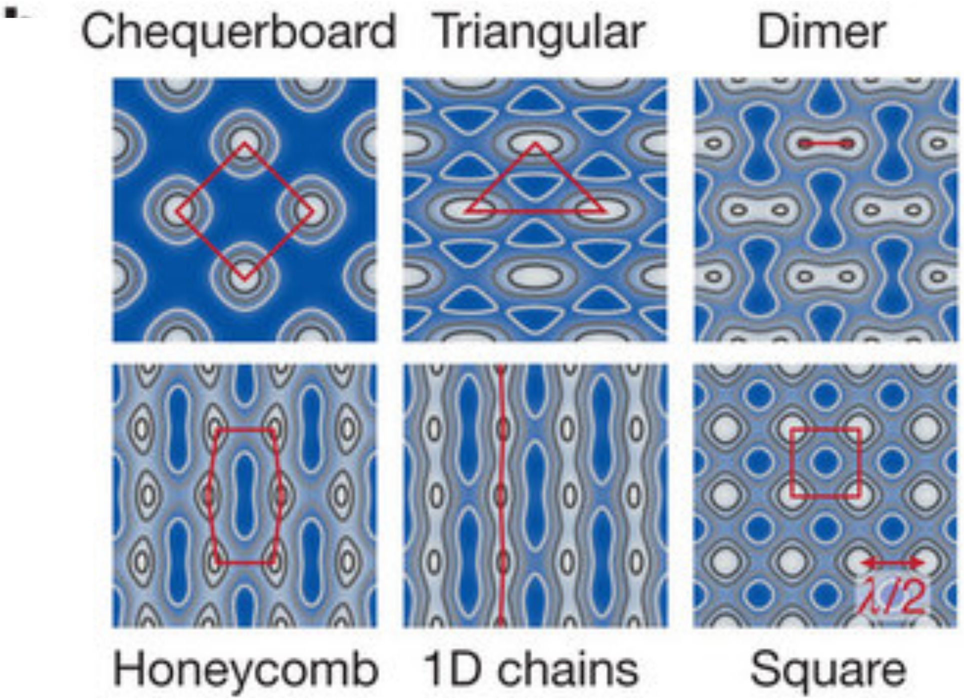
2D atomic lattices

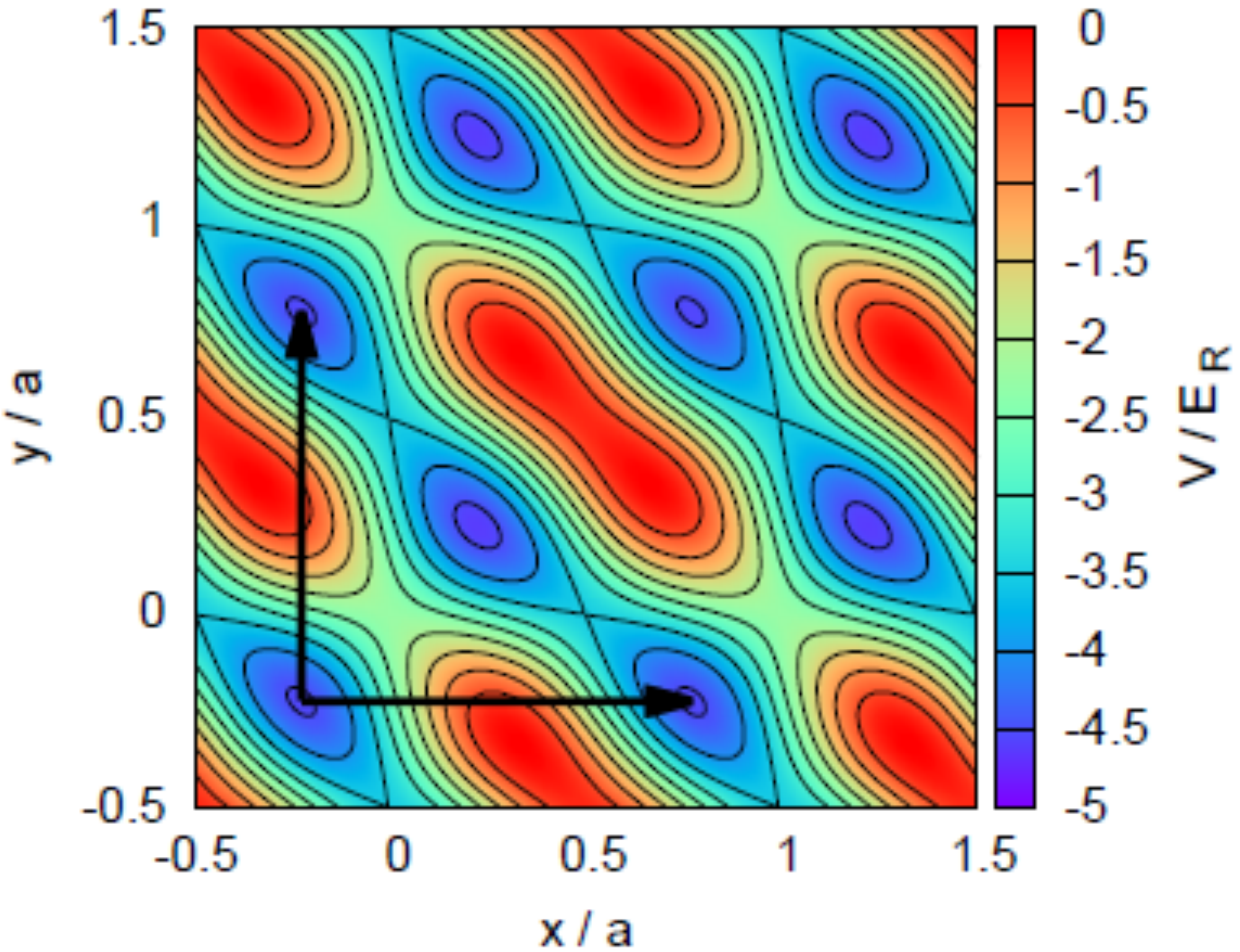




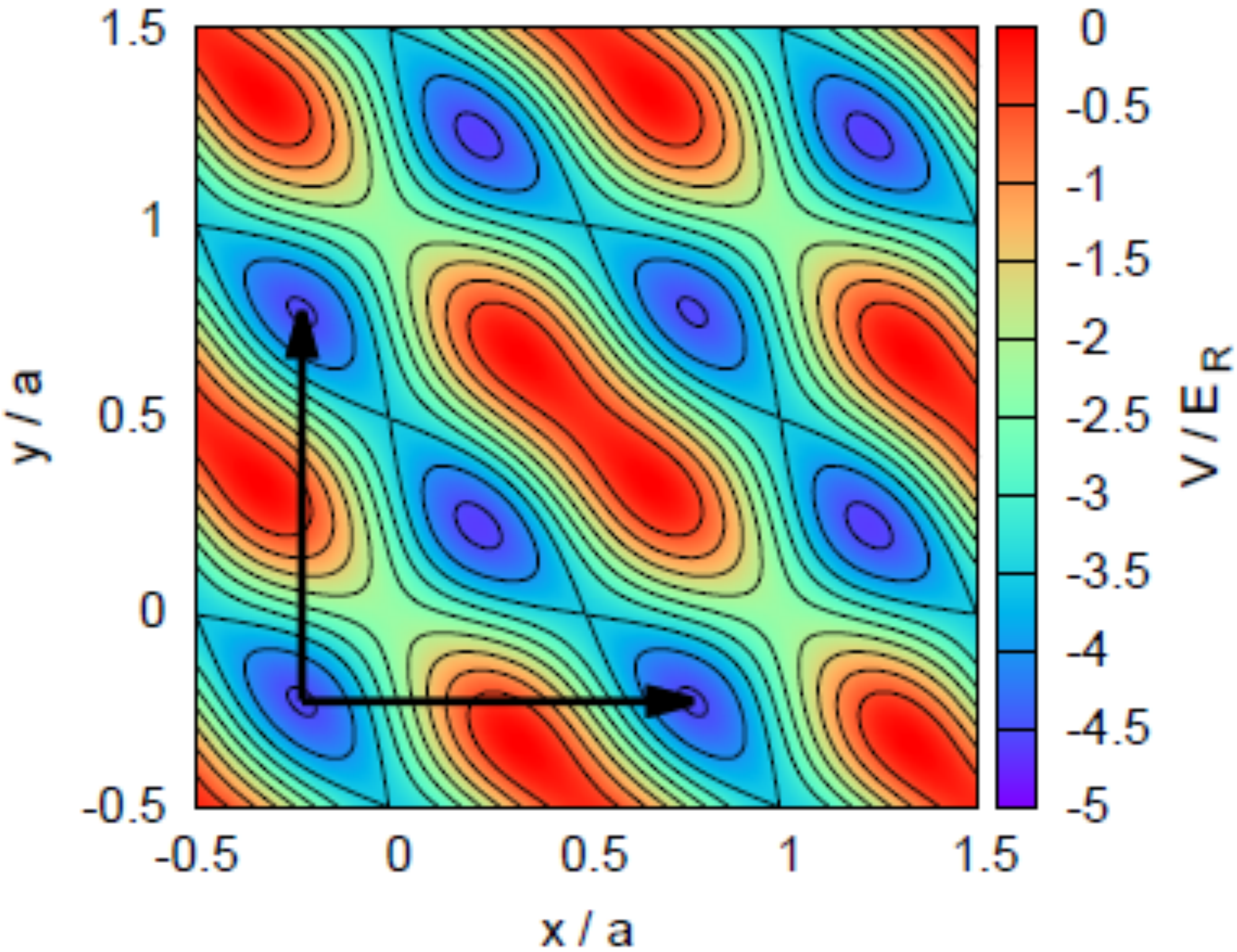
$$V(x,y) = -\frac{V_{\bar{X}}}{2}\cos(k_R(x+y)+\theta) - \frac{V_X}{2}\cos(k_R(x+y)) - \frac{V_Y}{2}\cos(k_R(x-y))$$

$$-\sqrt{V_X V_Y}\left(\cos(k_R x) + \cos(k_R y)\right) - \frac{1}{2}\left(V_{\bar{X}} + V_X + V_Y\right)$$

Tarruell et al., Nature 483, 202 (2012)

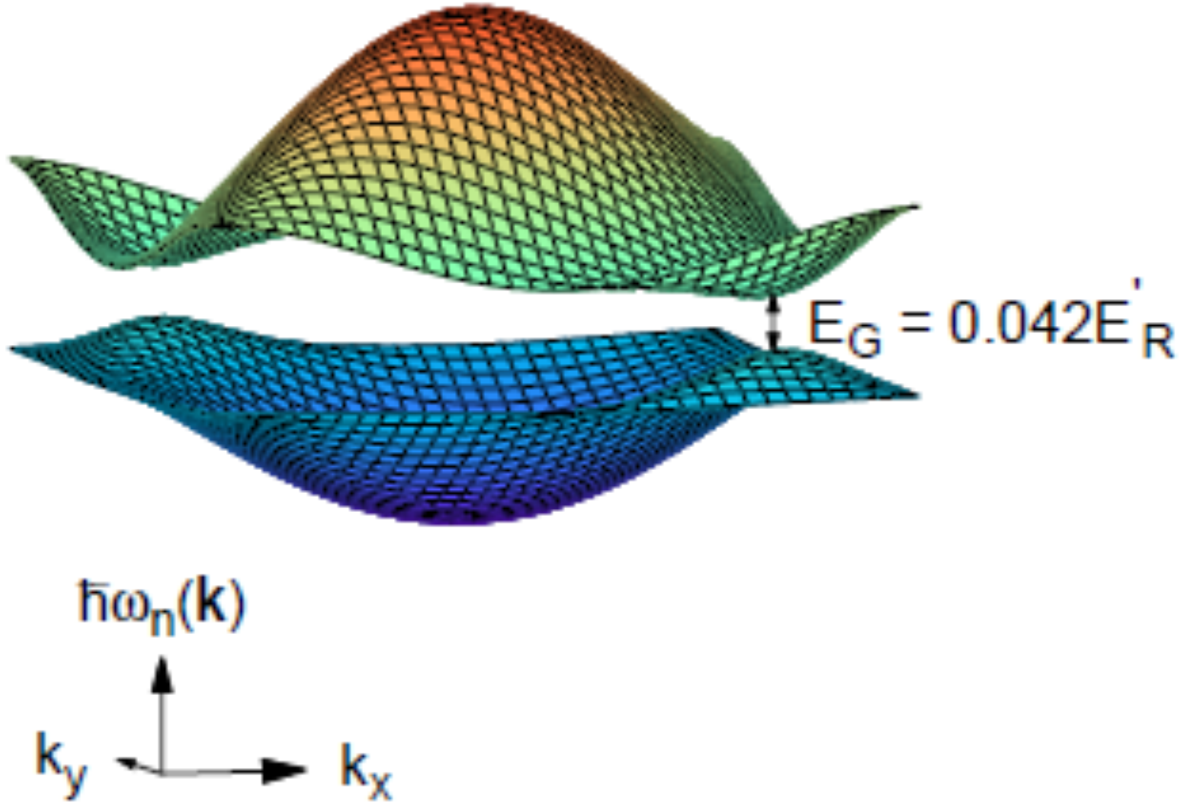


$V_X = 0.25 E_R$ $\quad$ $V_{\bar{X}} = 3.5 E_R$ $\quad$ $V_Y = 1.0 E_R$



$V_X = 0.25 E_R \qquad V_{\bar{X}} = 3.5 E_R \qquad V_Y = 1.0 E_R$

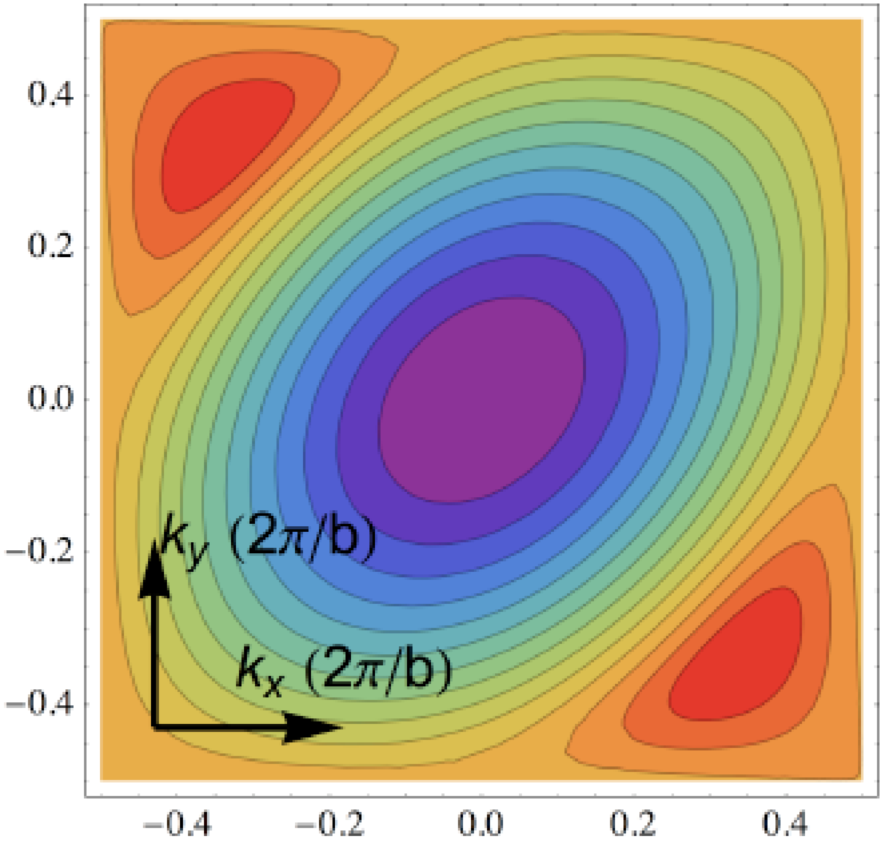
$\theta = 1.02\pi$



$V_X = 0.25E_R \qquad V_{\bar{X}} = 3.5E_R \qquad V_Y = 1.0E_R$

$\theta = 1.02\pi$

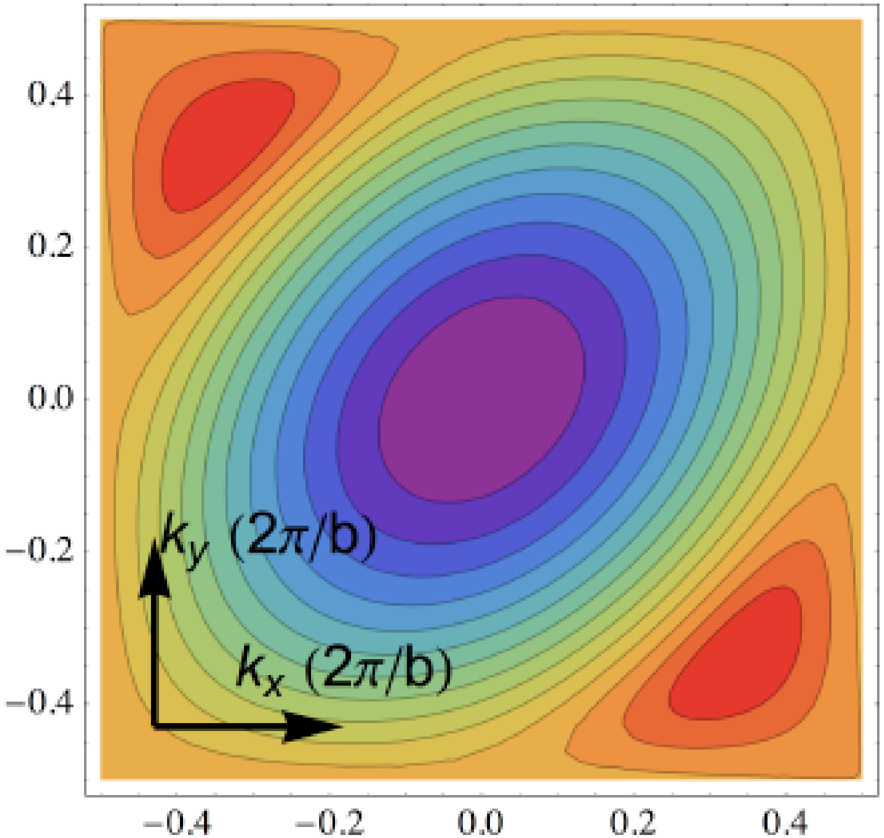
Lowest band



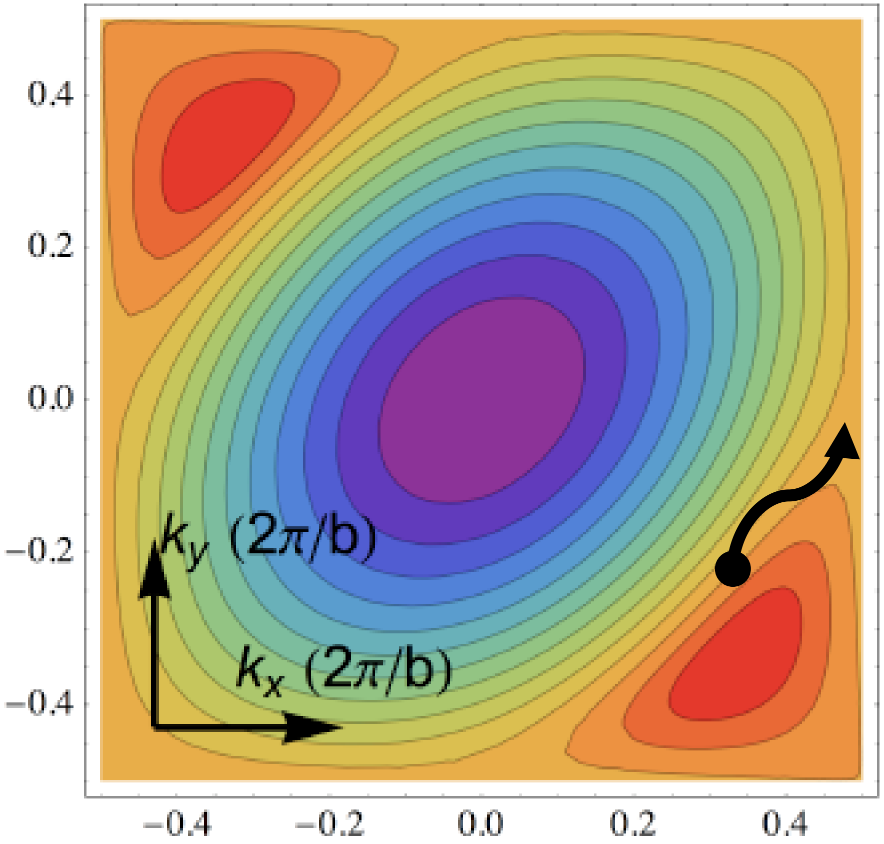
Lowest band

$$\phi_{n\mathbf{k}}(\mathbf{r}) = \frac{u_{n\mathbf{k}}(\mathbf{r})}{2\pi} e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$u_{n\mathbf{k}}(\mathbf{r}+\mathbf{R}) = u_{n\mathbf{k}}(\mathbf{r})$$



Lowest band



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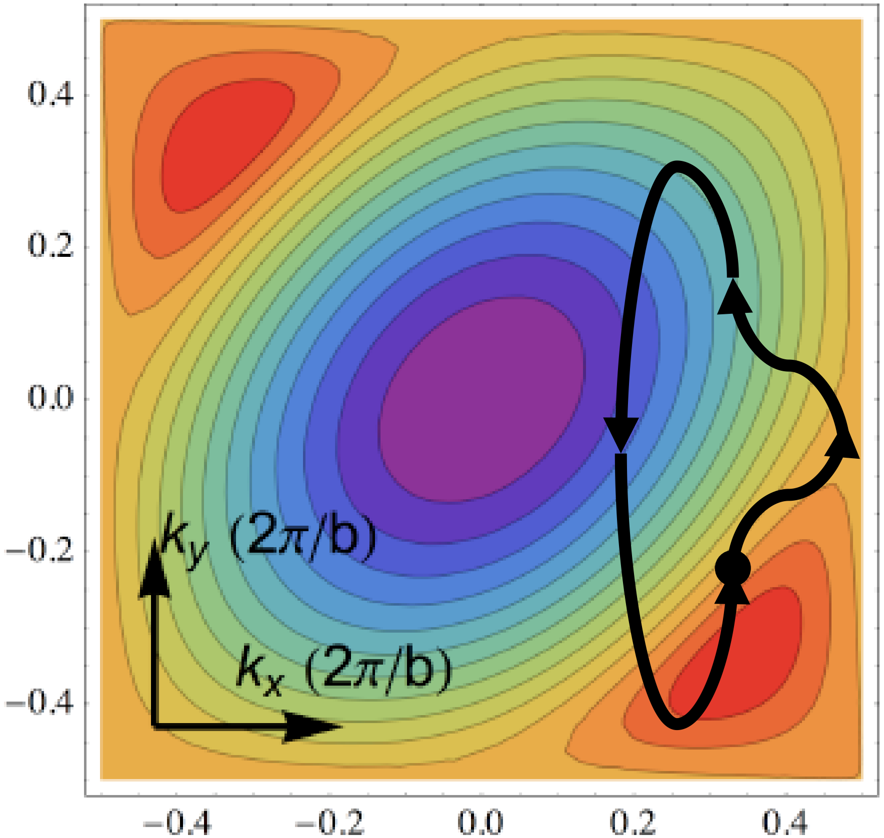
$$u_{n\mathbf{k}}(\mathbf{r}+\mathbf{R}) = u_{n\mathbf{k}}(\mathbf{r})$$

$$i\frac{\partial u_{n\mathbf{k}}(\mathbf{r})}{\partial \mathbf{k}} = \sum_m u_{m\mathbf{k}}(\mathbf{r})\xi_{mn}(\mathbf{k})$$

$$\xi_{mn}(\mathbf{k}) = \frac{\hbar}{im}\frac{\mathbf{p}_{mn}(\mathbf{k})}{E_{mn}(\mathbf{k})}$$

for $\mathrm{m} \neq n$

Lowest band



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Lowest band

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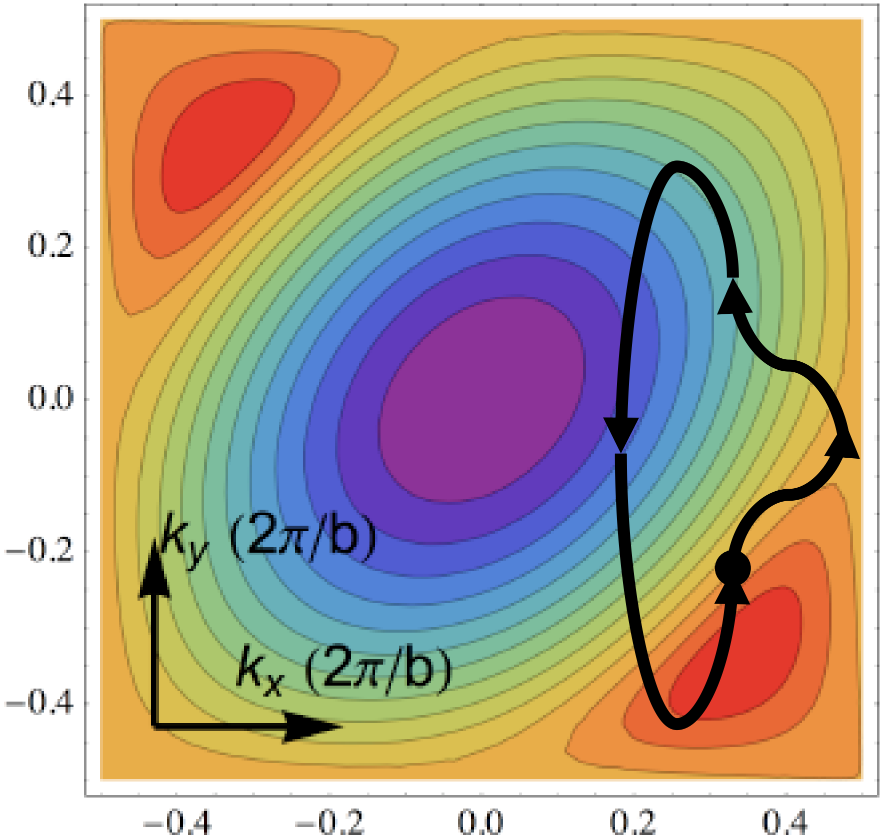

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for m $\neq n$

Berry phase

$$\boxed{\xi_{nn}(\mathbf{k})}$$

Lowest band

$$\phi_{n\mathbf{k}}(\mathbf{r}) = \frac{u_{n\mathbf{k}}(\mathbf{r})}{2\pi} e^{i\mathbf{k}\cdot\mathbf{r}}$$

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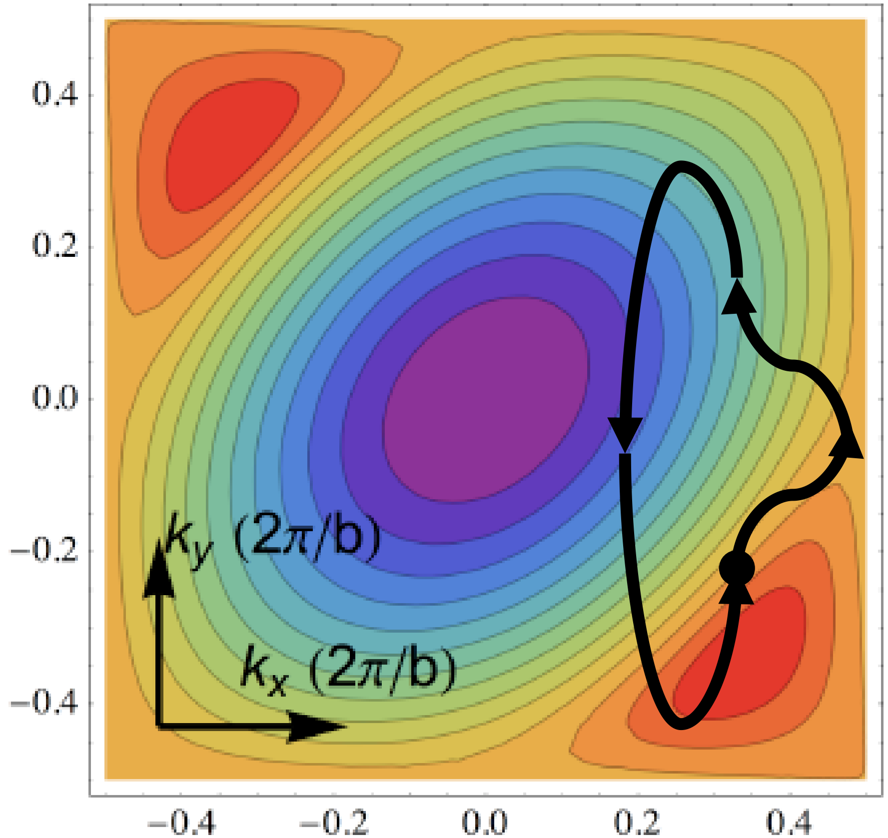

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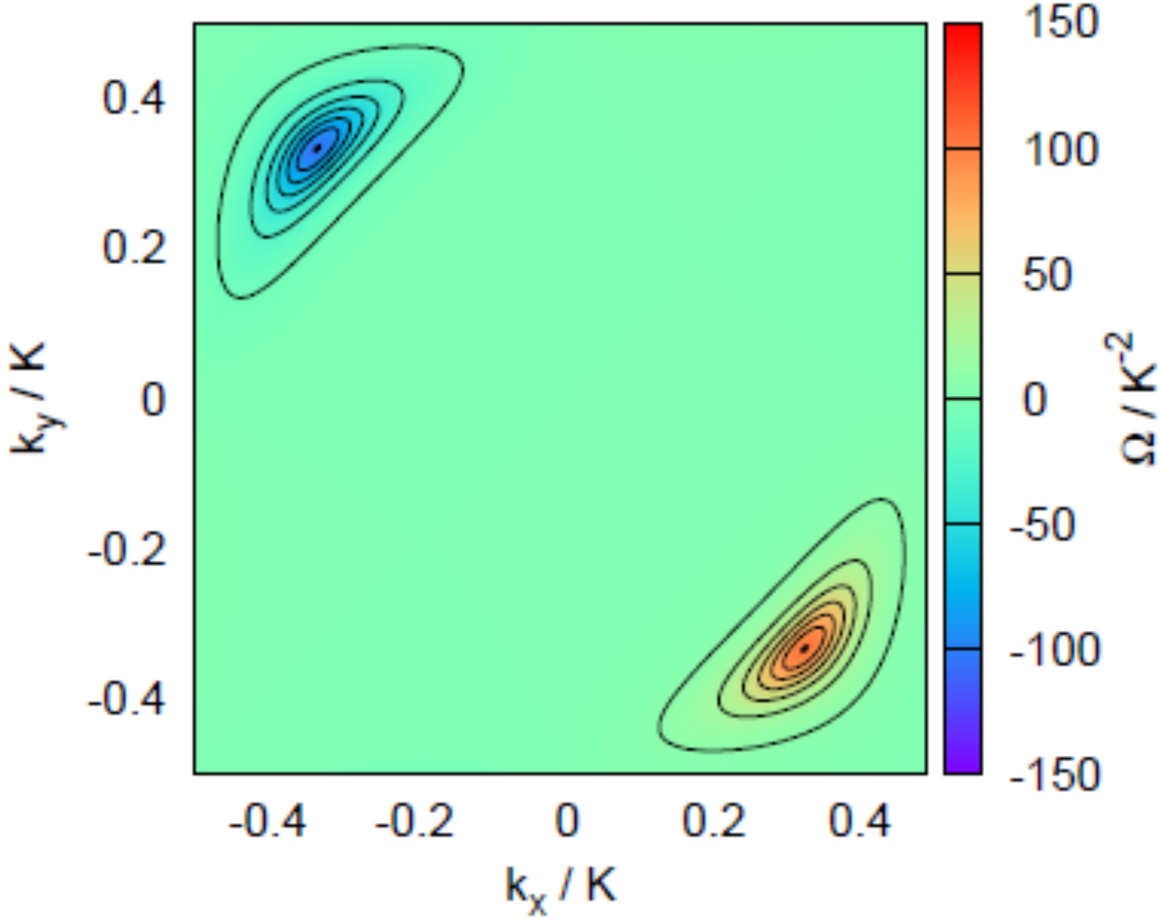
$$\xi_{mn}(\mathbf{k}) = \frac{\hbar}{im}\frac{\mathbf{p}_{mn}(\mathbf{k})}{E_{mn}(\mathbf{k})}$$

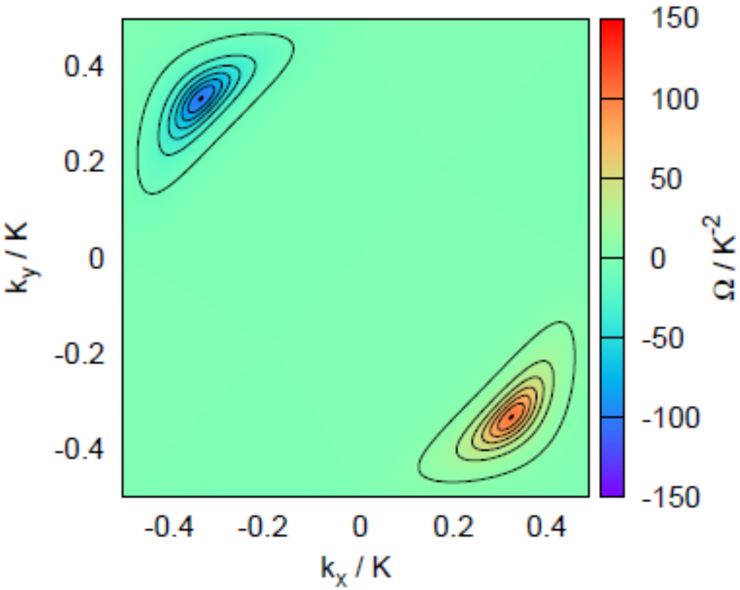
for m $\neq n$

Berry curvature

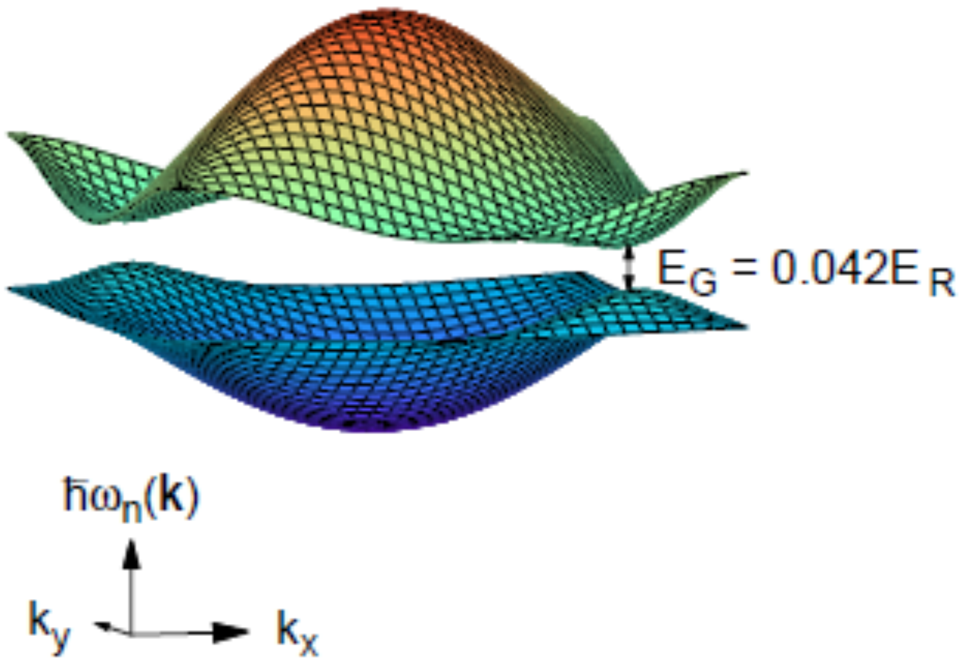
$$\boxed{\Omega_n(\mathbf{k}) = \frac{\partial}{\partial \mathbf{k}} \times \xi_{nn}(\mathbf{k})}$$



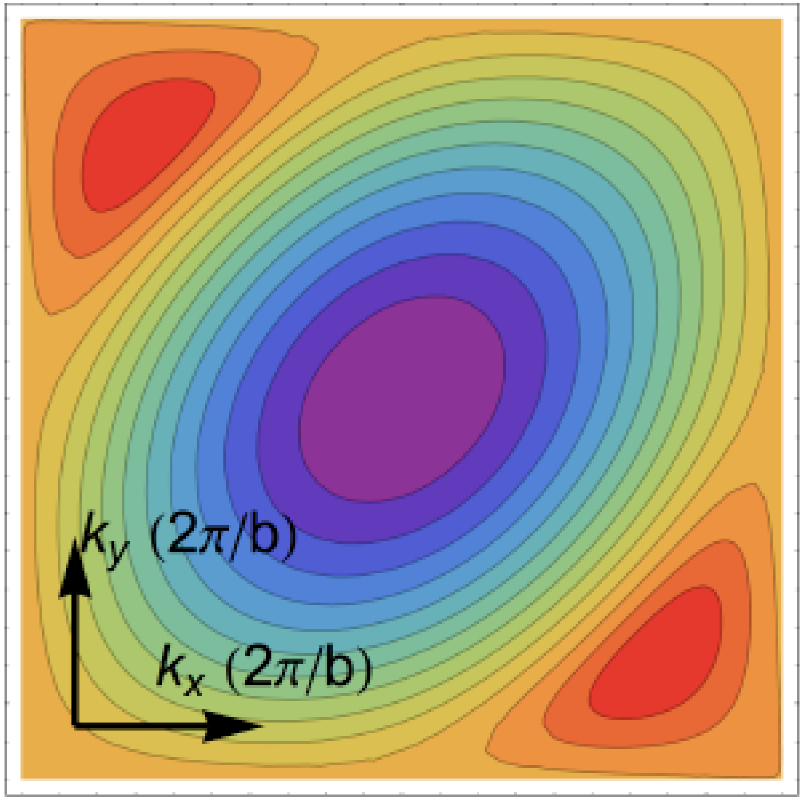
0.4
0.2
0
-0.2
-0.4
k_y / K
-0.4
-0.2
0
0.2
0.4
k_x / K
150
100
50
0
-50
-100
-150
Ω / K^{-2}

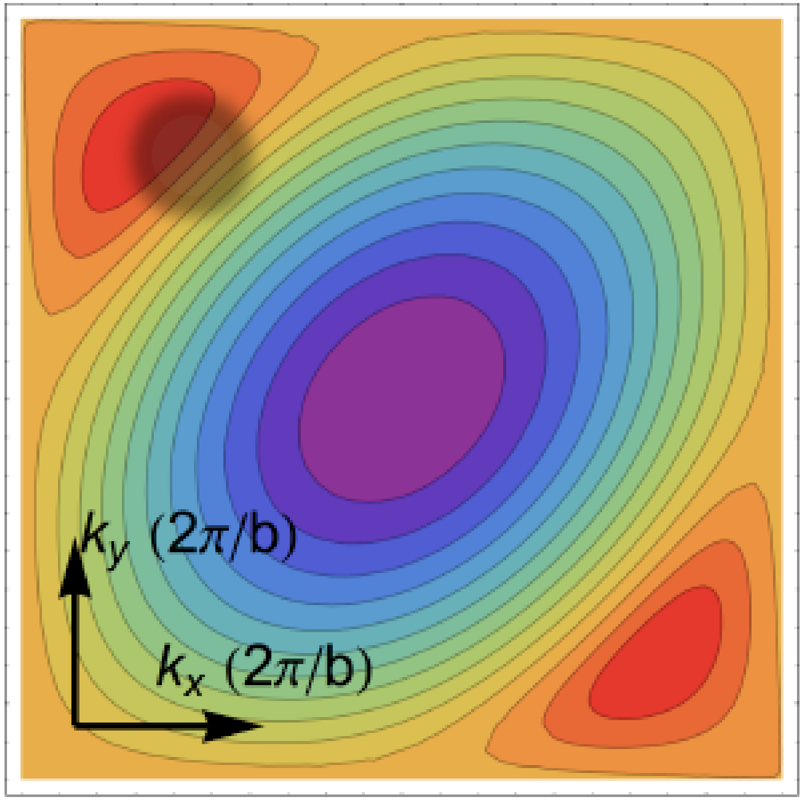


0.4
0.2
0
-0.2
-0.4
k_y / K
-0.4
-0.2
0
0.2
0.4
k_x / K
150
100
50
0
-50
-100
-150
Ω / K^-2



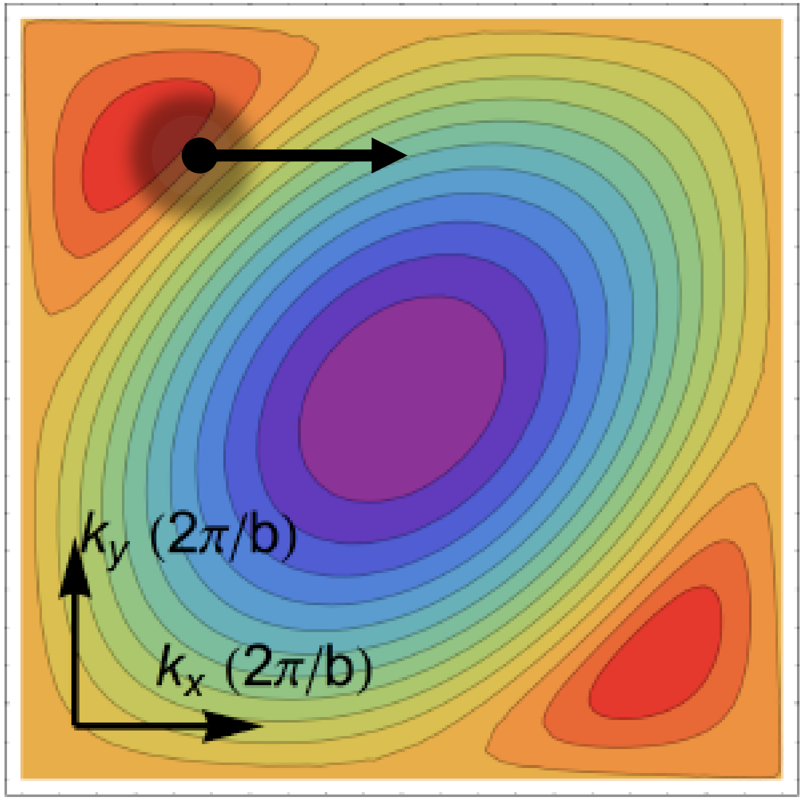
E_G = 0.042E_R
ħω_n(k)
k_y
k_x


k_y (2π/b)
k_x (2π/b)

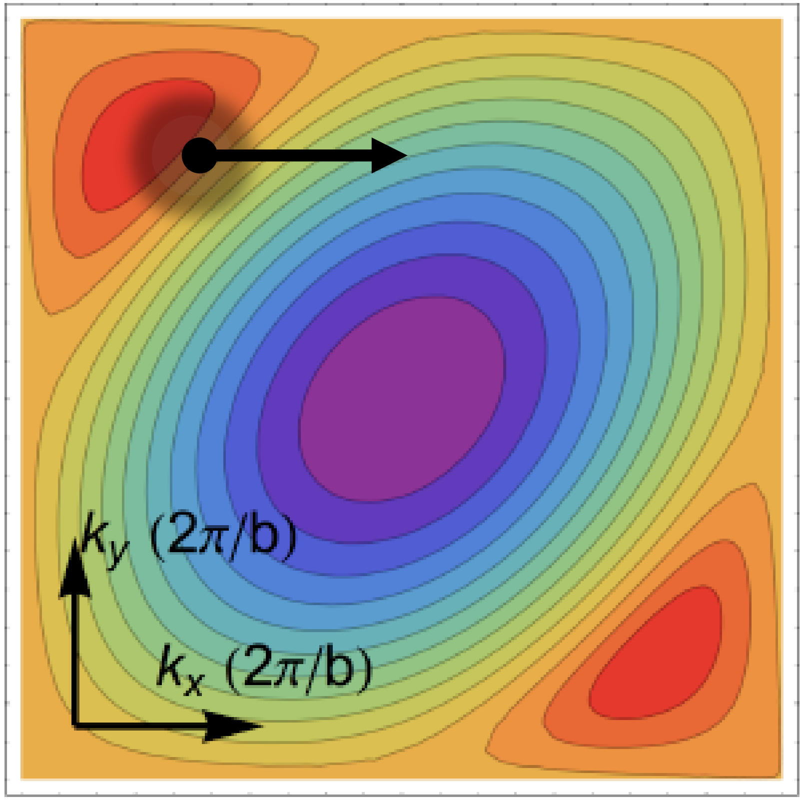


k_y (2π/b)
k_x (2π/b)

$$\langle \mathbf{k} \rangle \to \langle \mathbf{k} \rangle + \frac{\mathbf{F}}{\hbar} t$$



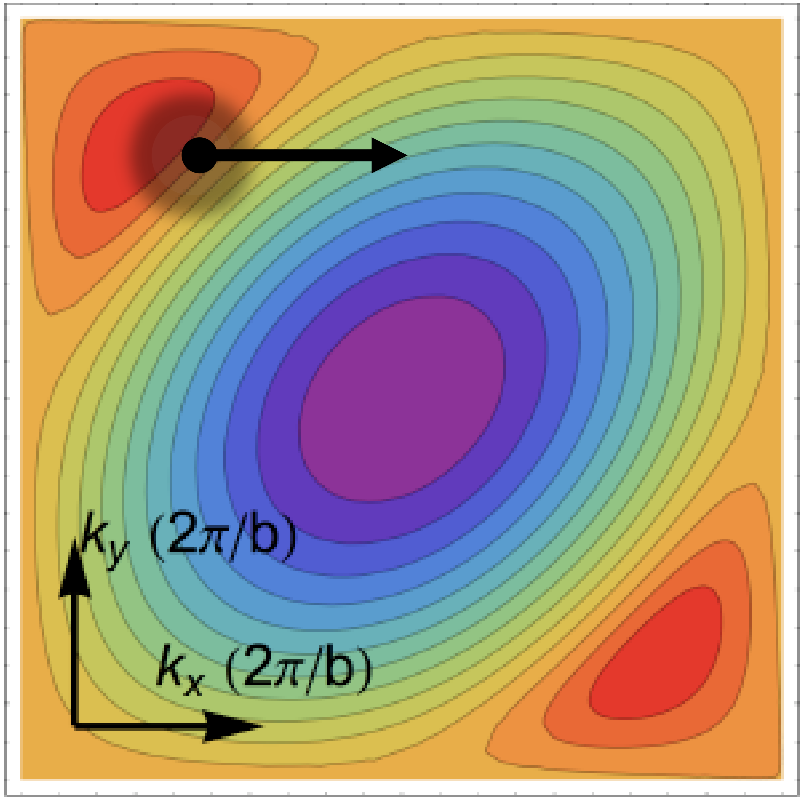
$$\langle \mathbf{k} \rangle \to \langle \mathbf{k} \rangle + \frac{\mathbf{F}}{\hbar} t$$



$$\frac{d\langle \mathbf{r} \rangle}{dt} = \mathbf{v}_g$$

$$\mathbf{v}_g = \left\langle \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \right\rangle$$

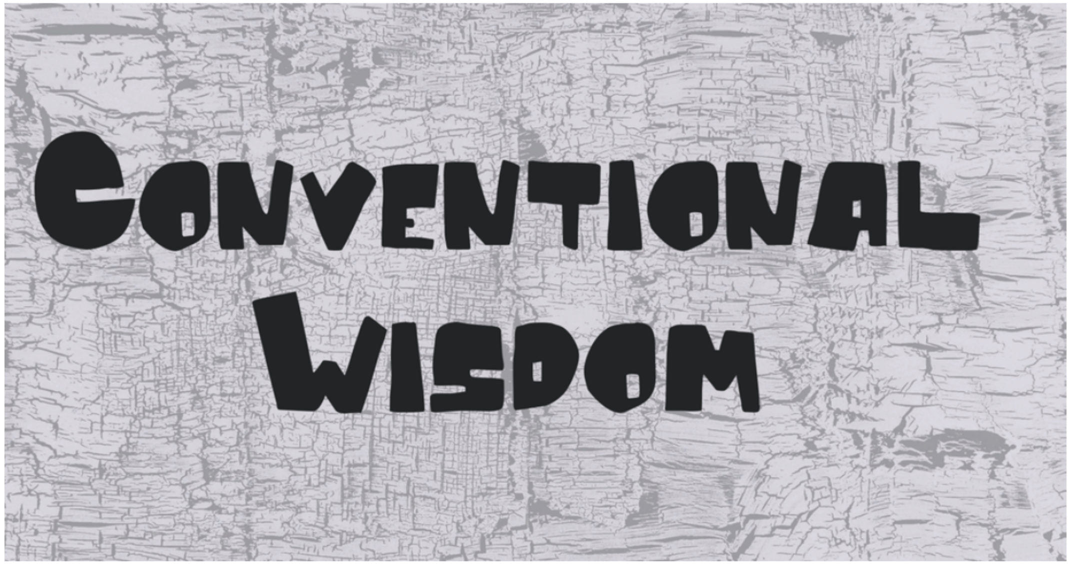
$$\langle \mathbf{k} \rangle \to \langle \mathbf{k} \rangle + \frac{\mathbf{F}}{\hbar} t$$



$$\frac{d\langle \mathbf{r} \rangle}{dt} = \mathbf{v}_g + \mathbf{v}_a$$

$$\mathbf{v}_g = \left\langle \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \right\rangle$$

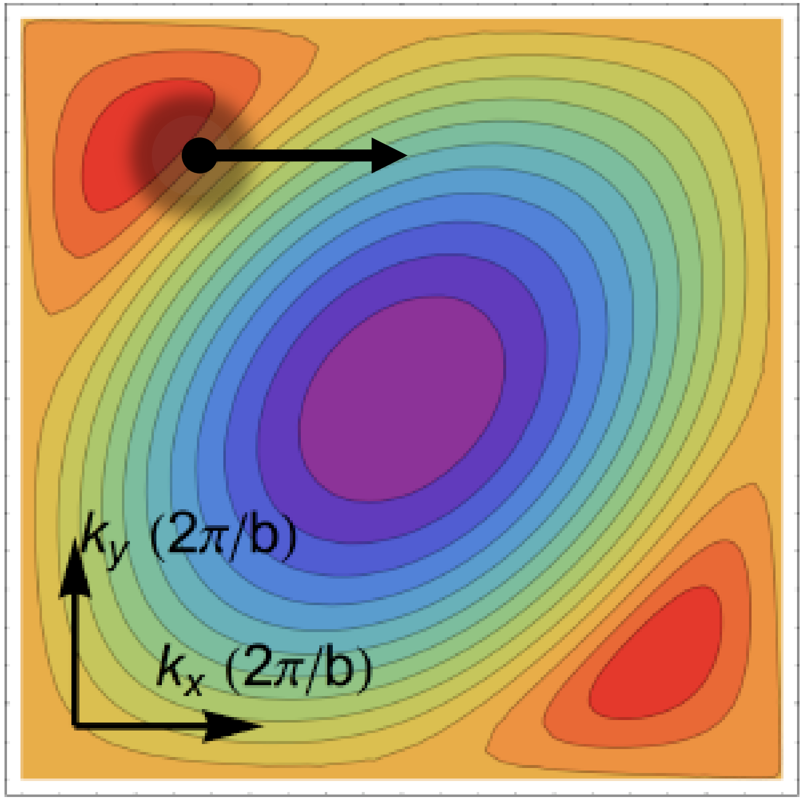
$$\mathbf{v}_a = \frac{1}{\hbar} \langle \mathbf{\Omega}(\mathbf{k}) \rangle \times \mathbf{F}$$



Effective mass and Bloch oscillations

The anomalous velocity

$$\langle \mathbf{k} \rangle \rightarrow \langle \mathbf{k} \rangle + \frac{\mathbf{F}}{\hbar} t$$



$$\frac{d\langle \mathbf{r} \rangle}{dt} = \mathbf{v}_g + \mathbf{v}_a$$

$$\frac{d^2 \langle r^a \rangle}{dt^2} = \left(\frac{1}{\hbar} \frac{\partial^2 \omega(\mathbf{k})}{\partial k^a \partial k^b} \right) F^b$$

$$\mathbf{v}_g = \left\langle \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \right\rangle$$

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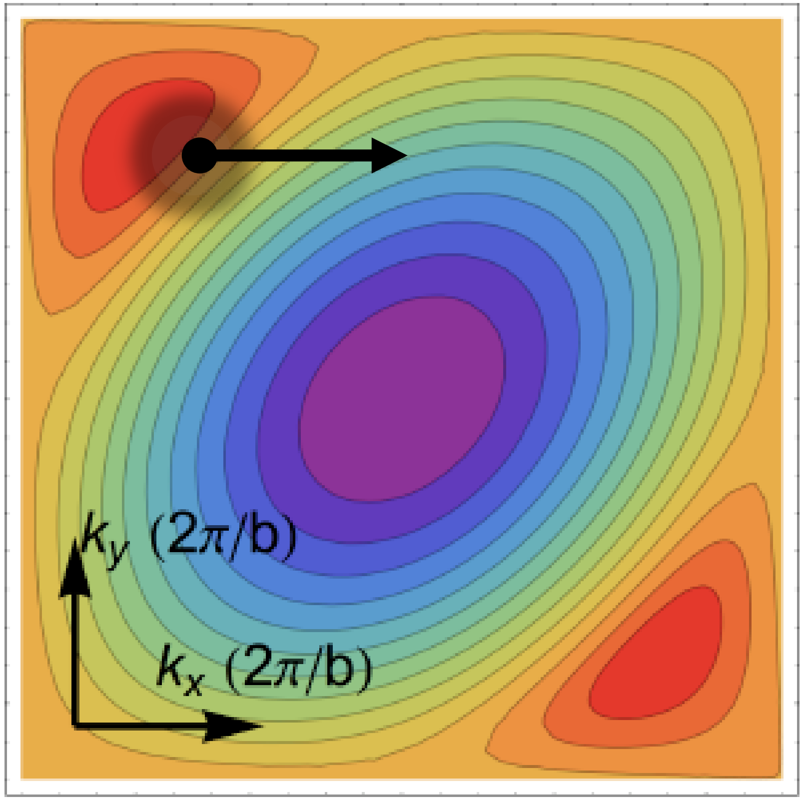
$$\langle \mathbf{k} \rangle \to \langle \mathbf{k} \rangle + \frac{\mathbf{F}}{\hbar} t$$



$$\frac{d\langle \mathbf{r} \rangle}{dt} = \mathbf{v}_g + \mathbf{v}_a$$

$$\frac{d^2 \langle r^a \rangle}{dt^2} = \left(\frac{1}{\hbar} \frac{\partial^2 \omega(\mathbf{k})}{\partial k^a \partial k^b} \right) F^b$$

$$= \left[\frac{1}{m^*(\mathbf{k})} \right]^{ab} F^b$$

$$\mathbf{v}_g = \left\langle \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \right\rangle$$

$$\mathbf{v}_a = \frac{1}{\hbar} \langle \boldsymbol{\Omega}(\mathbf{k}) \rangle \times \mathbf{F}$$

Outline

The conventional wisdom

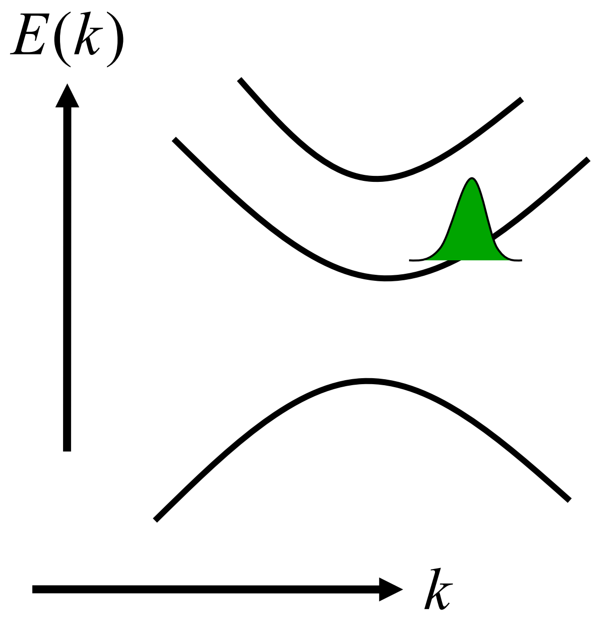
The conventional wisdom is wrong!

When the conventional wisdom is not so wrong

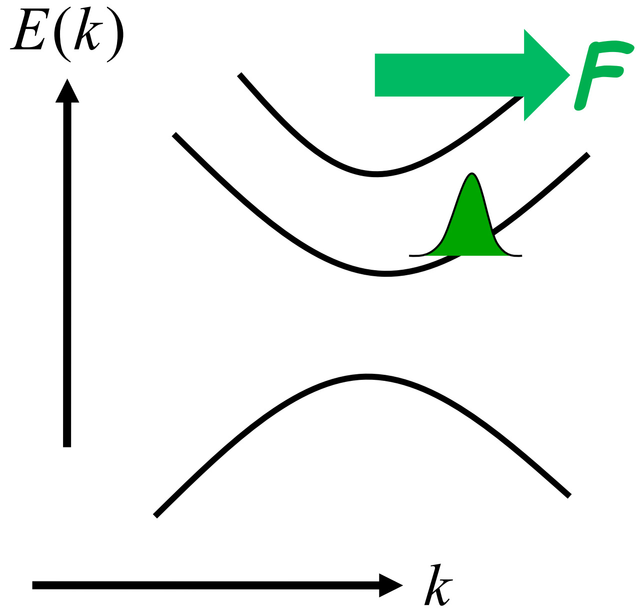
Beyond the conventional wisdom

Experiments and possible experiments

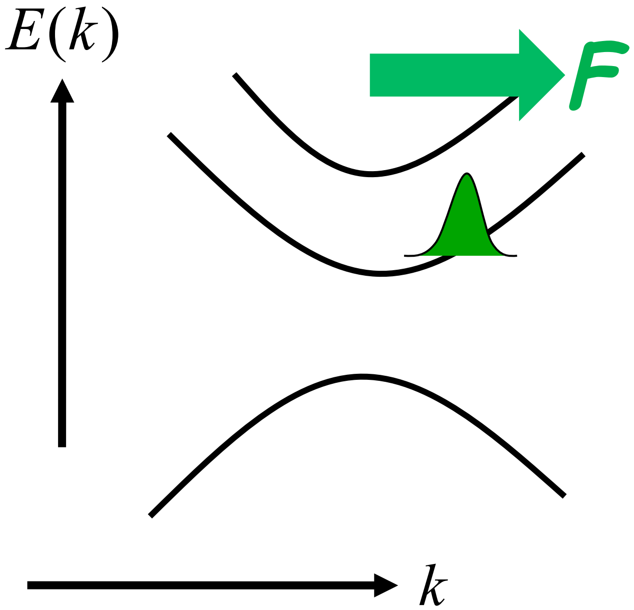
The moral(s) of the story



E(k)
k



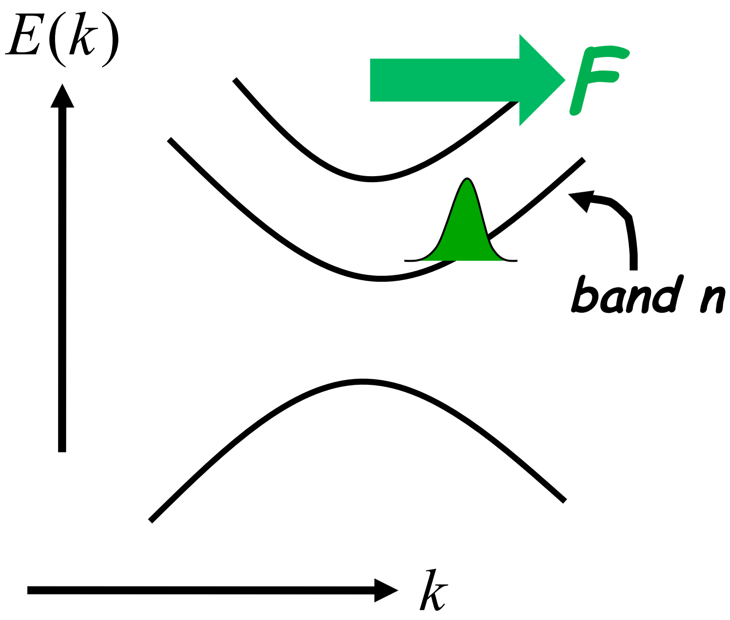
$E(k)$
F
k



$$H = H_o - xF$$

$$H_o = \frac{p^2}{2m} + V(x)$$

$$V(x) = V(x+b)$$



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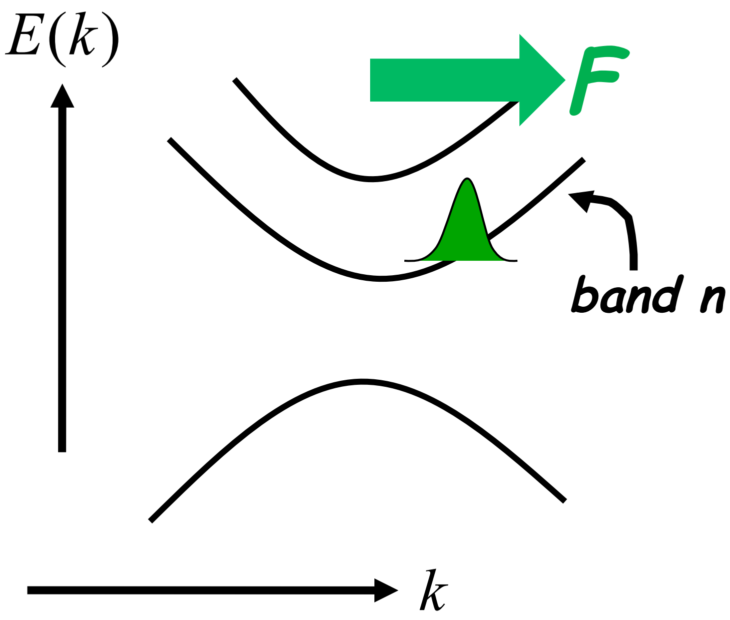
$$V(x) = V(x+b)$$

$$|\psi(0)\rangle = \int dk\, c(k)\, |nk\rangle$$

Bloch states



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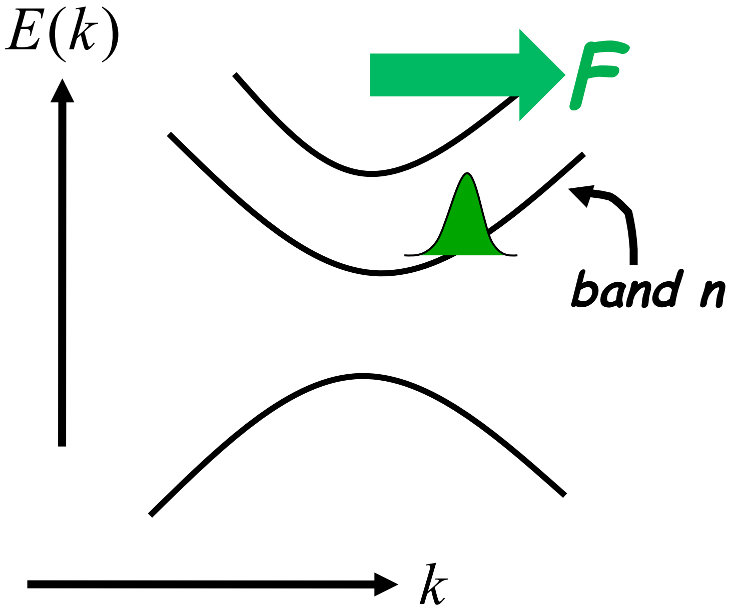

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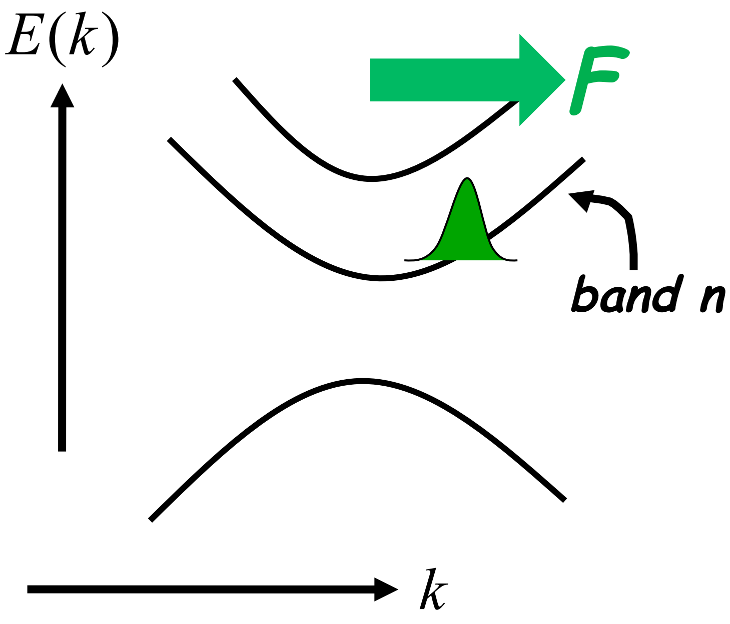
Bloch states

$$\langle\psi(t)|\,a\,|\psi(t)\rangle \equiv \frac{d^2}{dt^2}\langle\psi(t)|\,x\,|\psi(t)\rangle \quad = \frac{1}{m}\frac{d}{dt}\langle\psi(t)|\,p\,|\psi(t)\rangle$$



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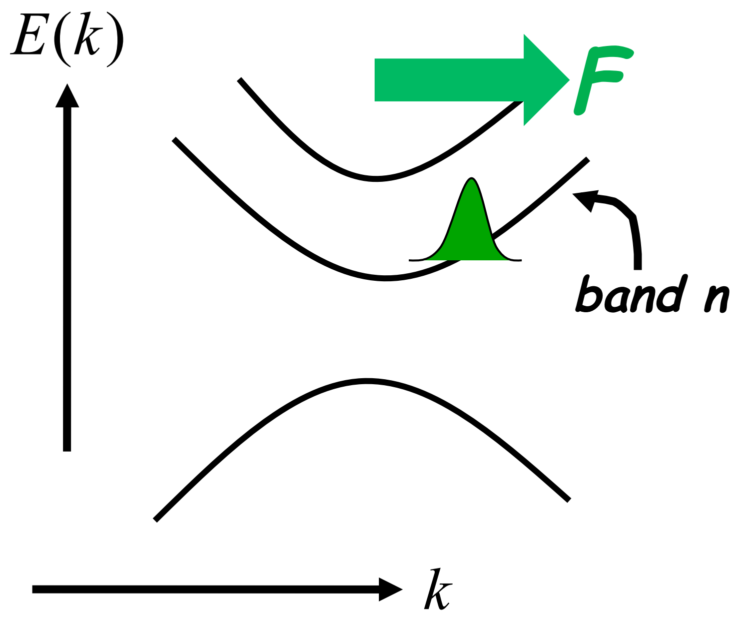
$$\frac{d}{dt}\langle\psi(t)|\,p\,|\psi(t)\rangle = \frac{1}{i\hbar}\langle\psi(t)|\big[p,H\big]|\psi(t)\rangle$$



$$H = H_o - xF$$

$$H_o = \frac{p^2}{2m} + V(x)$$

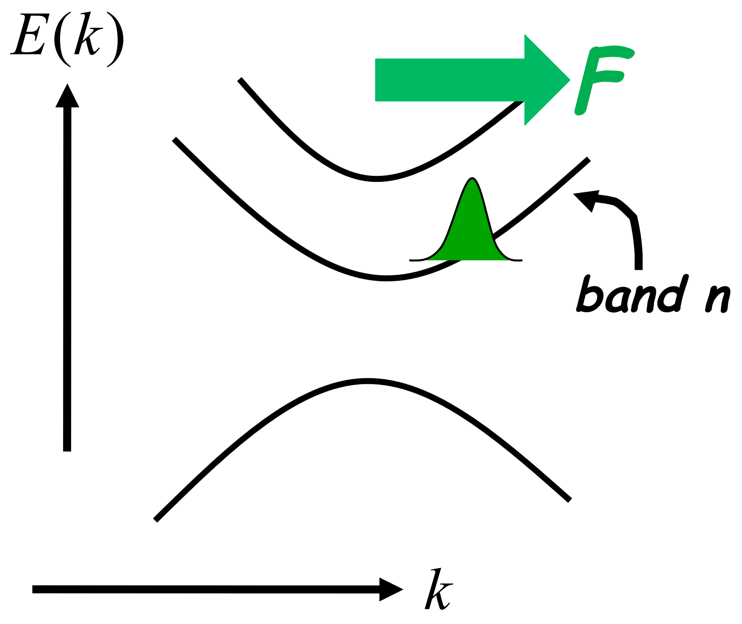
$$V(x) = V(x+b)$$

$$|\psi(0)\rangle = \int dk\; c(k)\,|nk\rangle$$

Bloch states

$$\langle \psi(t)|\,a\,|\psi(t)\rangle \equiv \frac{d^2}{dt^2}\langle \psi(t)|\,x\,|\psi(t)\rangle \quad = \frac{1}{m}\frac{d}{dt}\langle \psi(t)|\,p\,|\psi(t)\rangle$$

$$\frac{d}{dt}\langle \psi(t)|\,p\,|\psi(t)\rangle = \frac{1}{i\hbar}\langle \psi(t)|\left[p,H\right]|\psi(t)\rangle = \frac{1}{i\hbar}\langle \psi(t)|\left[p,H_o\right]|\psi(t)\rangle + F$$



$$H = H_o - xF$$

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$$V(x) = V(x+b)$$

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Bloch states

$$H_o|nk\rangle = E_n(k)|nk\rangle$$

$$\langle nk\,|\,p\,|\,nk'\rangle = p_{nn}(k)\,\delta(k-k')$$

$$\langle\psi(t)\,|\,a\,|\,\psi(t)\rangle \equiv \frac{d^2}{dt^2}\langle\psi(t)\,|\,x\,|\,\psi(t)\rangle \quad = \frac{1}{m}\frac{d}{dt}\langle\psi(t)\,|\,p\,|\,\psi(t)\rangle$$

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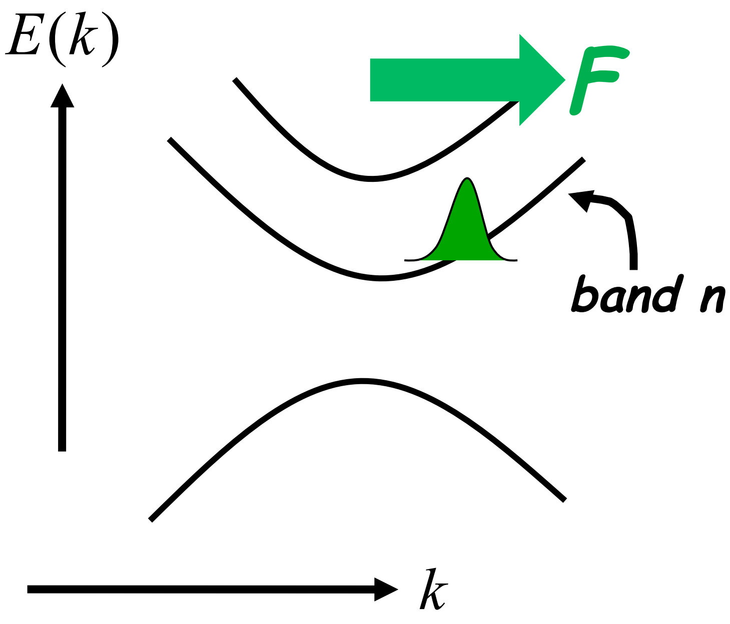

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Bloch states

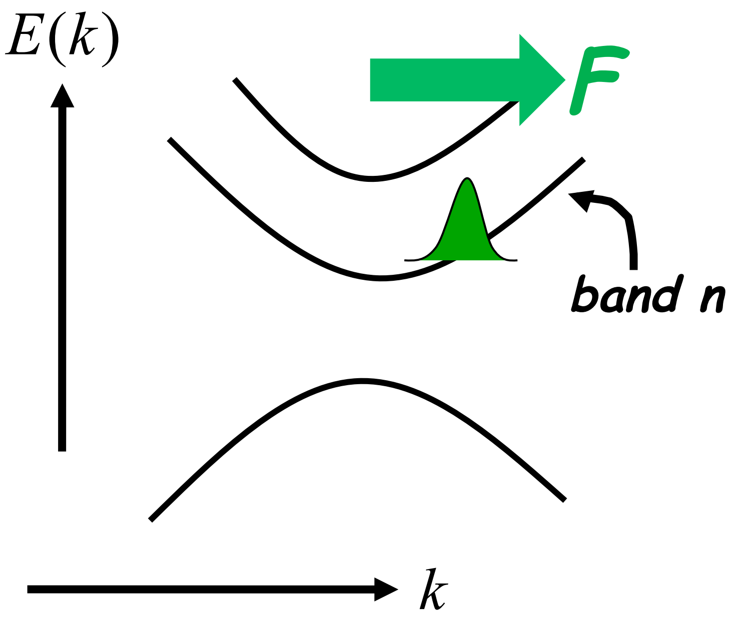
$$H_o |nk\rangle = E_n(k) |nk\rangle$$

$$\langle nk | p | nk'\rangle = p_{nn}(k)\, \delta(k-k')$$

$$\langle \psi(t) | a | \psi(t)\rangle \equiv \frac{d^2}{dt^2} \langle \psi(t) | x | \psi(t)\rangle \quad = \frac{1}{m}\frac{d}{dt}\langle \psi(t) | p | \psi(t)\rangle$$

$$\frac{d}{dt}\langle \psi(t) | p | \psi(t)\rangle = \frac{1}{i\hbar}\langle \psi(t) | \left[p, H\right] | \psi(t)\rangle = \frac{1}{i\hbar}\langle \psi(t) | \left[p, H_o\right] | \psi(t)\rangle + F$$

$$\langle \psi(0) | [p, H_o] | \psi(0)\rangle = 0$$



$$H = H_o - xF$$

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Bloch states

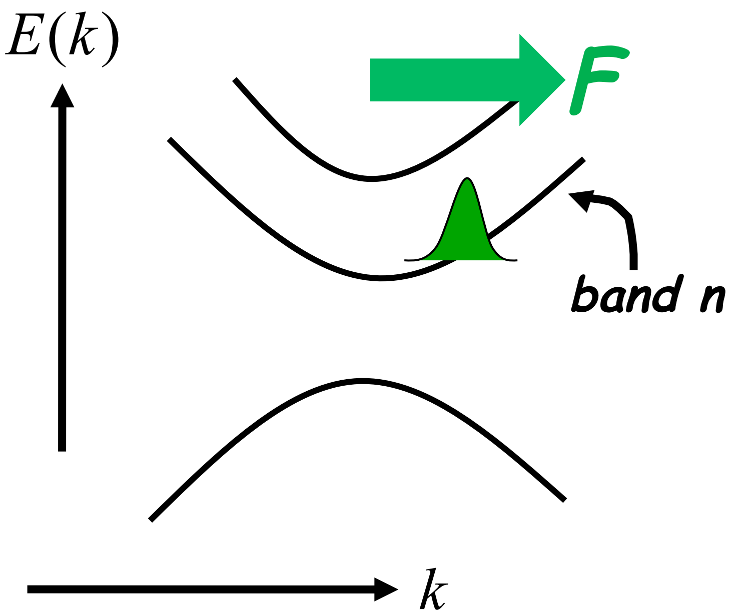
$$H_o |nk\rangle = E_n(k) |nk\rangle$$

$$\langle nk | p | nk' \rangle = p_{nn}(k)\, \delta(k-k')$$

$$\langle \psi(0) | a | \psi(0) \rangle = \frac{F}{m}$$

$$\frac{F}{\langle \psi(0) | a | \psi(0) \rangle} = m$$

Bare mass!



$$H = H_o - xF$$

$$H_o = \frac{p^2}{2m} + V(x)$$

$$V(x) = V(x+b)$$

$$|\psi(0)\rangle = \int dk\, c(k)|nk\rangle$$

Bloch states

$$H_o|nk\rangle = E_n(k)|nk\rangle$$

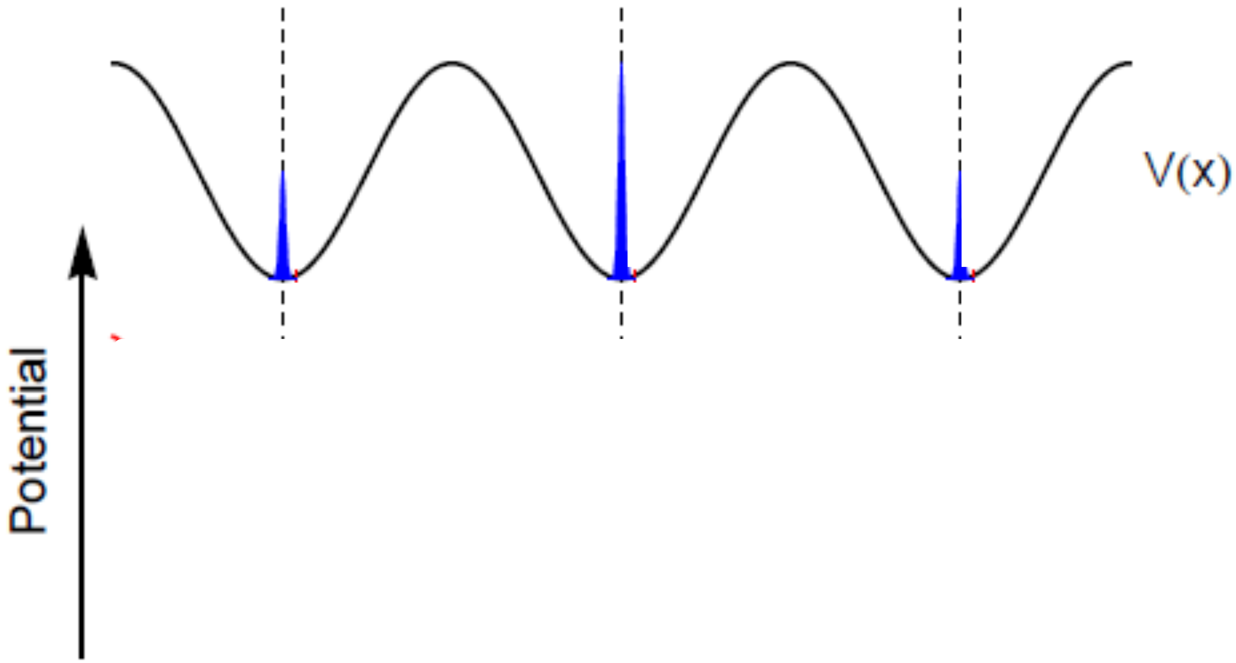
$$\langle nk\,|\,p\,|\,nk'\rangle = p_{nn}(k)\,\delta(k-k')$$

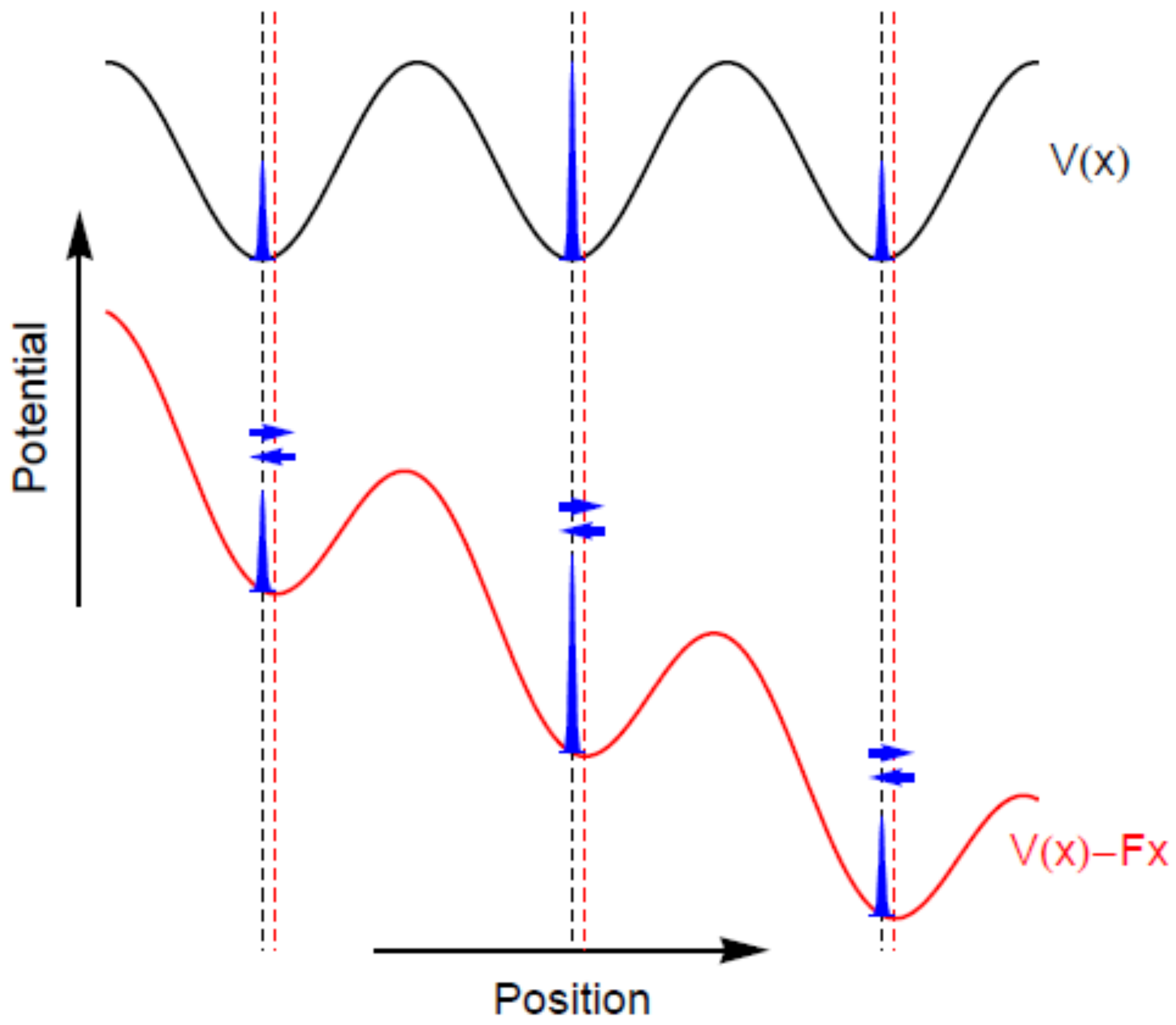
$$\langle\psi(0)\,|\,a\,|\,\psi(0)\rangle = \frac{F}{m}$$

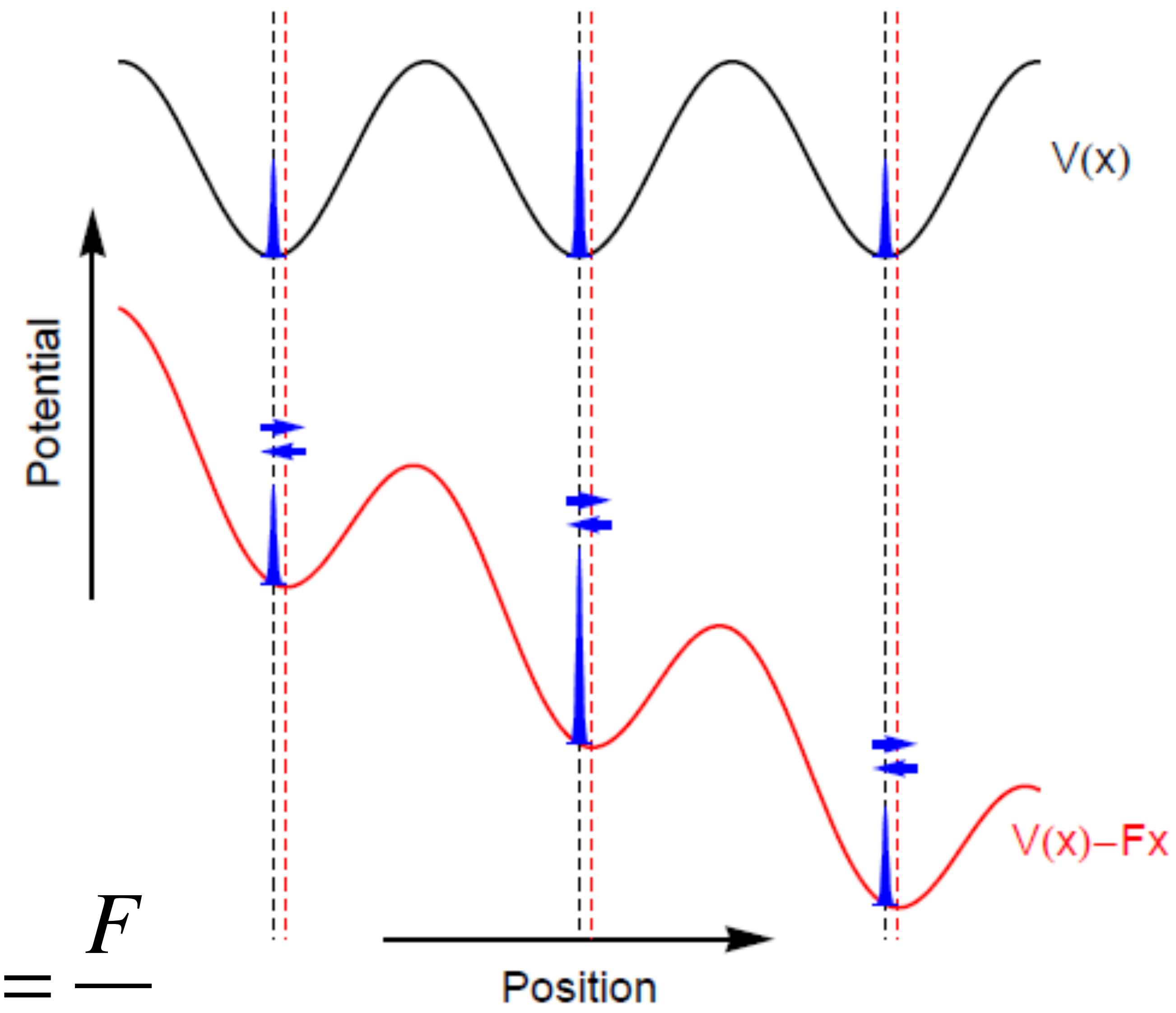
$$\frac{F}{\langle\psi(0)\,|\,a\,|\,\psi(0)\rangle} = m$$

Bare mass!

D. Pfirsch and E. Spenke, Z. Physik 137, 309 (1954)

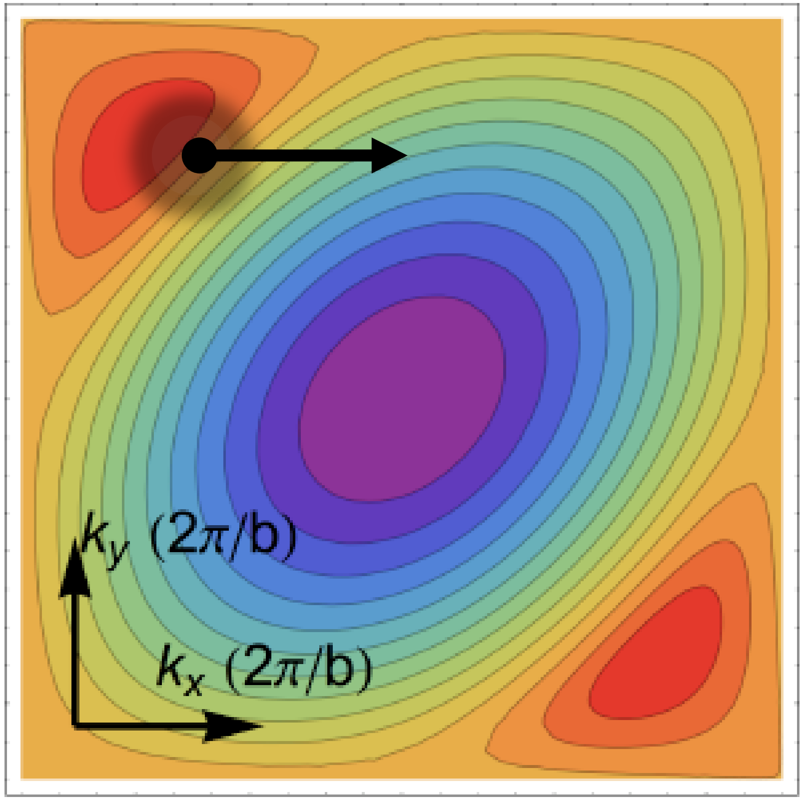

V(x)
Potential


V(x)
Potential
V(x)–Fx
Position



$$\frac{d\langle v\rangle}{dt}=\frac{F}{m}$$

apply ***F*****:** ***initially.....***



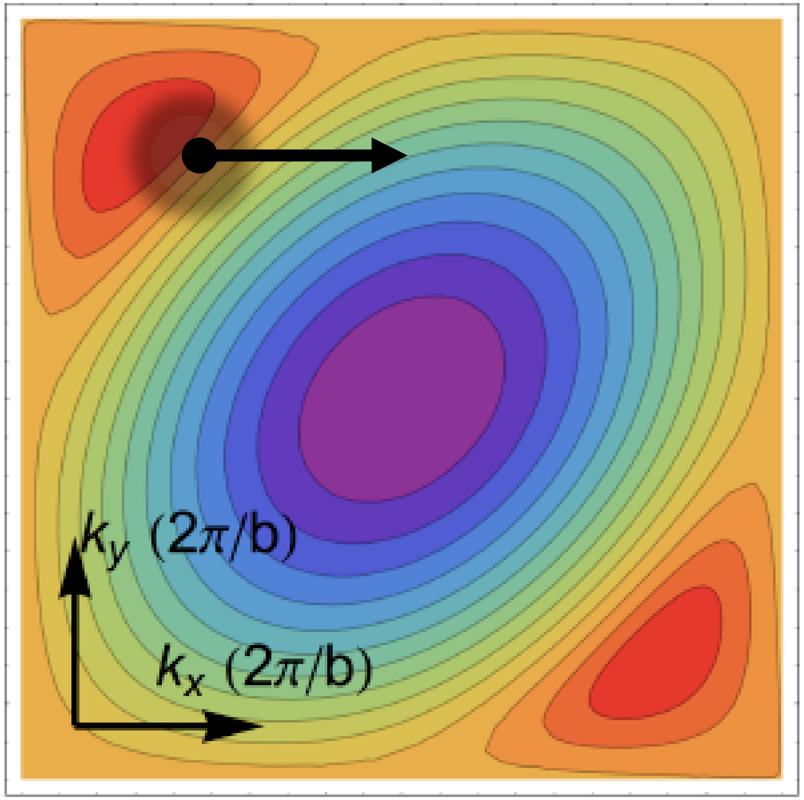
$$\frac{d\langle \mathbf{r} \rangle}{dt} = \mathbf{v}_g + \mathbf{v}_a$$

$$\frac{d^2\langle r^a \rangle}{dt^2} = \left(\frac{1}{\hbar} \frac{\partial^2 \omega(\mathbf{k})}{\partial k^a \, \partial k^b} \right) F^b$$

$$= \left[\frac{1}{m^*(\mathbf{k})} \right]^{ab} F^b$$

$$\mathbf{v}_g = \left\langle \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \right\rangle \qquad \mathbf{v}_a = \frac{1}{\hbar} \langle \boldsymbol{\Omega}(\mathbf{k}) \rangle \times \mathbf{F}$$

*apply **F**: **initially.....***



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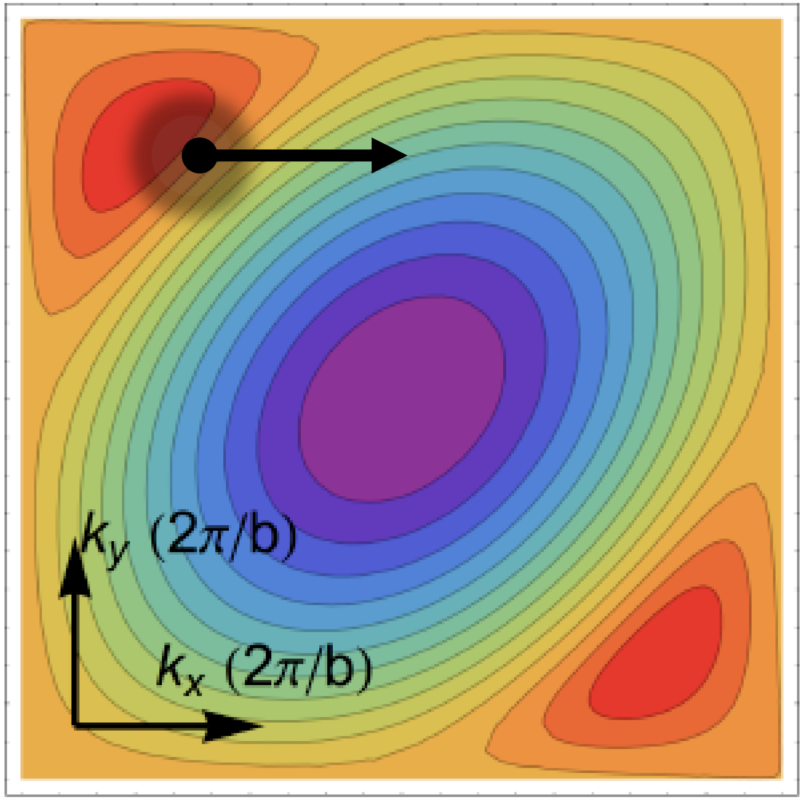
apply ***F***: ***initially…..***



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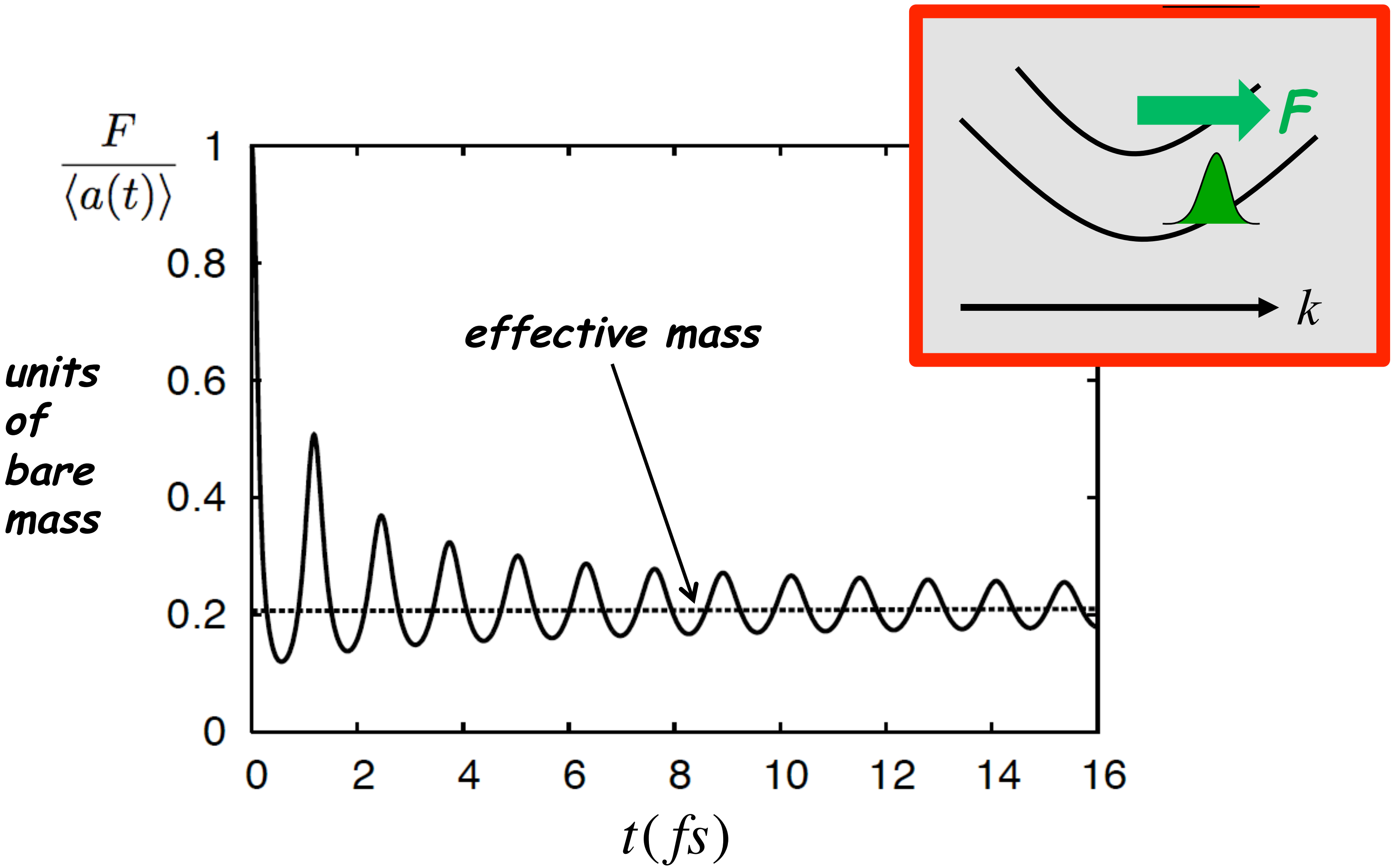
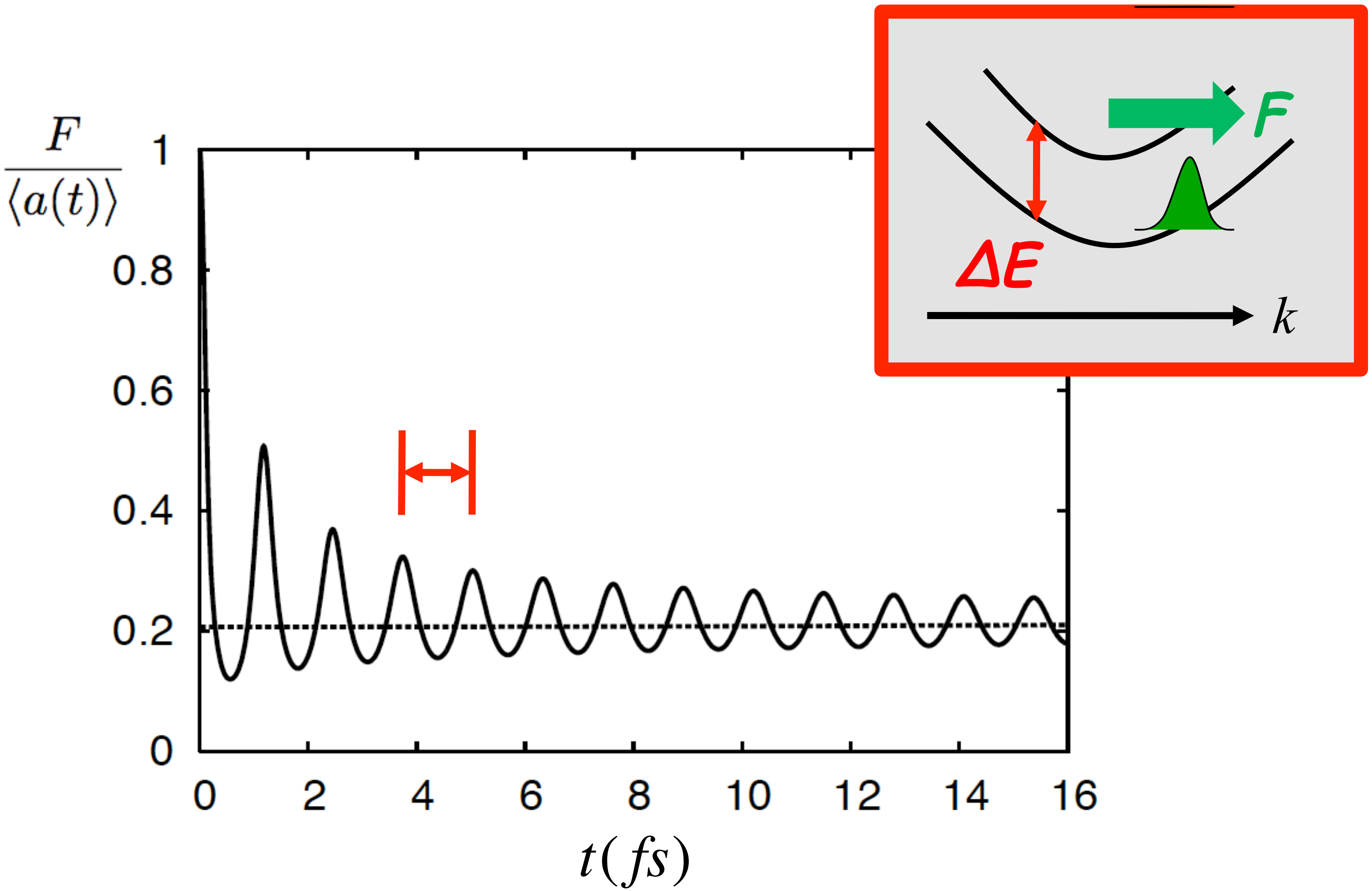
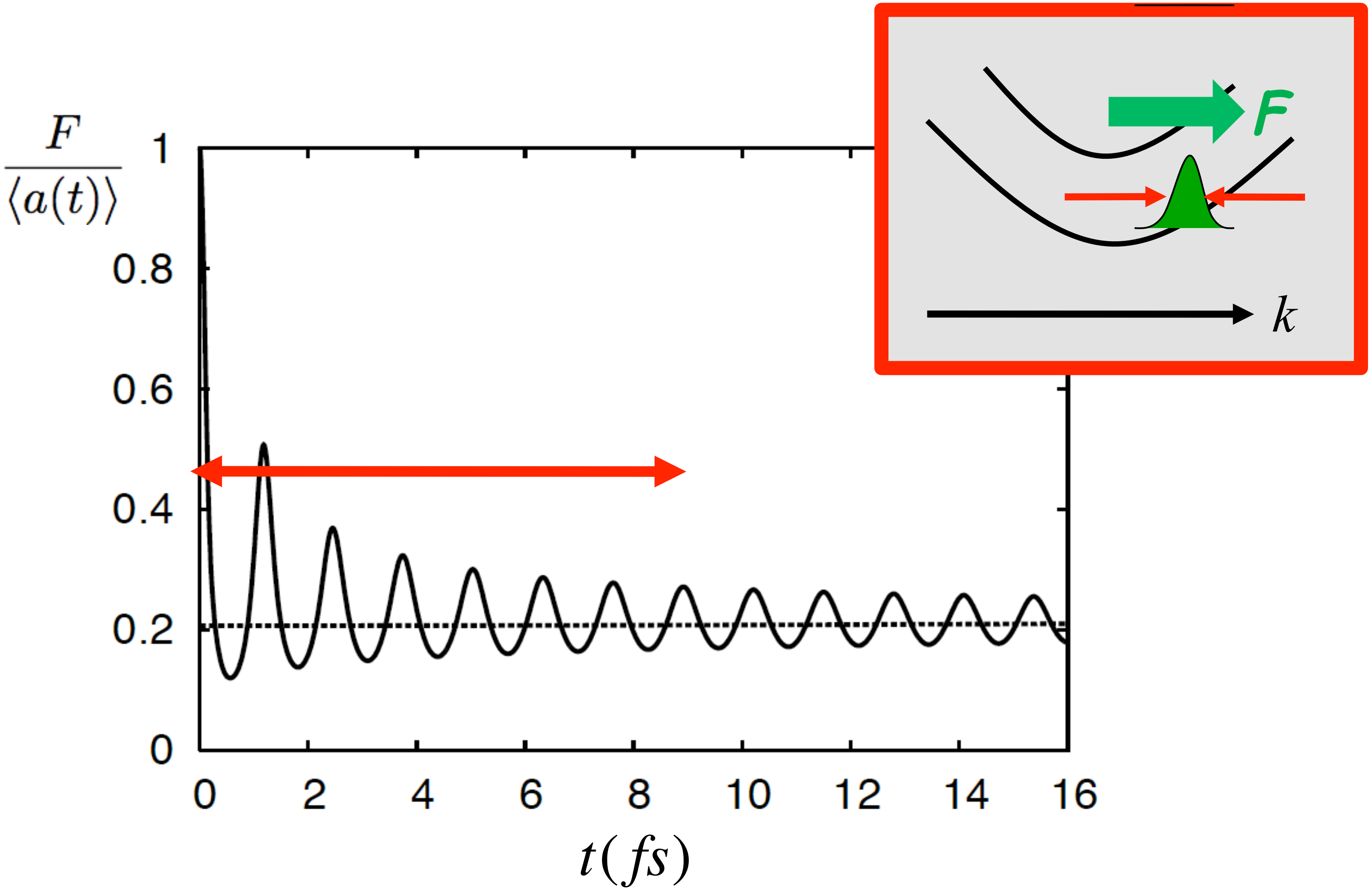


$$t_{Bloch} \approx 500\, fs$$

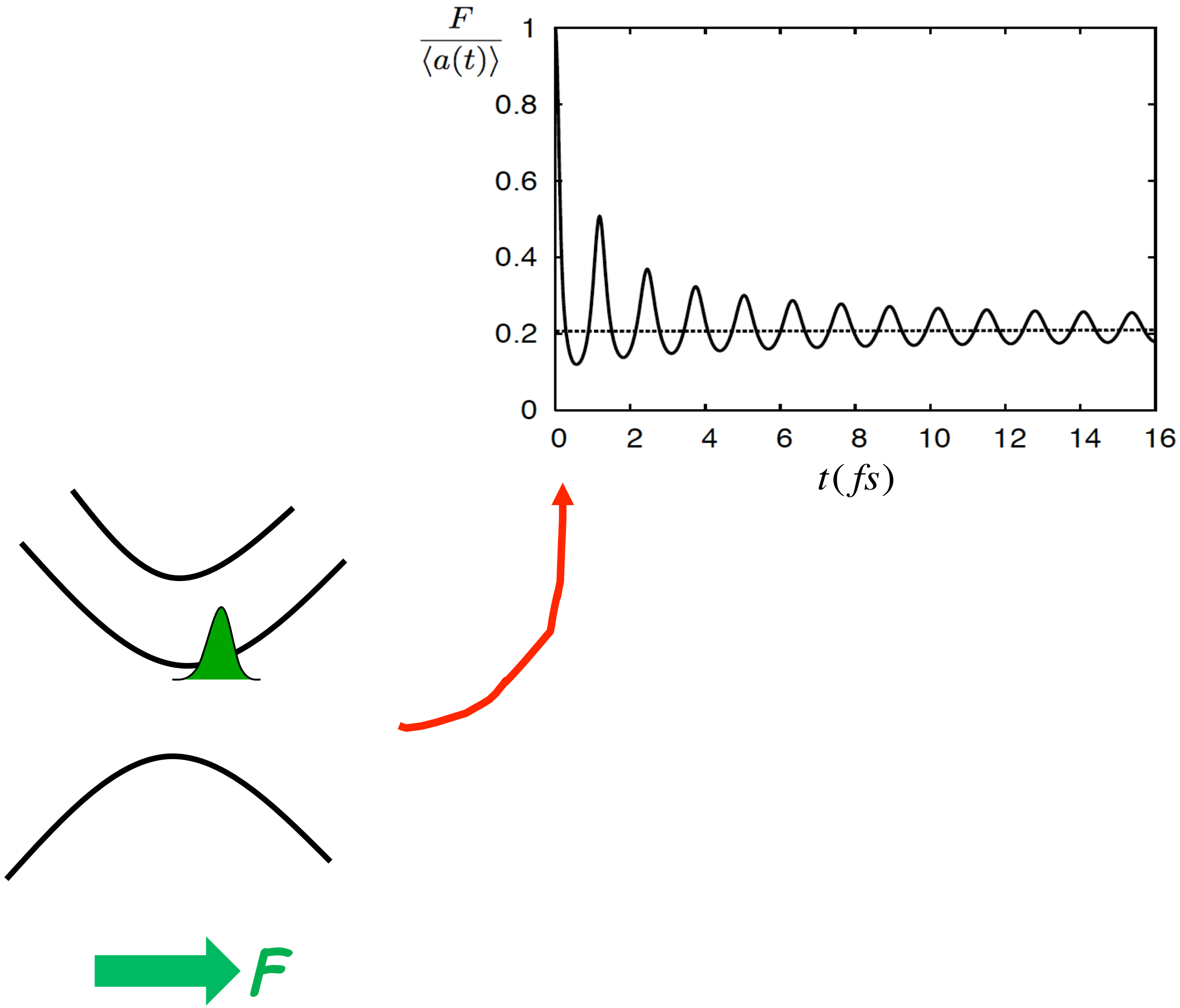


period(s) inversely proportional to energy difference(s)

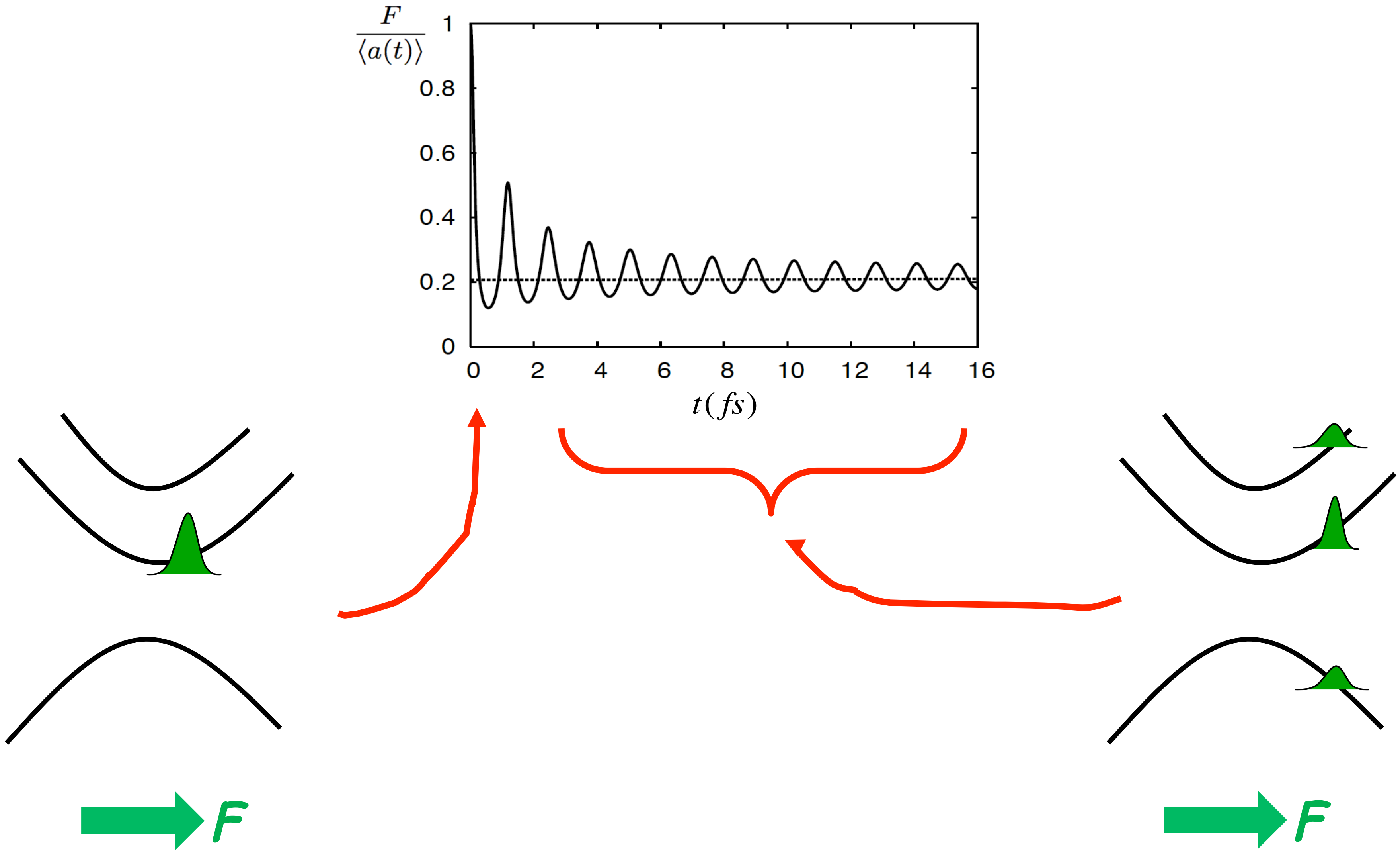


decay time increases with smaller spread in k

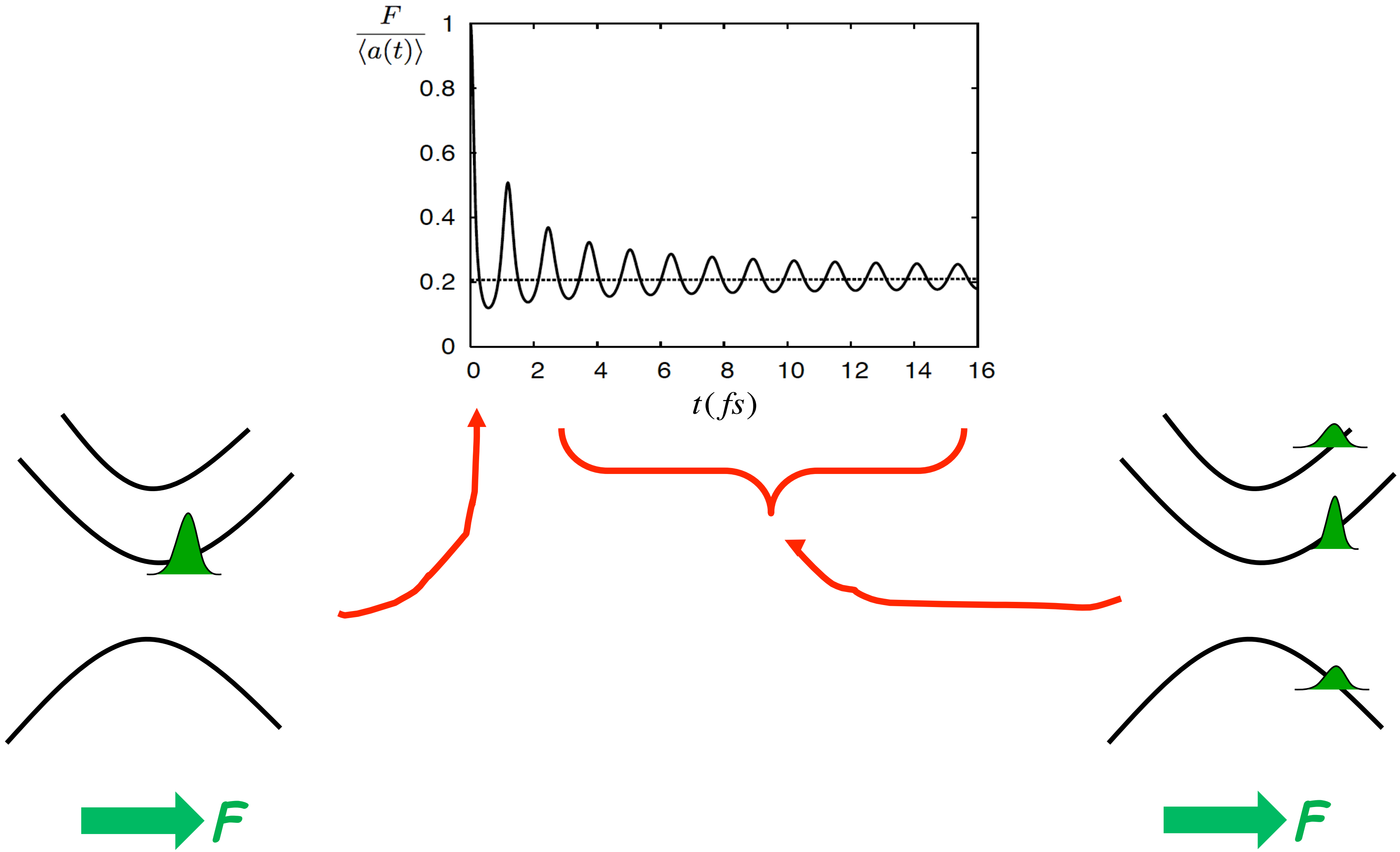
...and with increasing m and b



$\frac{F}{\langle a(t) \rangle}$
1
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16
$t(fs)$
F



$\frac{F}{\langle a(t)\rangle}$
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
$t(fs)$
F
F



$$\left[\frac{1}{m_n^*(\mathbf{k})}\right]^{ab} = \frac{1}{\hbar^2}\frac{\partial^2 E_u(\mathbf{k})}{\partial k^a \partial k^b} = \frac{\delta^{ab}}{m} + \frac{1}{m^2}\sum_{n'\neq n}\frac{p^a_{nn'}(\mathbf{k})p^b_{n'n}(\mathbf{k}) + p^b_{nn'}(\mathbf{k})p^a_{n'n}(\mathbf{k})}{E_{nn'}(\mathbf{k})}$$

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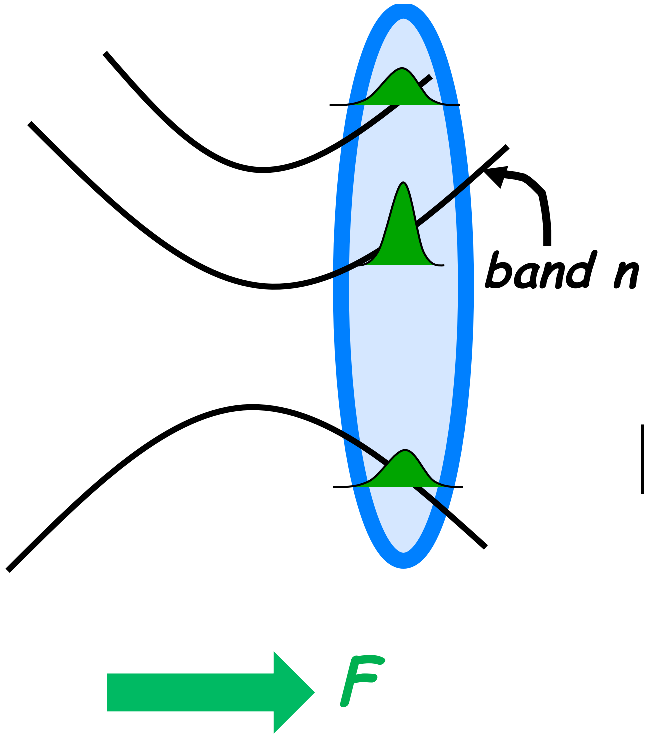


Wannier's (1960)
modified Bloch states

$$\overline{|n\mathbf{k}\rangle} = \sum_{n'} |n'\mathbf{k}\rangle U_{n'n}(\mathbf{k})$$

unitary transformation

$$|\psi_n(t)\rangle = \int d\mathbf{k}\, c_n(\kappa) e^{-i\phi_n(\mathbf{k},t)}\, \overline{|n\mathbf{k}\rangle}$$

$$\kappa = \mathbf{k} - \frac{\mathbf{F}t}{\hbar}$$

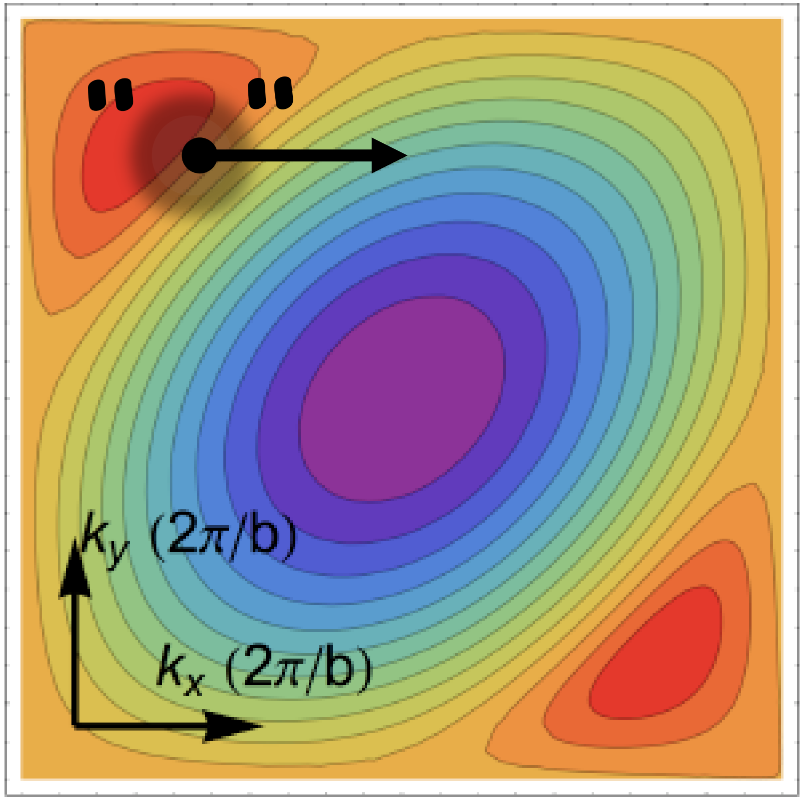
describes a wave packet moving with the effective mass, as long as Zener tunneling can be neglected

$$\xi_{n'n}(\mathbf{k}) = \frac{\hbar}{im}\frac{\mathbf{p}_{n'n}(\mathbf{k})}{E_{n'n}(\mathbf{k})}$$

for $n' \neq n$

$$\overline{|n\mathbf{k}\rangle} = |n\mathbf{k}\rangle + \sum_{n'\neq n} \frac{\xi_{n'n}(\mathbf{k})\cdot\mathbf{F}}{E_{n'n}(\mathbf{k})} |n'\mathbf{k}\rangle + \cdots$$

$$\langle \mathbf{k} \rangle \to \langle \mathbf{k} \rangle + \frac{\mathbf{F}}{\hbar} t$$



$$\frac{d\langle \mathbf{r} \rangle}{dt} = \mathbf{v}_g + \mathbf{v}_a$$

$$\frac{d^2 \langle r^a \rangle}{dt^2} = \left(\frac{1}{\hbar} \frac{\partial^2 \omega(\mathbf{k})}{\partial k^a \partial k^b} \right) F^b$$

$$= \left[\frac{1}{m^*(\mathbf{k})} \right]^{ab} F^b$$

$$\mathbf{v}_g = \left\langle \frac{\partial \omega(\mathbf{k})}{\partial \mathbf{k}} \right\rangle$$

$$\mathbf{v}_a = \frac{1}{\hbar} \langle \mathbf{\Omega}(\mathbf{k}) \rangle \times \mathbf{F}$$

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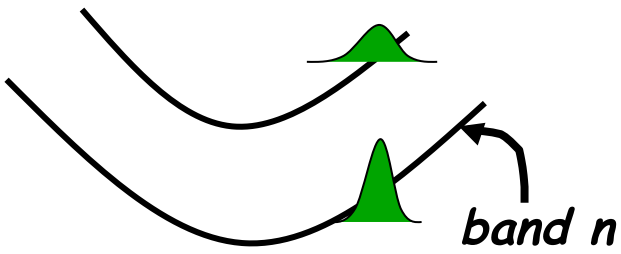
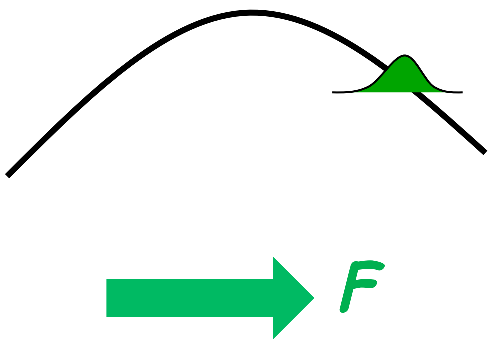
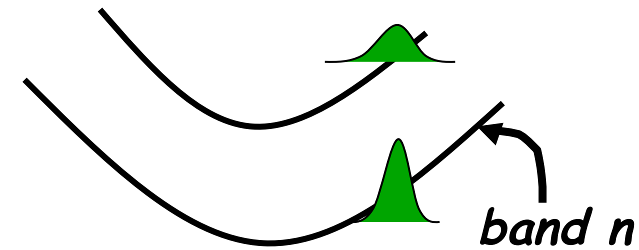
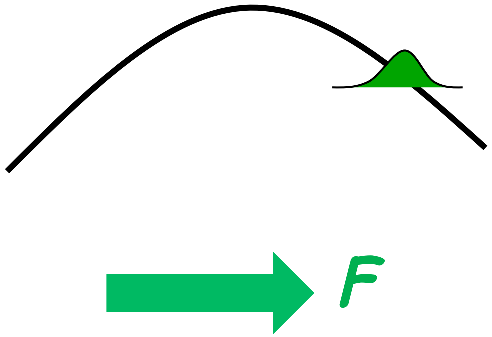




$$\left|\psi_n(t)\right\rangle = \int d\mathbf{k}\; c_n(\kappa) e^{-i\phi_n(\mathbf{k},t)} \overline{\left|n\mathbf{k}\right\rangle}$$

$$= \int d\mathbf{k}\; c_n(\kappa) e^{-i\phi_n(\mathbf{k},t)} \left[\left|n\mathbf{k}\right\rangle + \sum_{n'\neq n} \frac{\xi_{n'n}(\mathbf{k})\cdot\mathbf{F}}{E_{n'n}(\mathbf{k})} \left|n'\mathbf{k}\right\rangle + \cdots \right]$$





$$\left|\psi_n(t)\right\rangle = \int d\mathbf{k}\, c_n(\kappa) e^{-i\phi_n(\mathbf{k},t)} \overline{\left|n\mathbf{k}\right\rangle}$$

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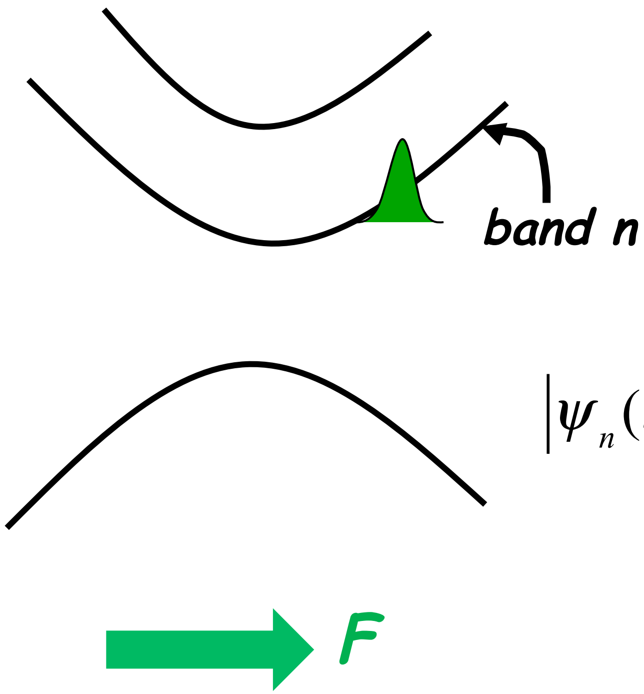
$$\left|\psi(t)\right\rangle = \left|\psi_n(t)\right\rangle + \sum_{n'\neq n} \left|\psi_{n'}(t)\right\rangle$$

t=0

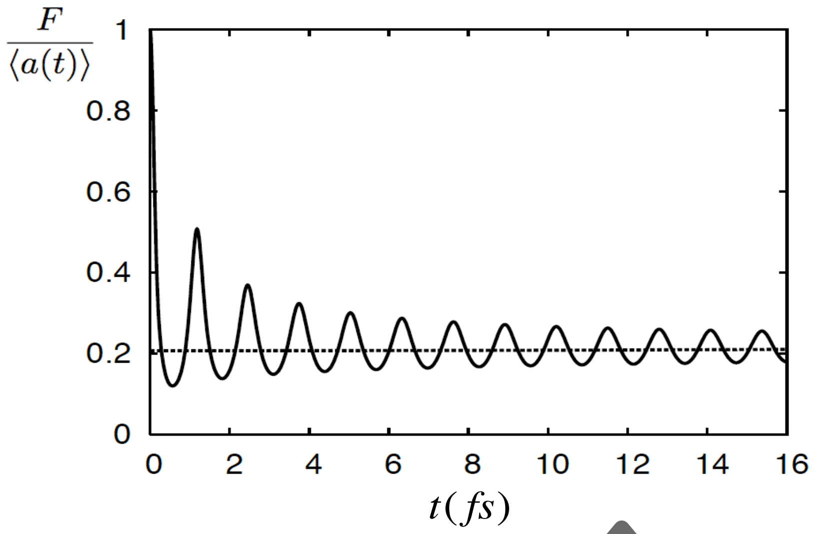
$$\left|\psi(0)\right\rangle = \int d\mathbf{k}\; c_n(\mathbf{k})e^{-i\phi_n(\mathbf{k},0)}\left|n\mathbf{k}\right\rangle$$

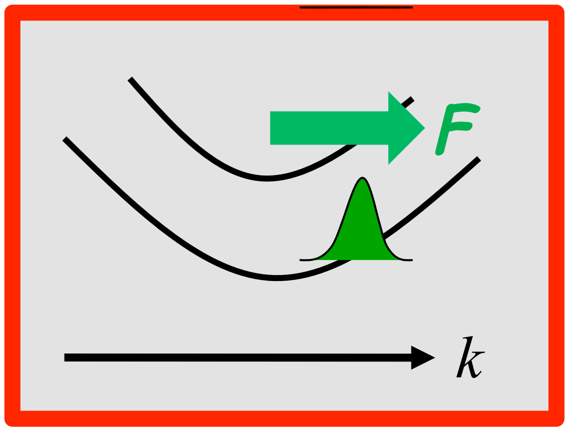


$$\left|\psi_n(t)\right\rangle = \int d\mathbf{k}\; c_n(\kappa)e^{-i\phi_n(\mathbf{k},t)}\,\overline{\left|n\mathbf{k}\right\rangle}$$

$$= \int d\mathbf{k}\; c_n(\kappa)e^{-i\phi_n(\mathbf{k},t)}\left[\left|n\mathbf{k}\right\rangle + \sum_{n'\neq n}\frac{\xi_{n'n}(\mathbf{k})\cdot\mathbf{F}}{E_{n'n}(\mathbf{k})}\left|n'\mathbf{k}\right\rangle + \cdots\right]$$

$$\left|\psi(t)\right\rangle = \left|\psi_n(t)\right\rangle + \sum_{n'\neq n}\left|\psi_{n'}(t)\right\rangle$$





femtoseconds for typical semiconductor systems

$$\langle a(t)\rangle \approx \frac{F}{m}\int dk\,|c_n(\kappa)|^2\left(\frac{m}{m_n^*(k)}+\frac{2}{m}\sum_{n'\neq n}\frac{E_{n'n}(k)}{E_{n'n}^2(\kappa)}\mathrm{Re}\left[p_{nn'}(k)p_{n'n}(\kappa)e^{i\gamma_{nn'}(\kappa,t)}\right]\right)$$

$$\kappa = k - \frac{Ft}{\hbar}$$

$$\gamma_{nn'}(\kappa,t) = \frac{1}{\hbar}\int_0^t E_{nn'}(\kappa+\frac{1}{\hbar}Ft')dt'$$

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Mathieu potential:

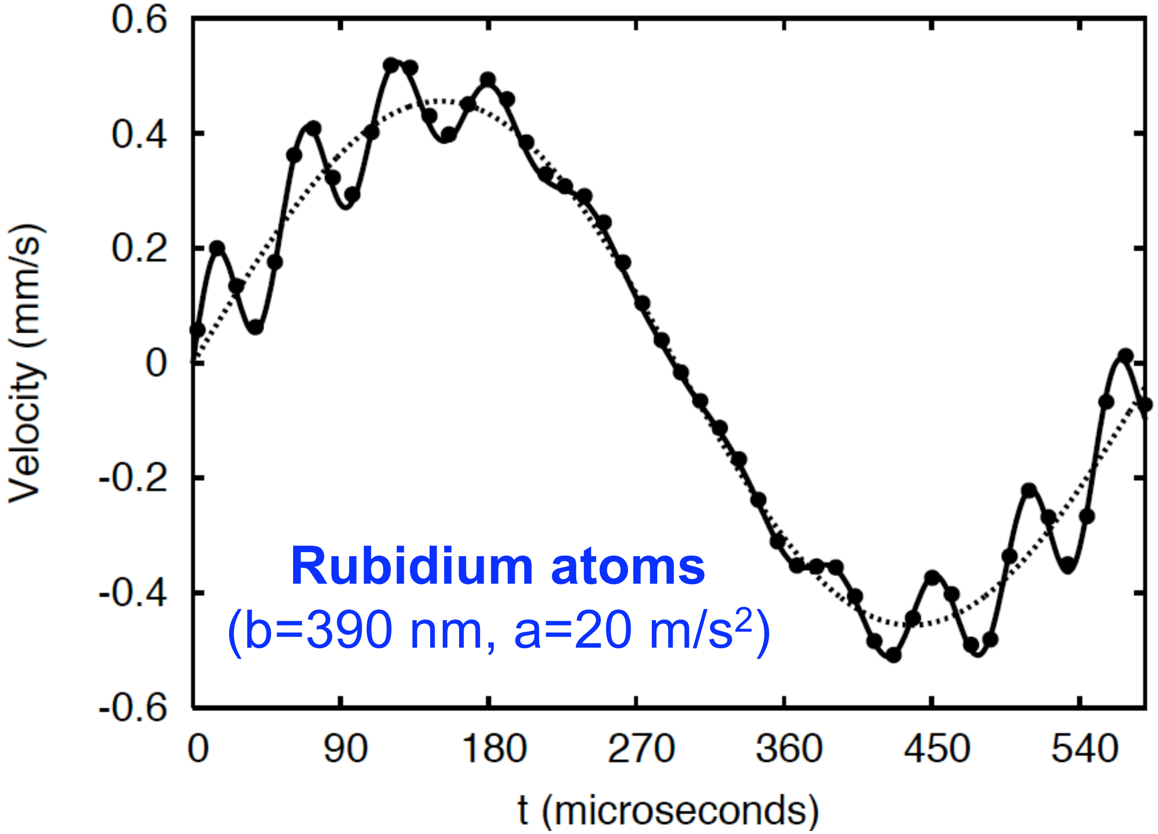
$$V(x) = sE_R \sin^2\left(\frac{\pi x}{b}\right)$$

$$E_R = \frac{\hbar^2 \pi^2}{2mb^2}$$

Gaussian wavepacket initially in centre of lowest band:

$$c_o(k) = \sqrt{\frac{1}{\sigma\sqrt{2\pi}}}\, e^{-\frac{k^2}{4\sigma^2}}$$

F. Duque Gomez and J.E. Sipe, Phys. Rev. A85, 053412 (2012)



$$s = 8 \qquad \sigma = 0.3\frac{\pi}{b} \qquad \tilde{F} \equiv \frac{bF}{\pi E_R} = 0.143$$

Mathieu potential:

$$V(x) = sE_R \sin^2\left(\frac{\pi x}{b}\right)$$

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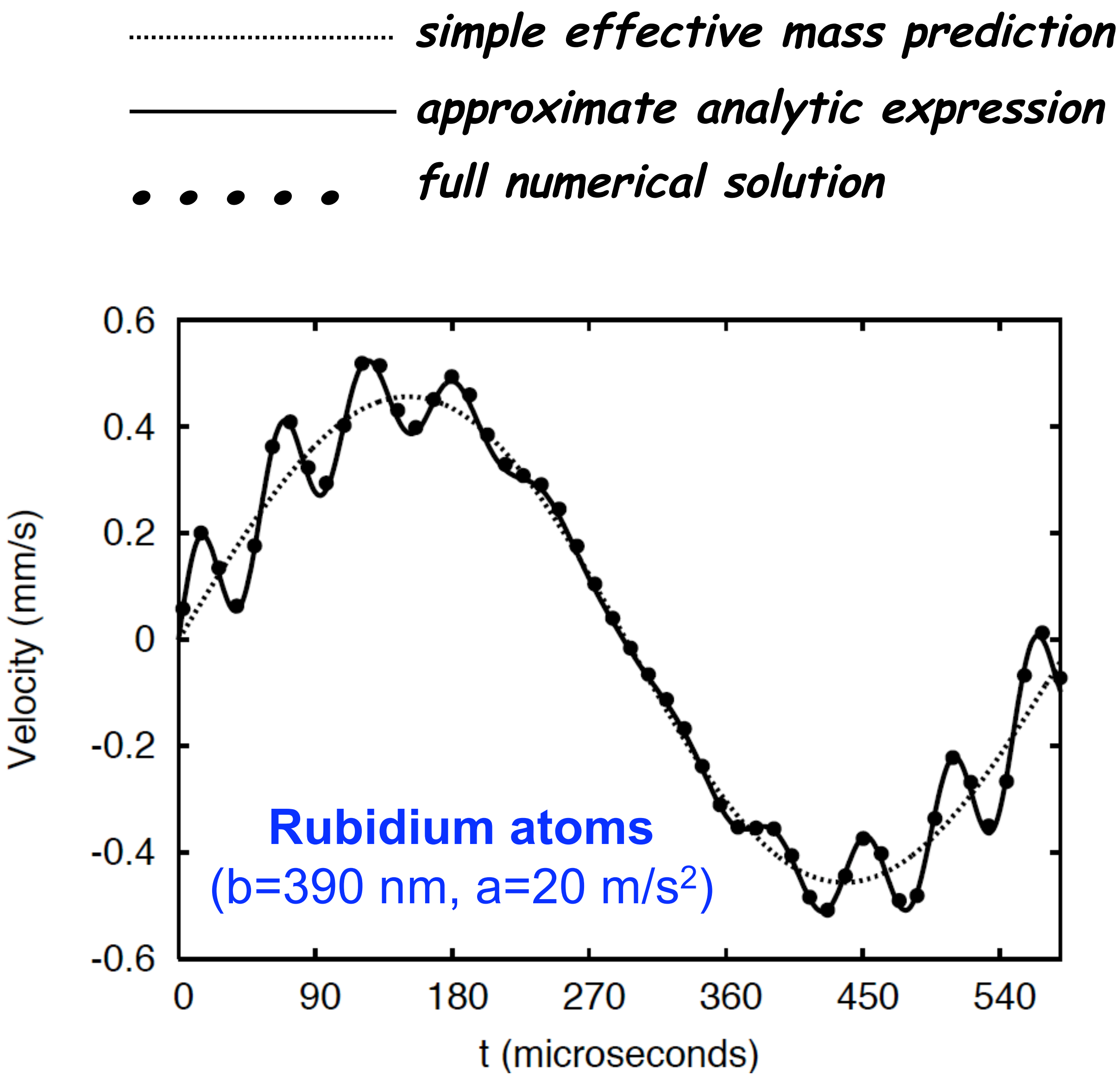
Gaussian wavepacket initially in centre of lowest band:

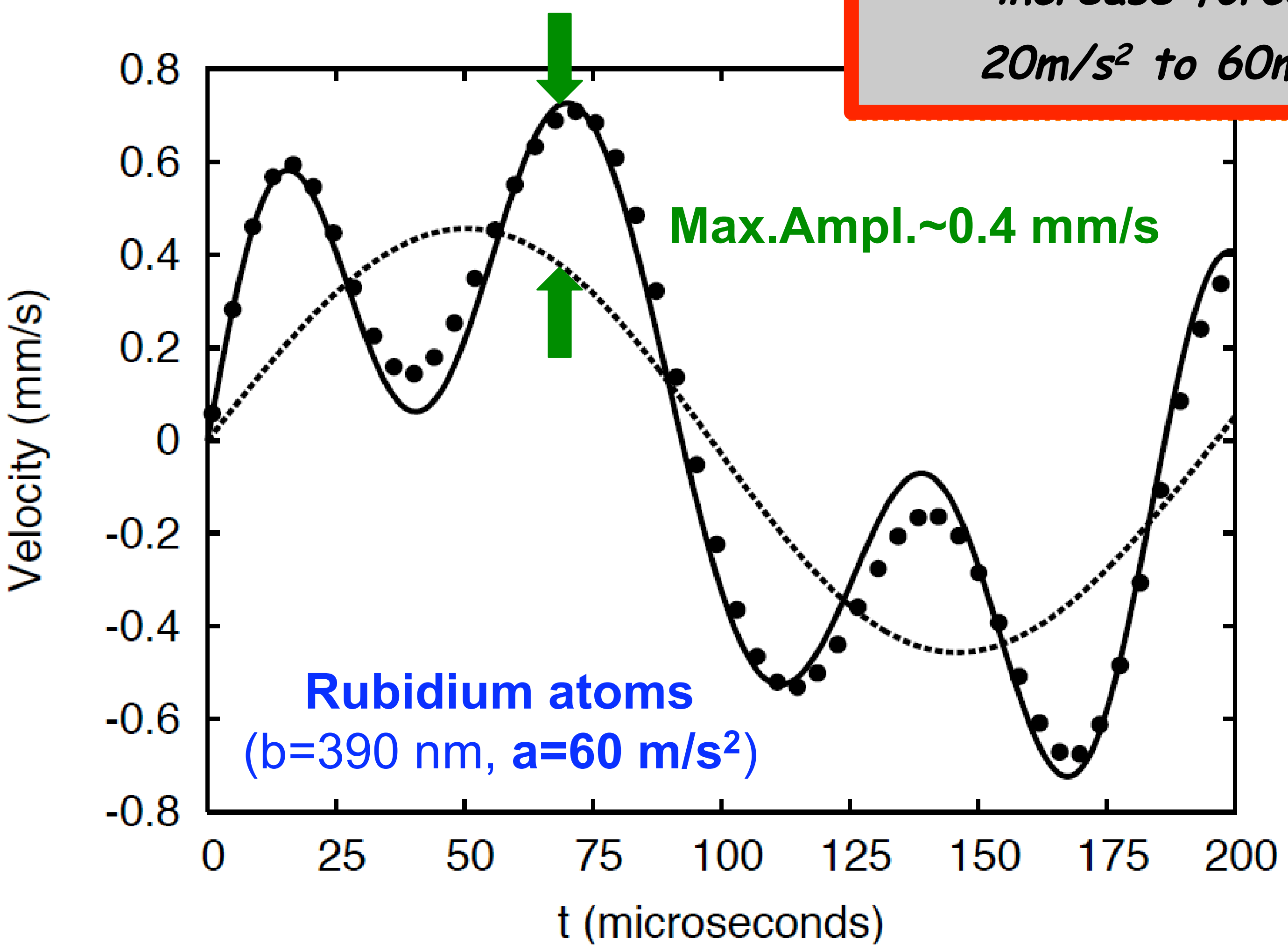
$$c_o(k) = \sqrt{\frac{1}{\sigma\sqrt{2\pi}}}\, e^{-\frac{k^2}{4\sigma^2}}$$



$$s = 8 \qquad \sigma = 0.3\frac{\pi}{b} \qquad \tilde{F} \equiv \frac{bF}{\pi E_R} = 0.143$$

increase force

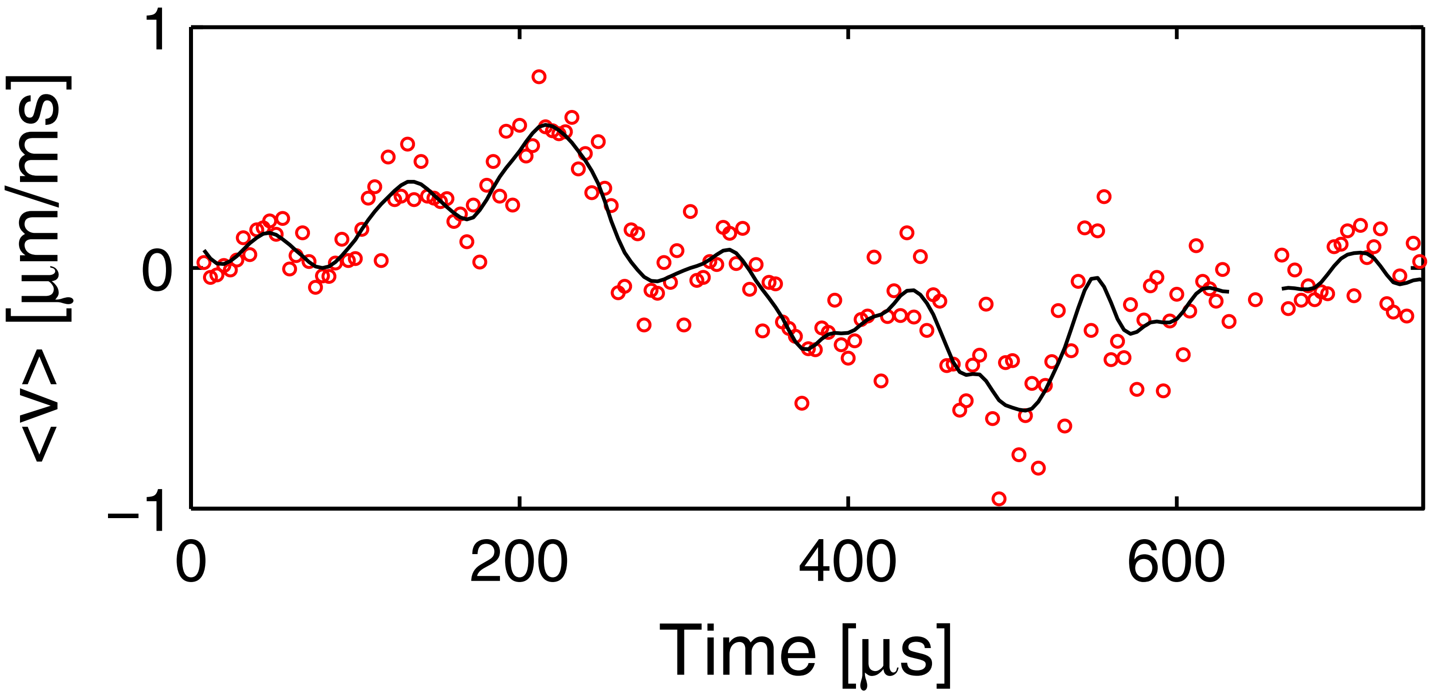
$20 m/s^2$ to $60 m/s^2$



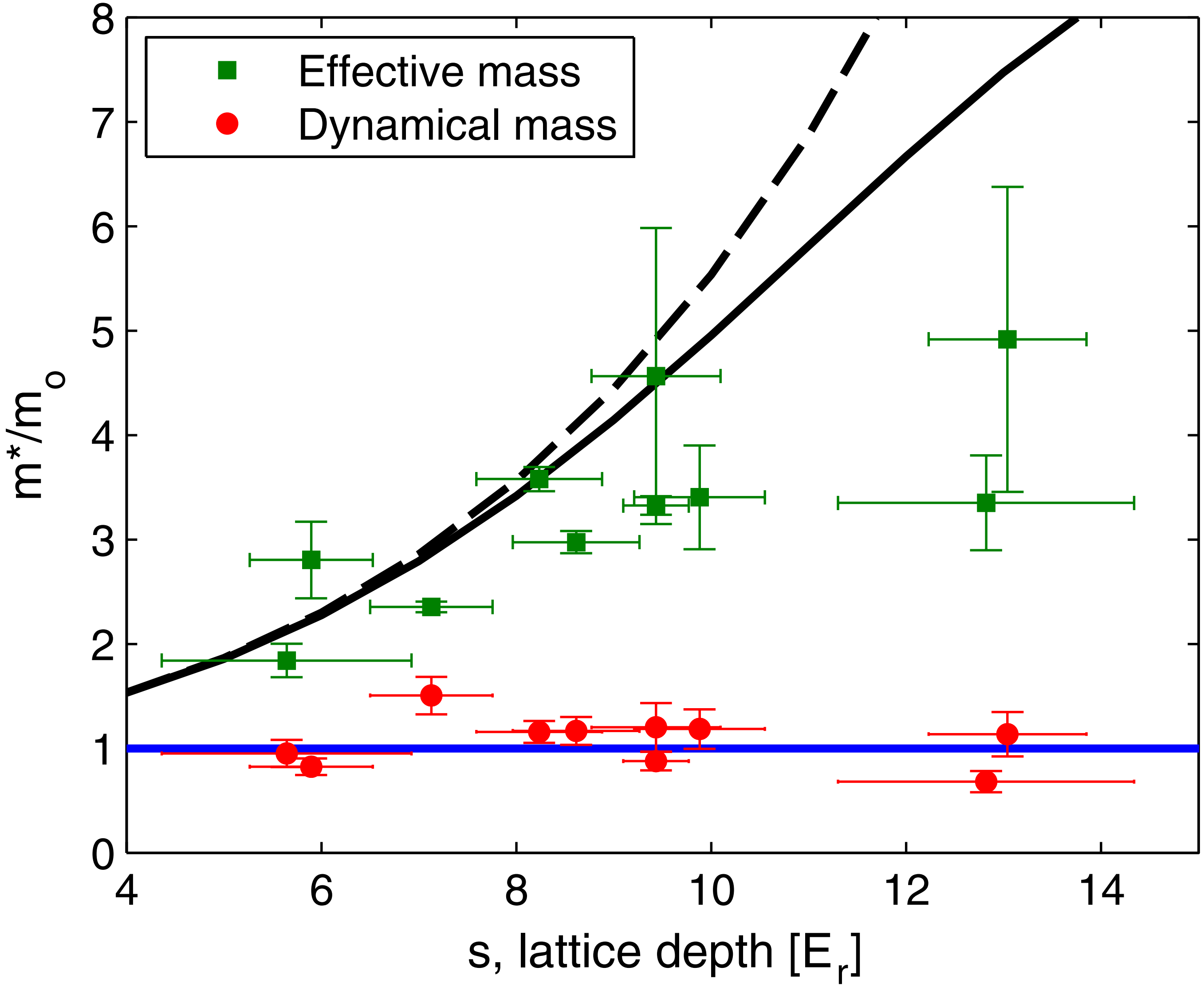
...still <1% Zener tunneling into other bands

Experiments

Rockson Chang et al.,
Phys. Rev. Lett. 112, 170404 (2014)



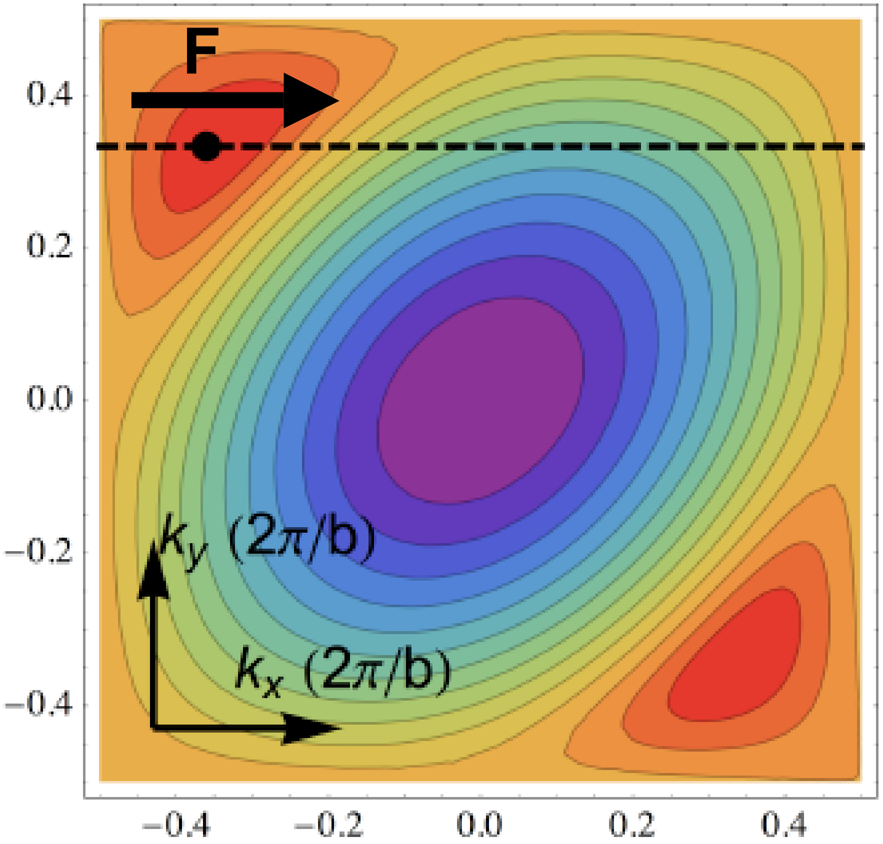
S = 9.4, F/m = 11.7 μm/ms^2



Effective mass
Dynamical mass
m*/m$_o$
s, lattice depth [E$_r$]
0
1
2
3
4
5
6
7
8
4
6
8
10
12
14

Experiments in 2D systems?

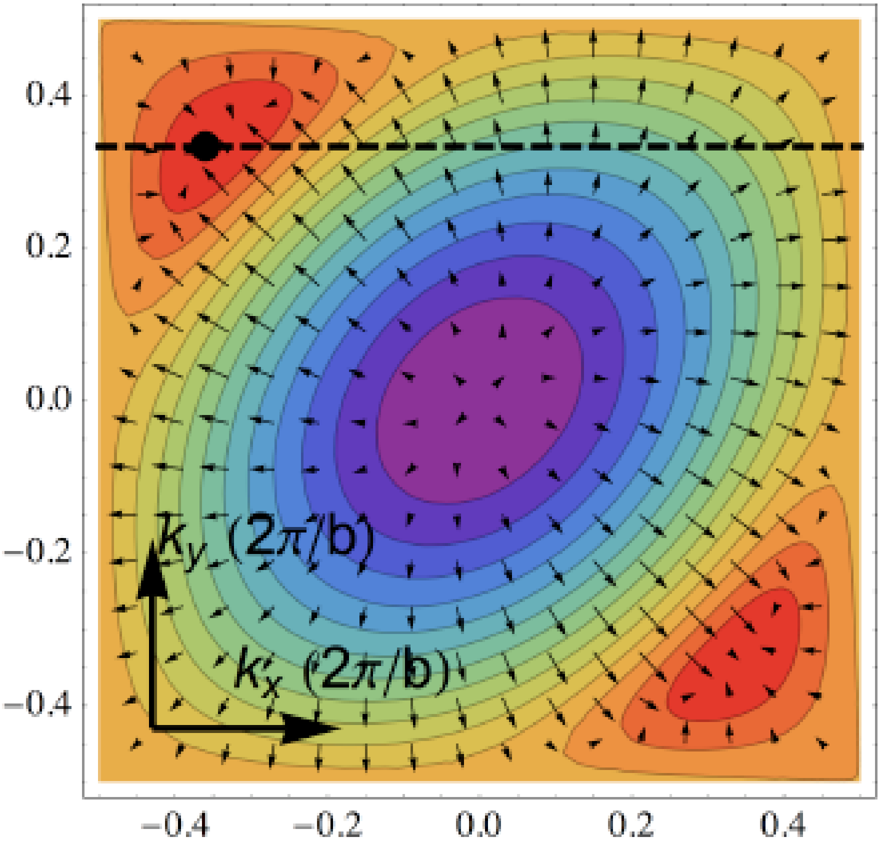
Y. Fang, Federico Duque-Gomez, and J.E. Sipe,
Phys. Rev. A90, 053407 (2014)



Load a wave packet in a band and apply a force

Experiments in 2D systems?

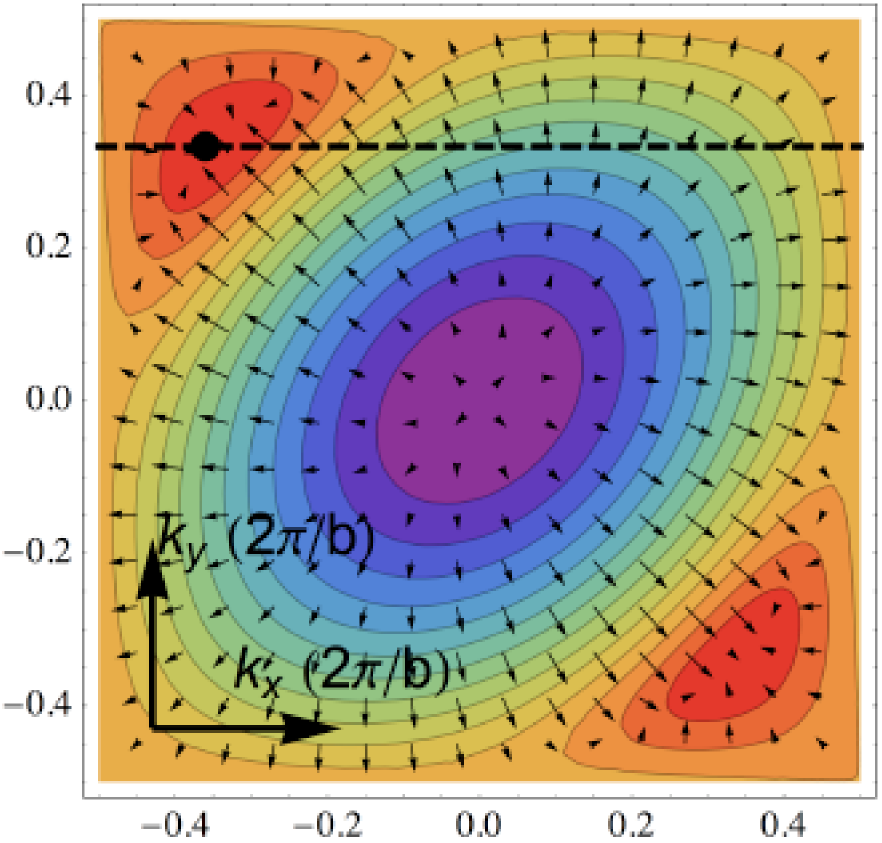
Y. Fang, Federico Duque-Gomez, and J.E. Sipe,
Phys. Rev. A90, 053407 (2014)

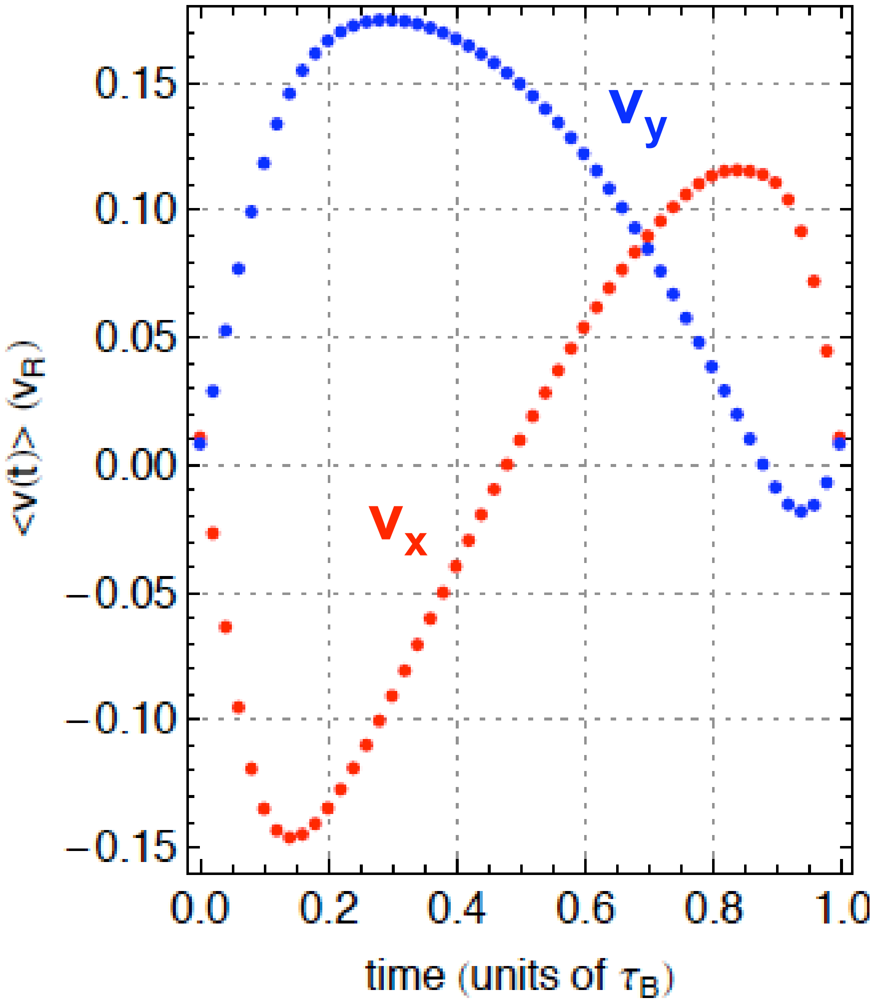


Load a wave packet in a band and apply a force

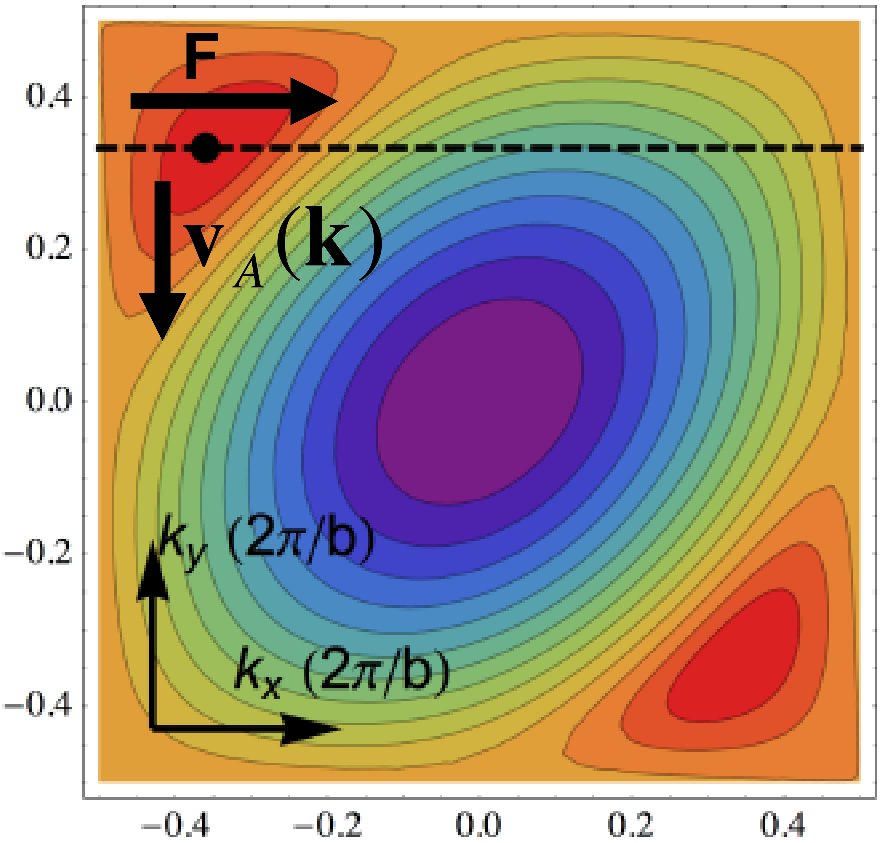
Simple effective mass picture

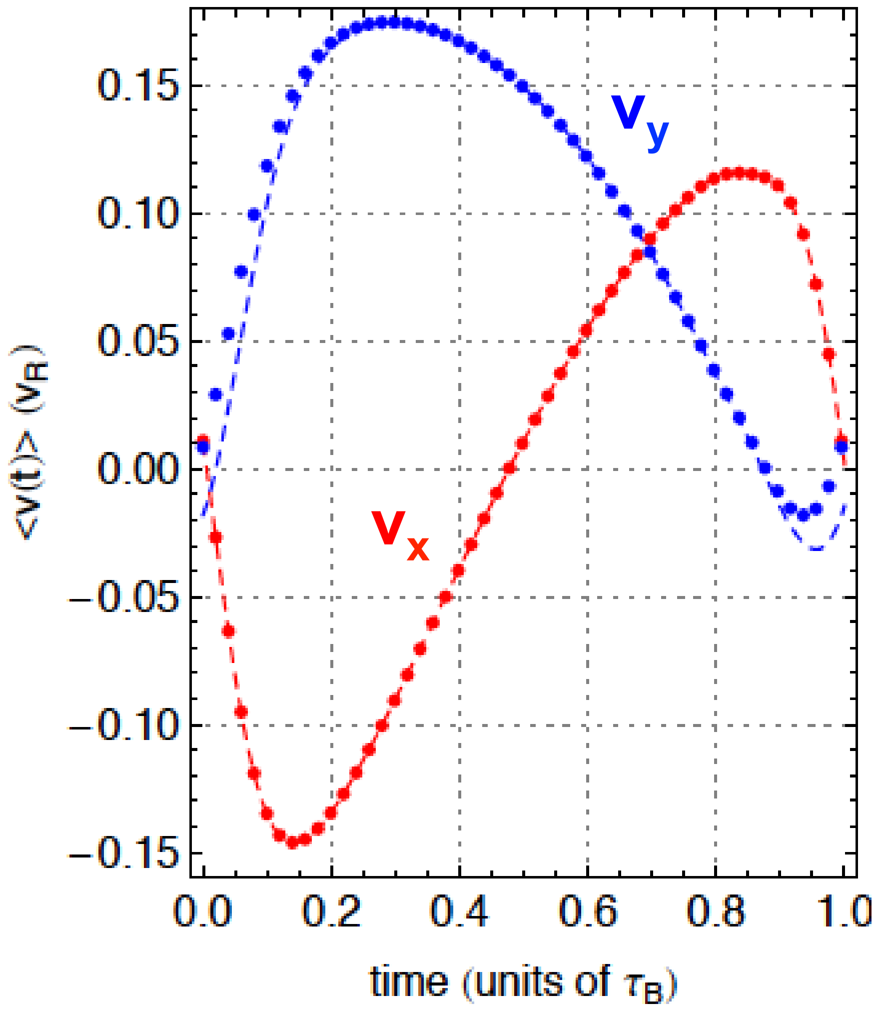
$$v_R = h / mb \qquad \tau_B = h / bF$$

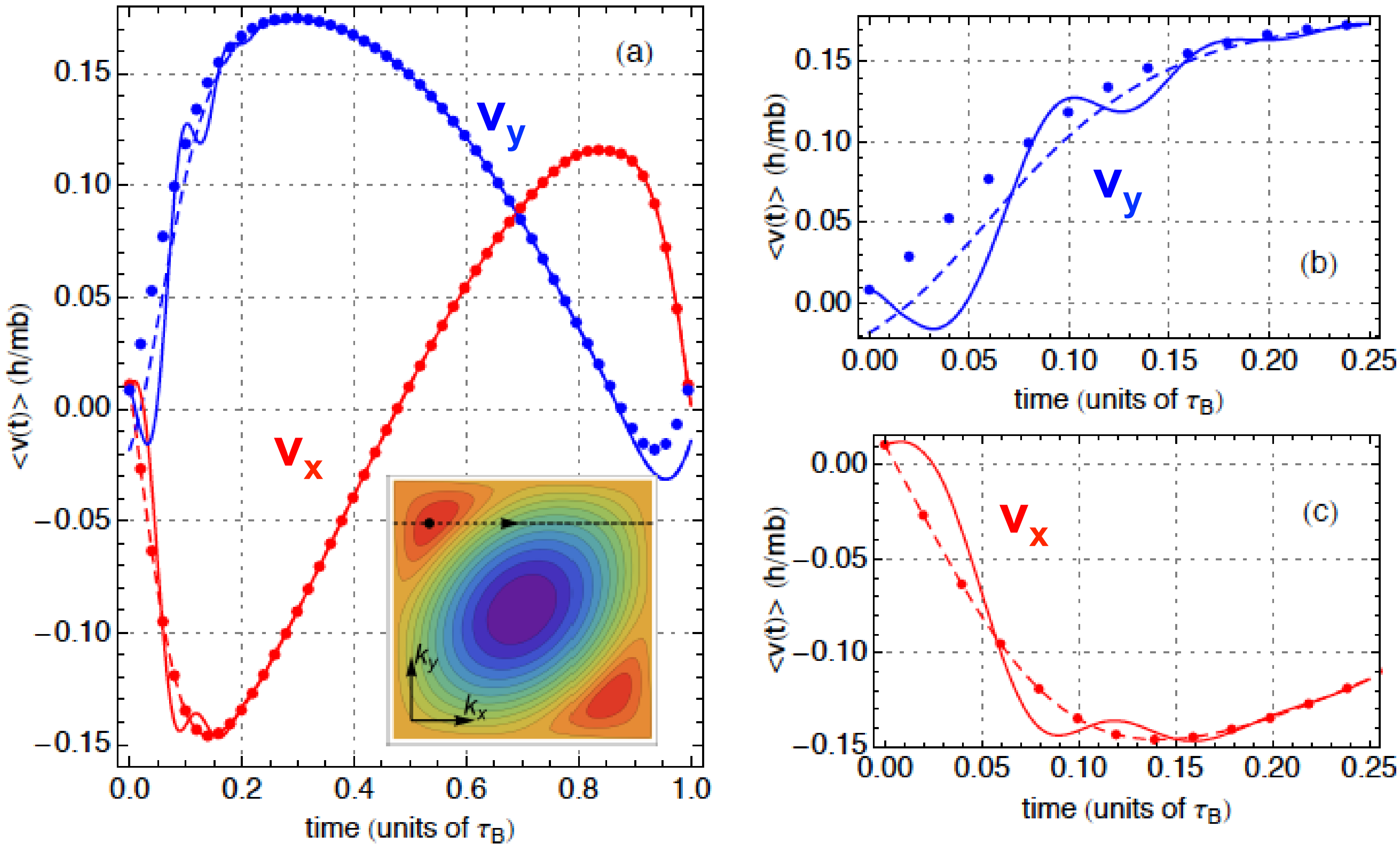




Simple effective mass
+ anomalous velocity picture







(a)
(b)
(c)
v_y
v_x
k_y
k_x
<v(t)> (h/mb)
time (units of τ_B)
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.0
0.2
0.4
0.6
0.8
1.0
0.25
0.20

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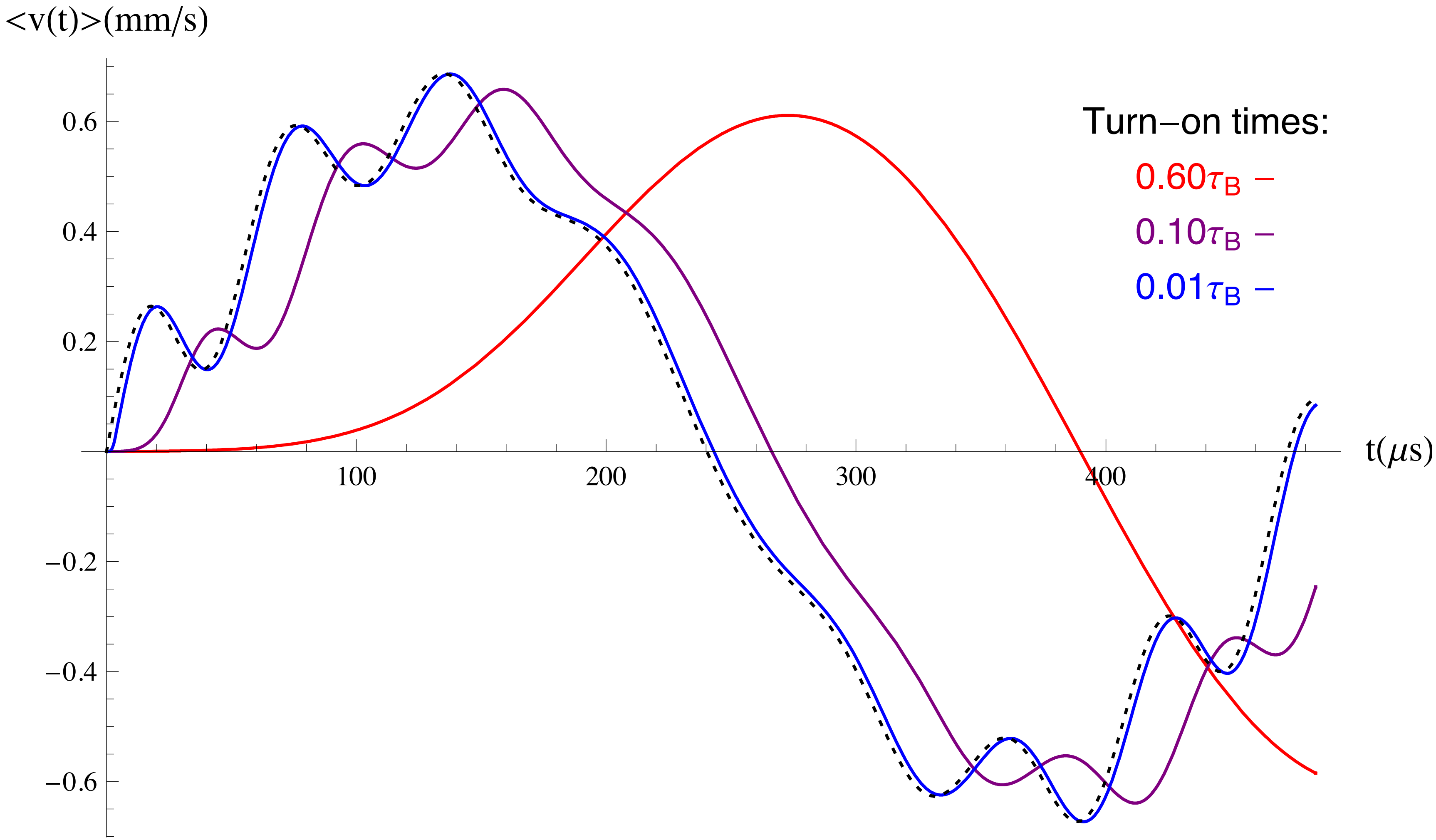
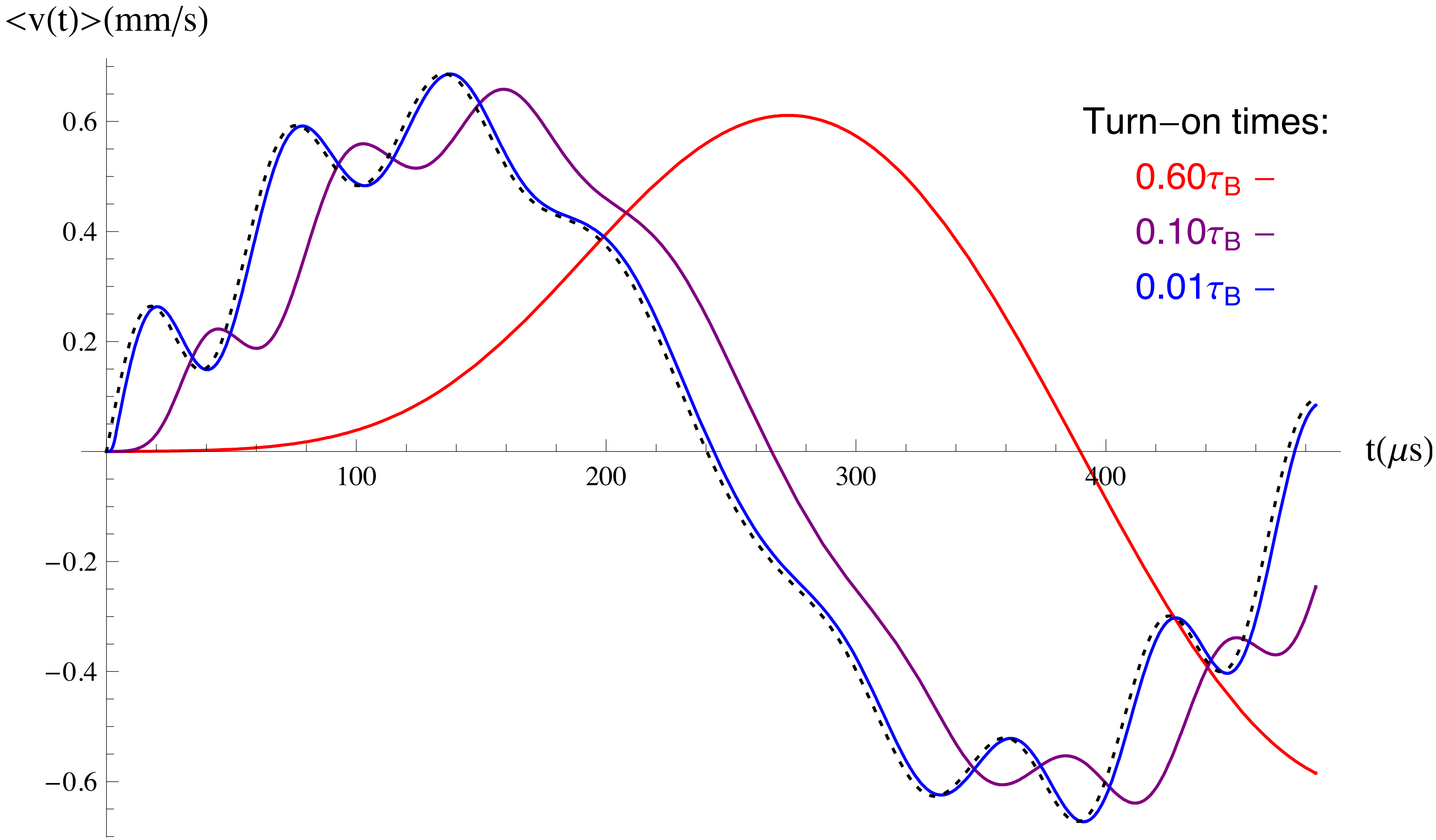
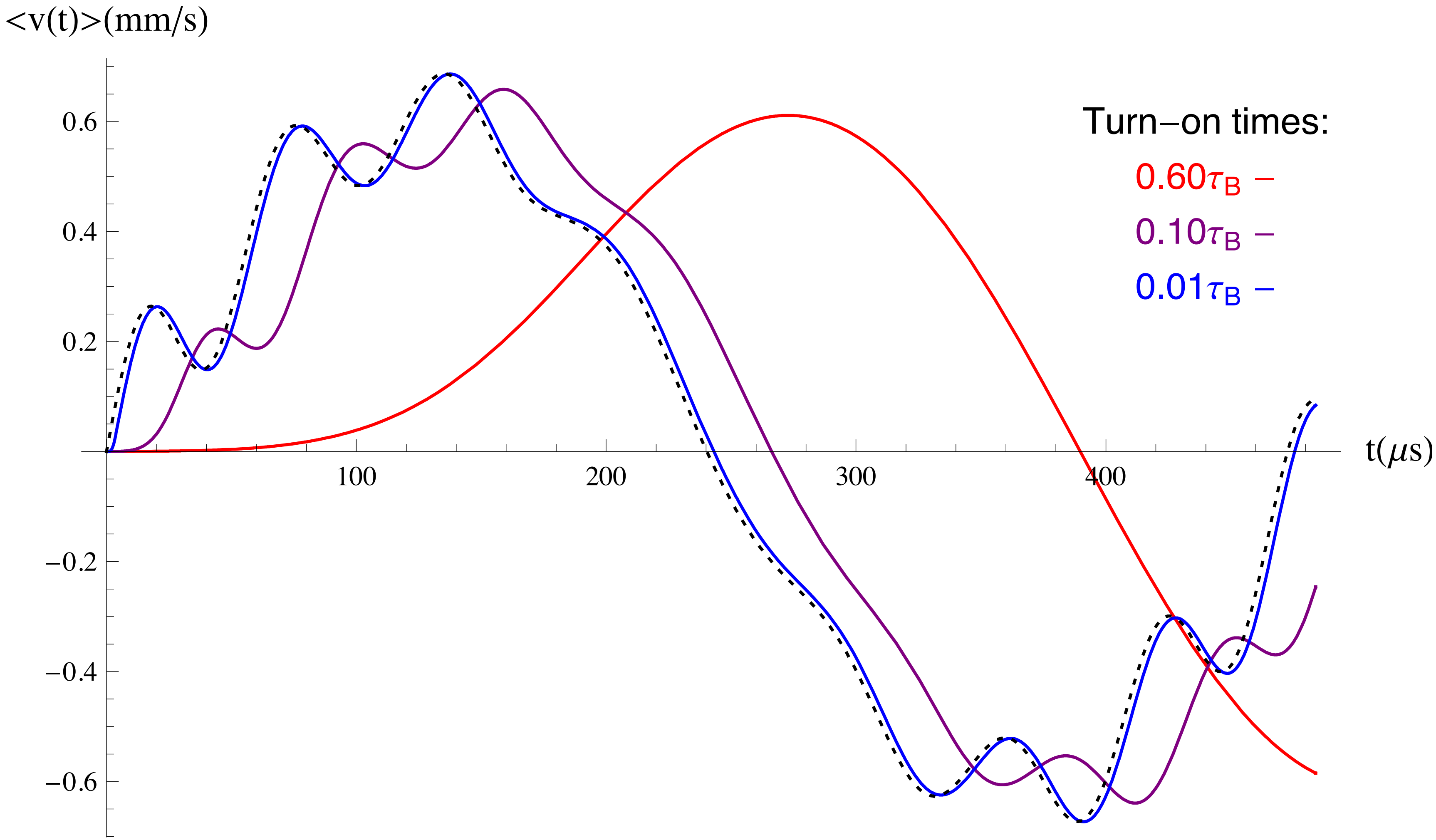


<v(t)>(mm/s)
0.6
0.4
0.2
−0.2
−0.4
−0.6
100
200
300
400
t(μs)
Turn−on times:
$0.60\tau_B$ –
$0.10\tau_B$ –
$0.01\tau_B$ –

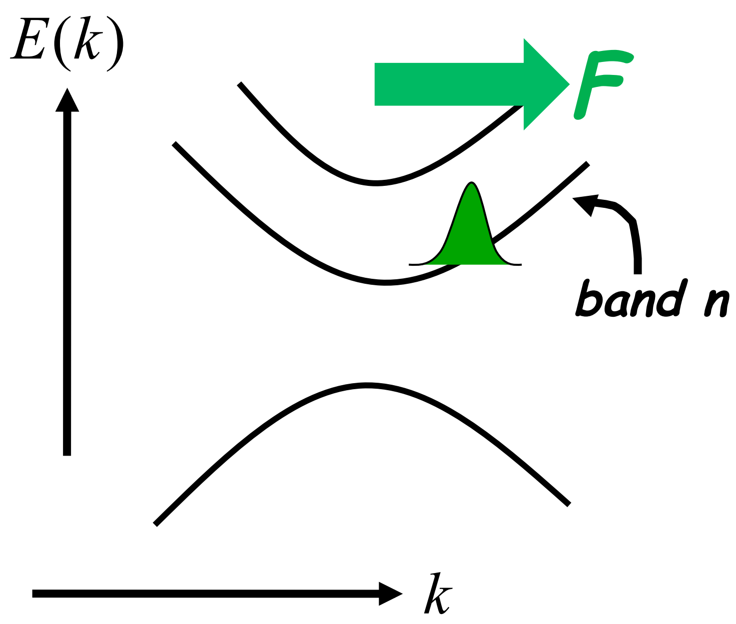


see the "emergence" of mass,



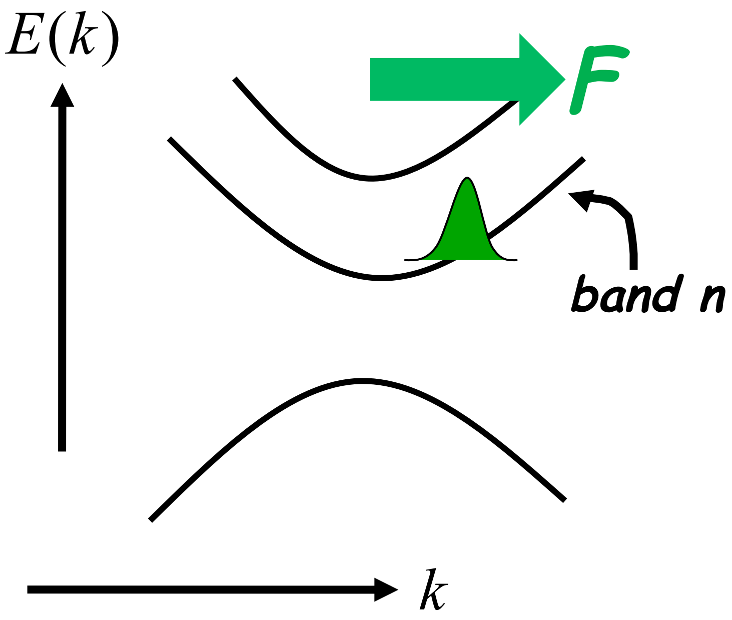
see the "emergence" of mass,

and other "single particle" quasi-particle properties



$$|\psi(0)\rangle = \int dk\; c(k)\,|nk\rangle$$

Bloch states

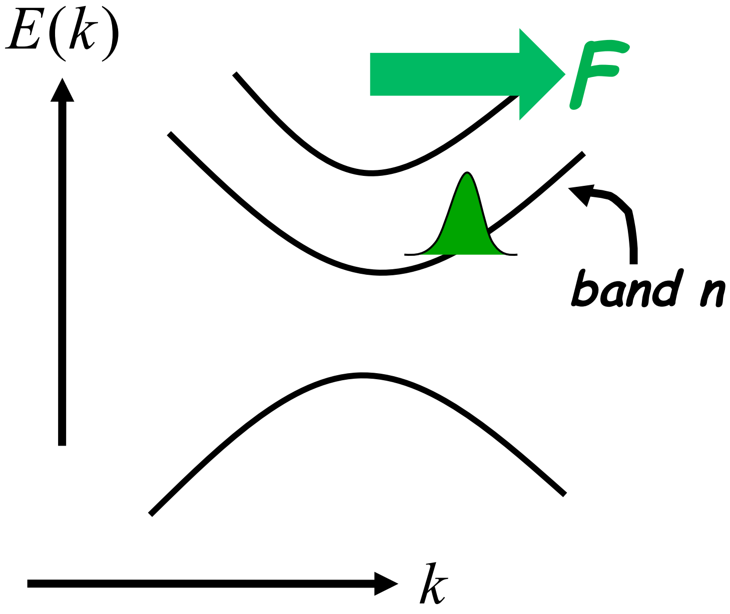


$$\kappa = \mathbf{k} - \frac{\mathbf{F}t}{\hbar}$$

$$\left|\psi(0)\right\rangle = \int dk\; c(k)\left|nk\right\rangle$$

Bloch states

$$\left[\frac{1}{m^*(t)}\right] = \int_{BZ} d\mathbf{k}\; \left|c(\kappa)\right|^2 \left(\left[\frac{1}{m^{(n)}(\mathbf{k},t)}\right] - \mathrm{K}_{sym}^{(n)}(\mathbf{k},t)\right)$$



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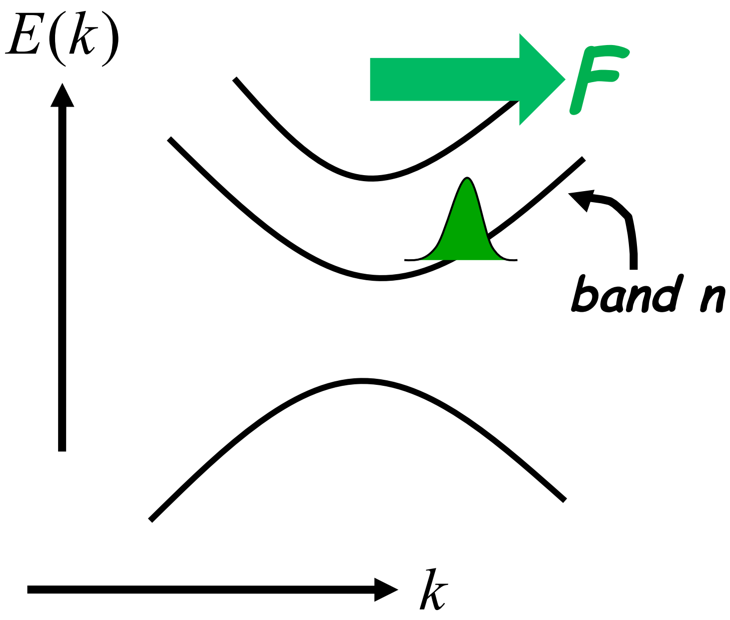
$$\Omega(t) = \int_{BZ} d\mathbf{k}\ |c(\kappa)|^2 \left(\Omega^{(n)}(\mathbf{k}) - \Lambda^{(n)}(\mathbf{k},t)\right)$$



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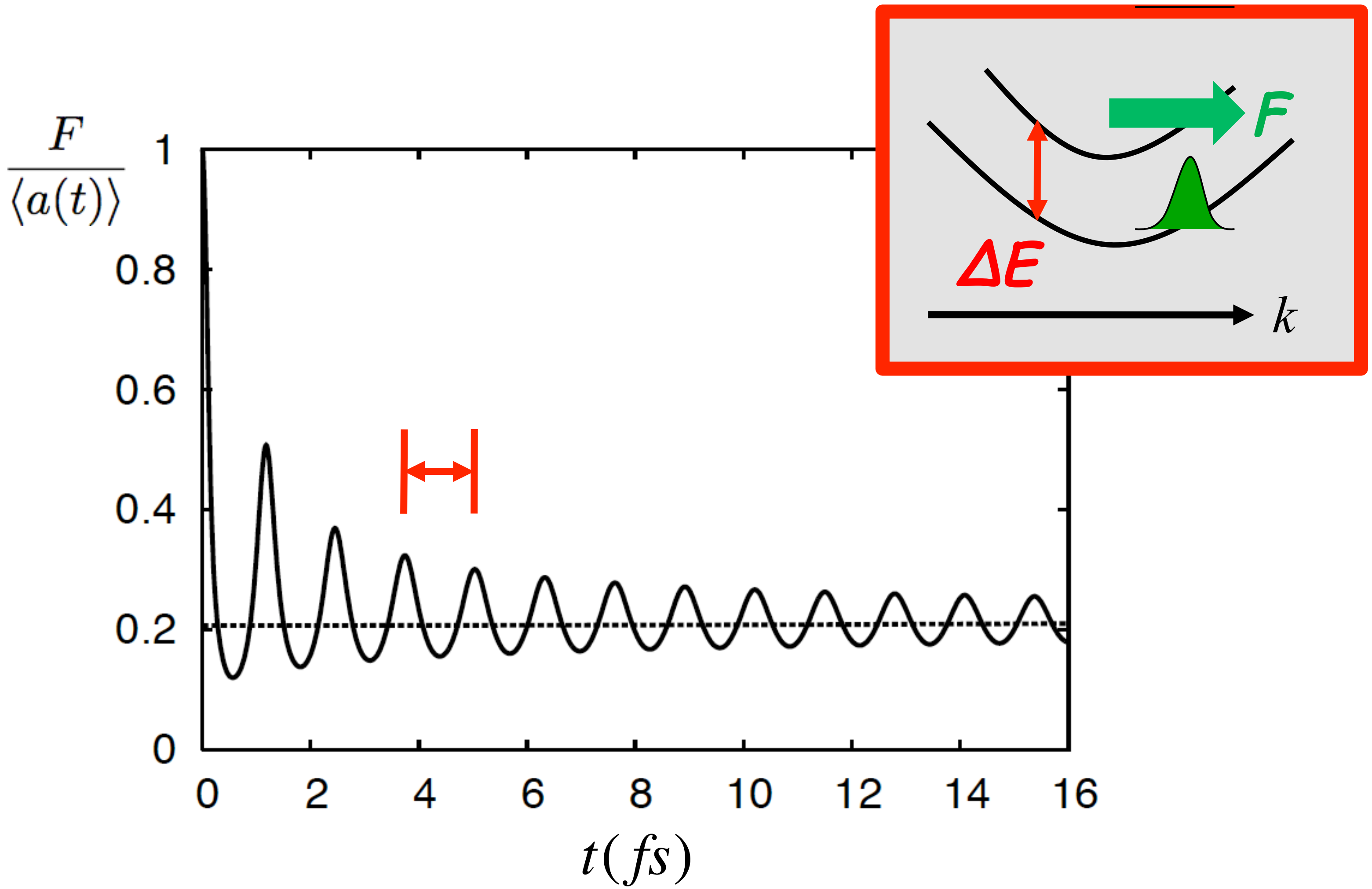
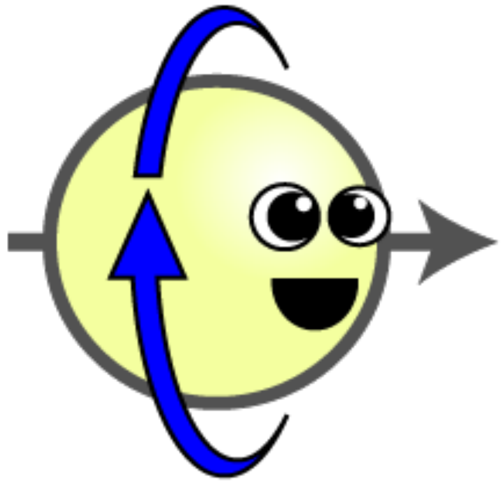
$$\Omega(t) = \int_{BZ} d\mathbf{k}\; |c(\kappa)|^2 \left(\Omega^{(n)}(\mathbf{k}) - \Lambda^{(n)}(\mathbf{k},t)\right)$$

$$\mathbf{a}_{anomalous}(t) = \int_{BZ} d\mathbf{k}\; |c(\kappa)|^2 \left(-\mathrm{K}^{(n)}_{antisym}(\mathbf{k},t)\right)$$



period(s) inversely proportional
to energy difference(s)

Attosecond control of pulses should allow the study of the emergence of "single-particle" quasi-particle properties

What about materials like graphene?

How affected by electron-electron,
electron-phonon interactions?


SUE THE BASTARDS