

# **A dream comes true: Room-temperature superconductivity**

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Universita' degli Studi dell'Aquila



Vostok base in Antarctica

In 1983 a temperature of  $-89.2\text{ }^{\circ}\text{C}$  was registered

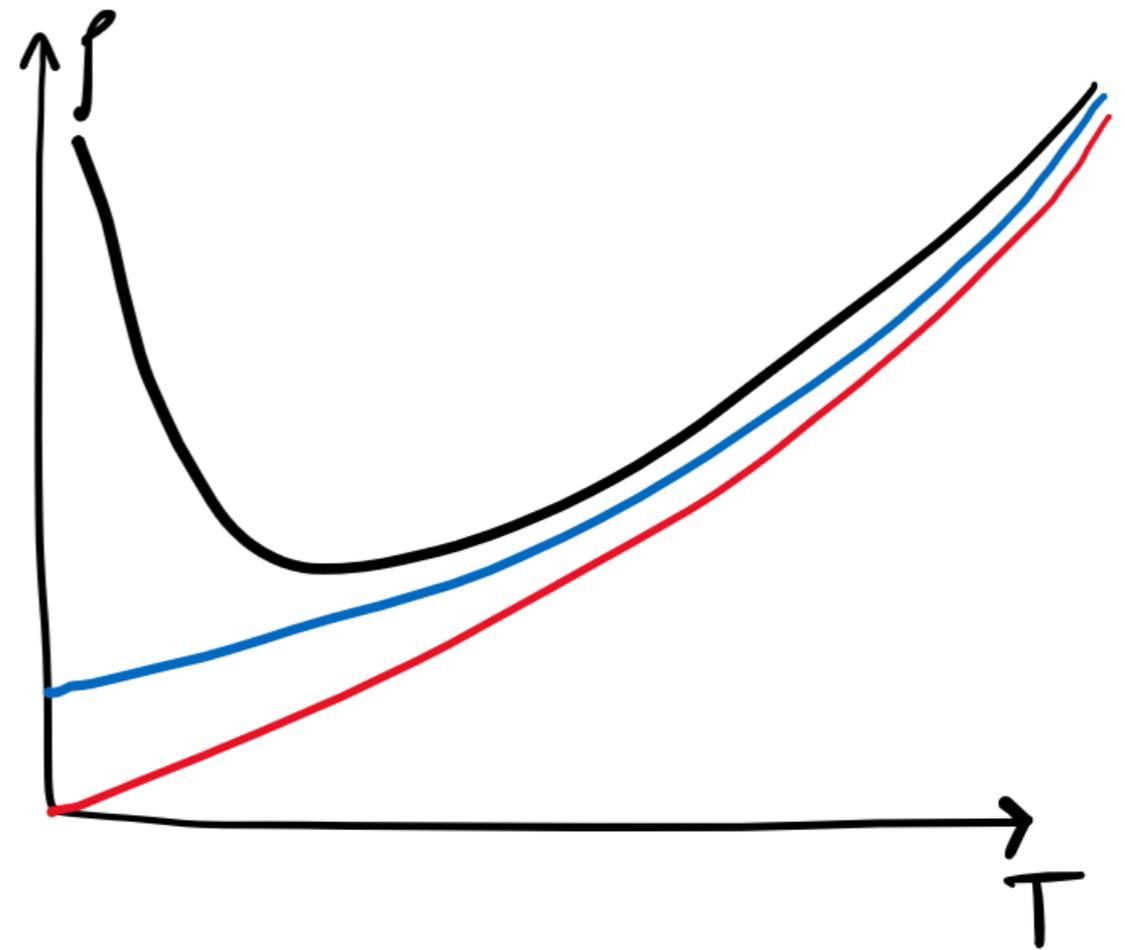


## Fontana Luminosa, L'Aquila

In January 2017 a temperature of  $-12.0\text{ }^{\circ}\text{C}$  was registered

What happens to the electrical resistivity when we lower the temperature?

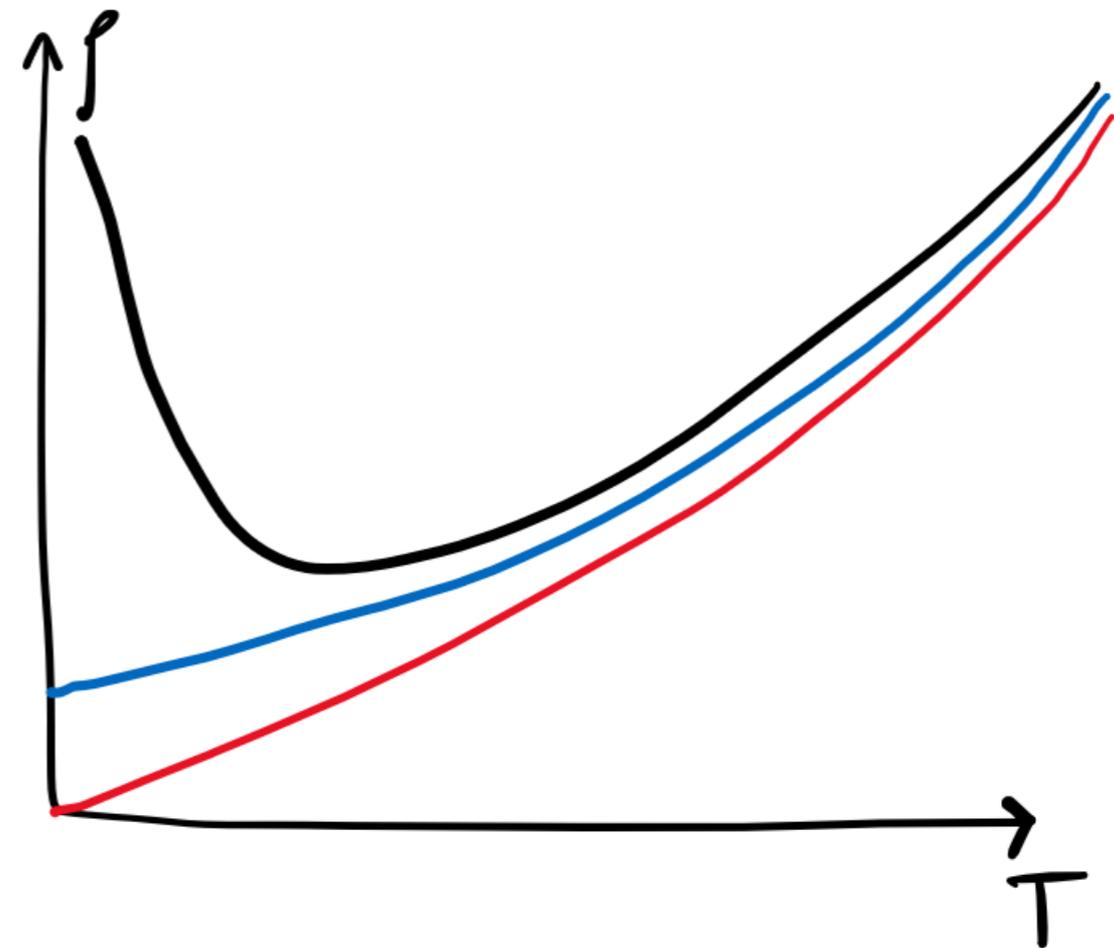
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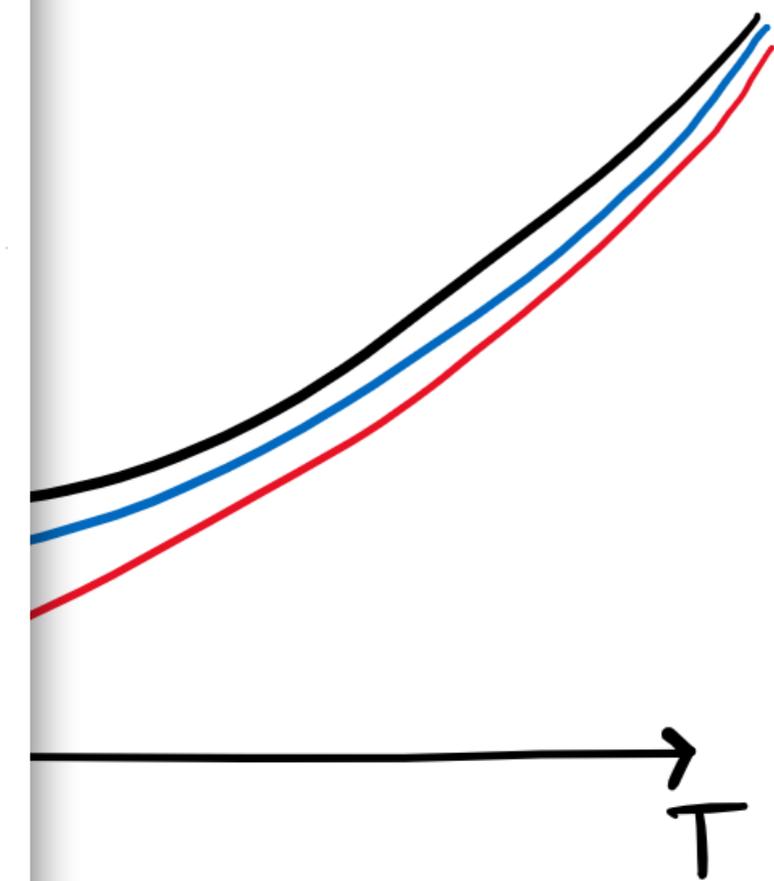
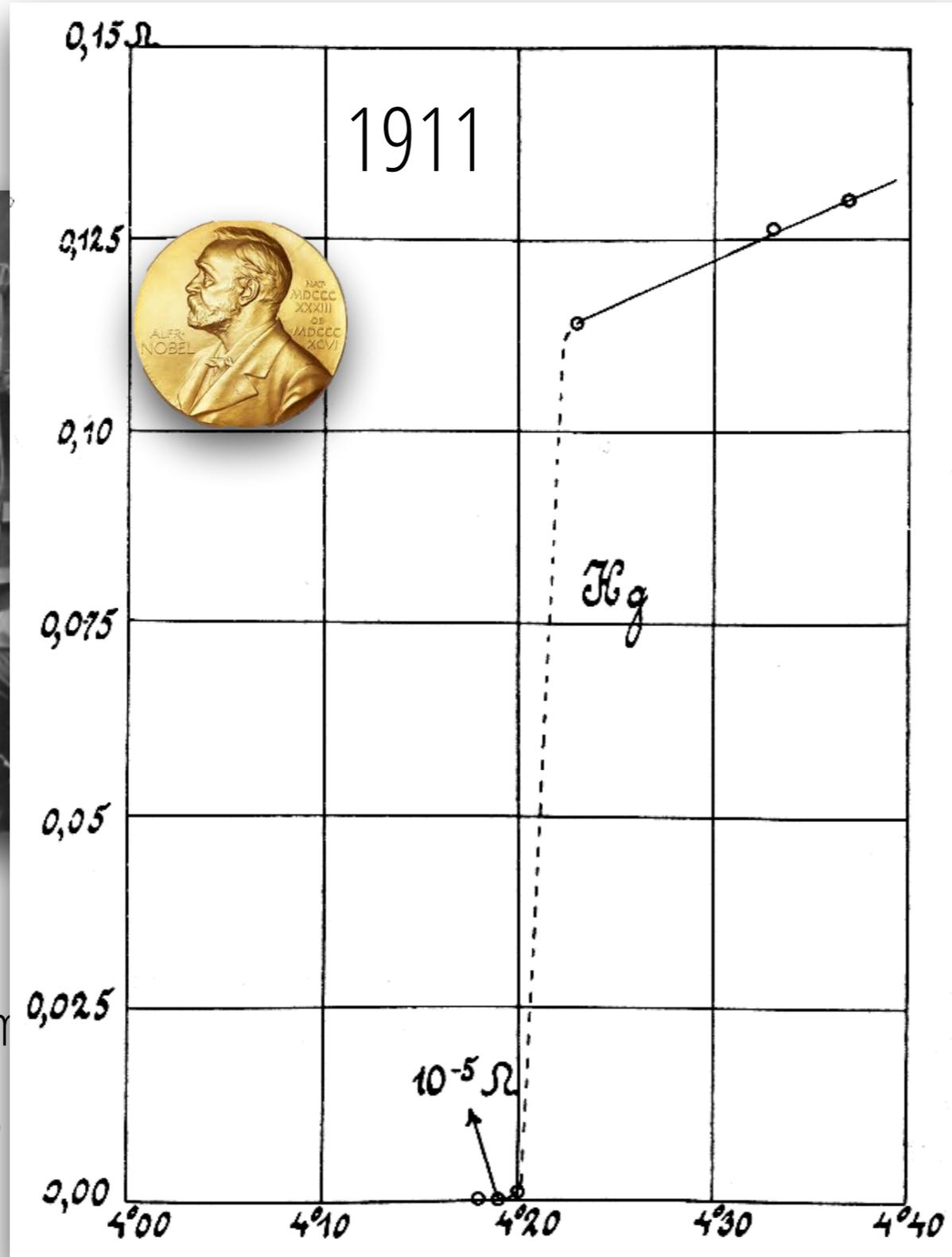
In 1908 liquid helium was realised by Kamerling Onnes



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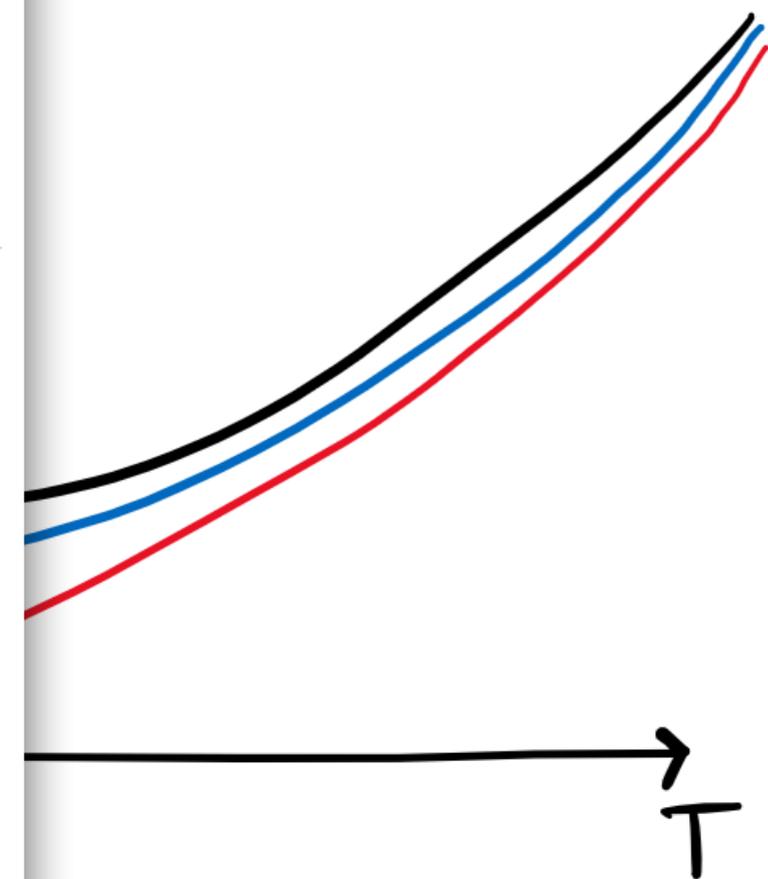
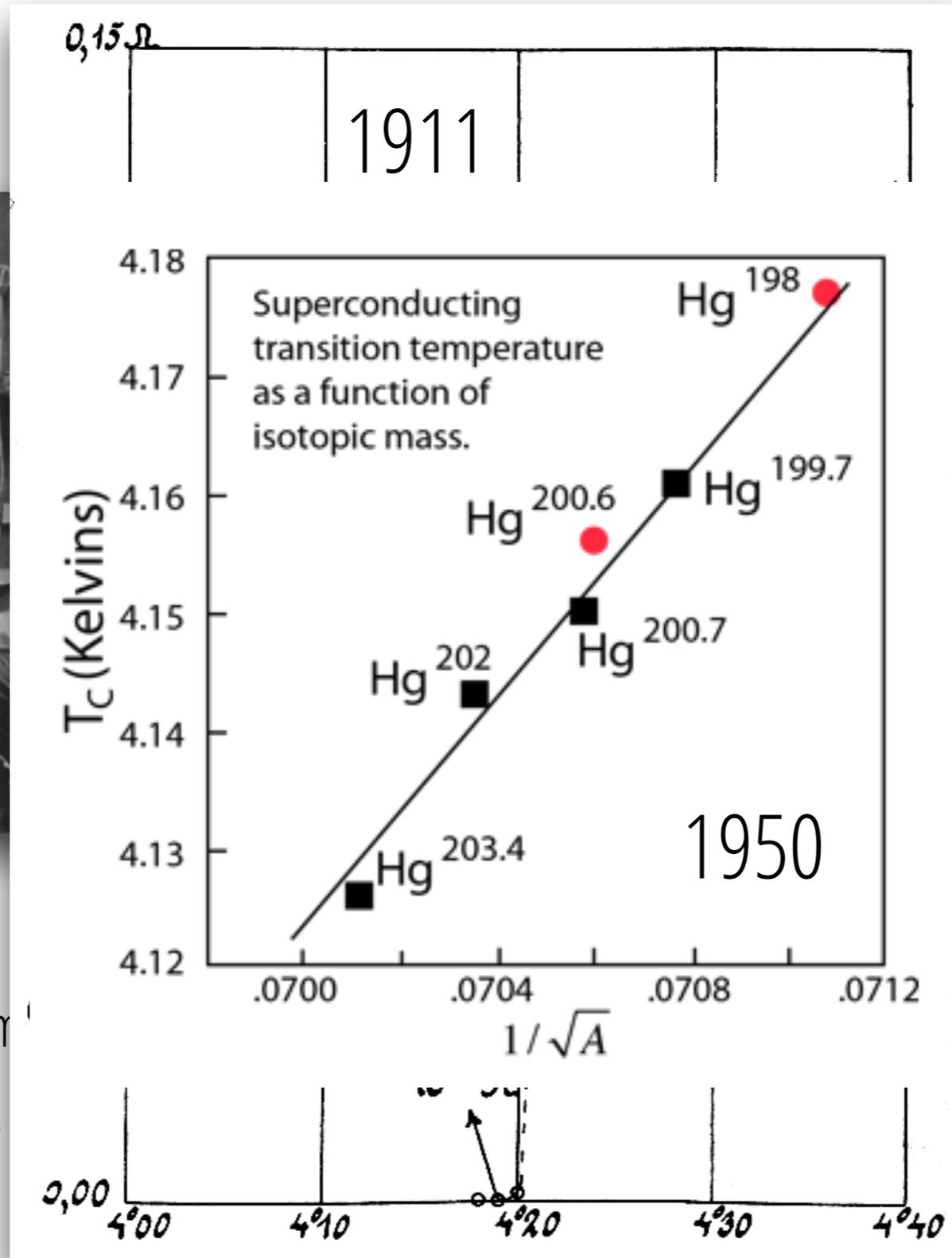
In 1908 liquid helium  
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# Superconductors

It is a common phenomenon

Persistent current: no Joule effect

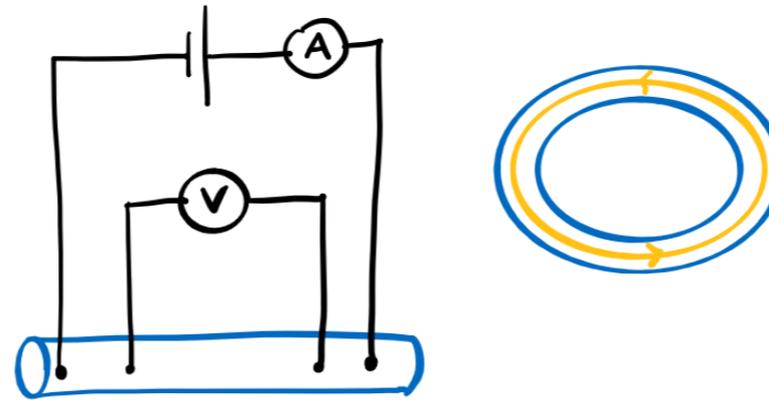
KNOWN SUPERCONDUCTIVE ELEMENTS

■ BLUE = AT AMBIENT PRESSURE  
■ GREEN = ONLY UNDER HIGH PRESSURE

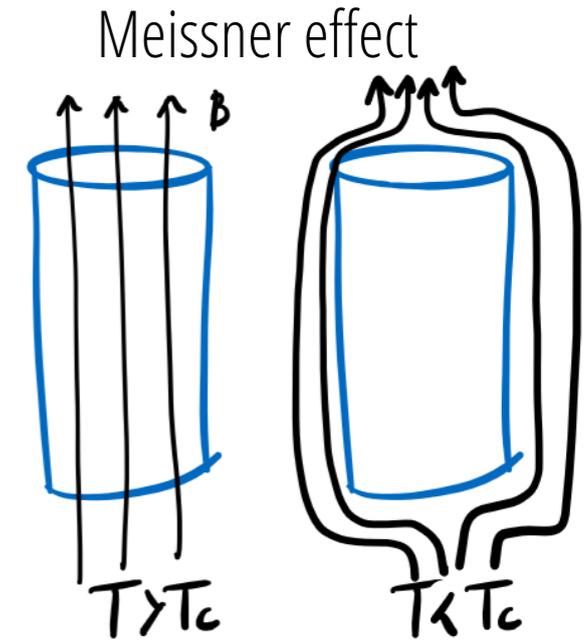
1	2																	10
1	2																	10
3	4																	10
11	12																	18
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	
87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	
117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	

\* Lanthanide Series: Ce, Pr, Nd, Pm, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, Lu  
+ Actinide Series: Th, Pa, U, Np, Pu, Am, Cm, Bk, Cf, Es, Fm, Md, No, Lr

SUPERCONDUCTORS.ORG



$J_c = 50 \text{ kA}$



$H_c = 13-18 \text{ Tesla}$

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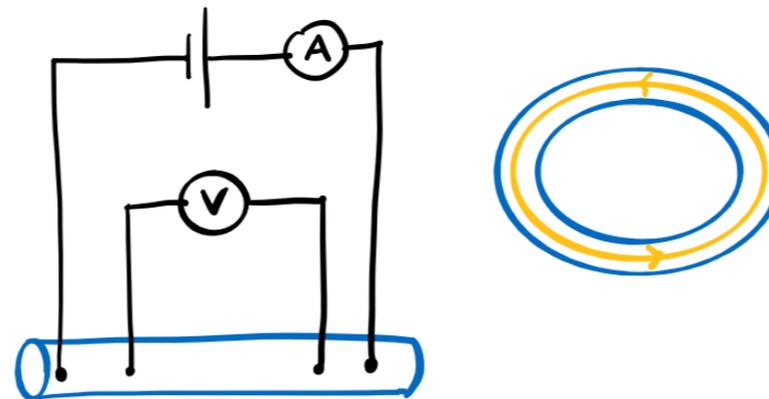
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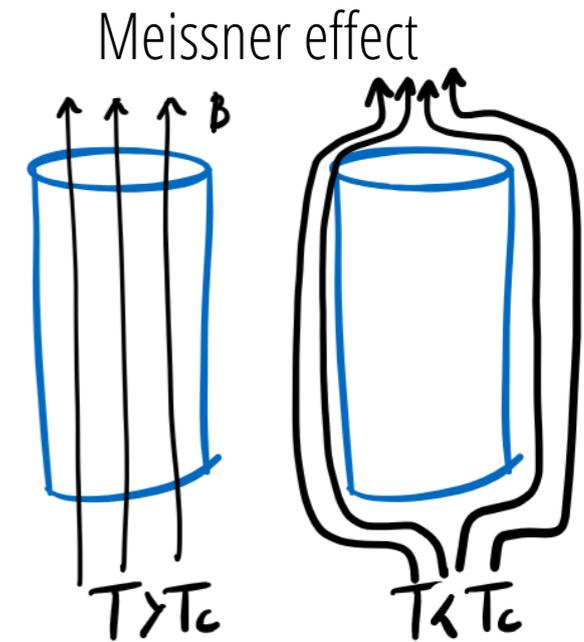
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1	H																	2	He
2	Li	Be											B	C	N	O	F	Ne	
3	Na	Mg					Al	Si	P	S	Cl	Ar							
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	
6	Cs	Ba	*La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
7	Fr	Ra	+Ac	Rf	Ha	106	107	108	109	110	111	112							
		* Lanthanide Series		59	60	61	62	63	64	65	66	67	68	69	70	71			
		+ Actinide Series		90	91	92	93	94	95	96	97	98	99	100	101	102	103		
				Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

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## ITER

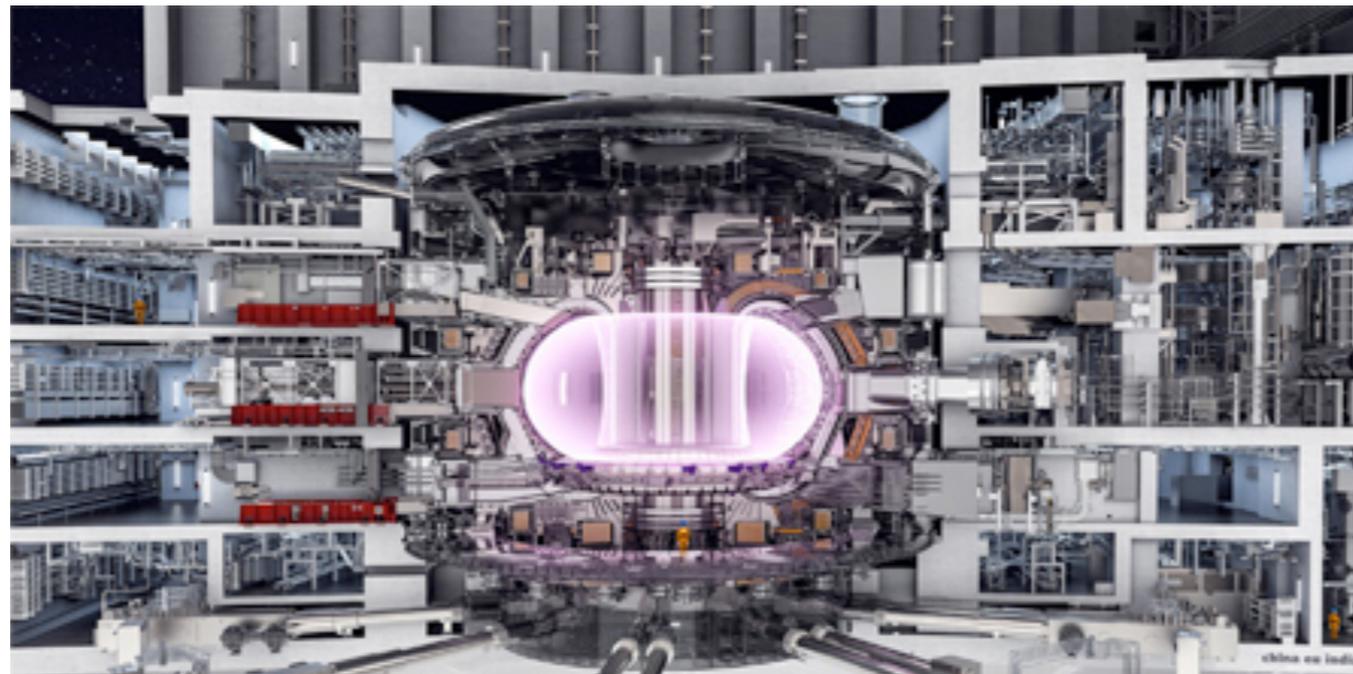
**Nb<sub>3</sub>Sn@ 4K (ASG)**

100.000 Km of cables

41 GJ energy

13.5 Tesla

80 kA



Input power: 620 MW

Output power: 500-700 MW

Large fraction for refrigeration

## Theory of Superconductivity\*

J. BARDEEN, L. N. COOPER,<sup>†</sup> AND J. R. SCHRIEFFER<sup>‡</sup>  
*Department of Physics, University of Illinois, Urbana, Illinois*

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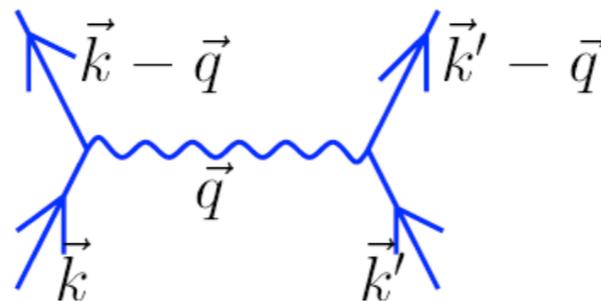
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### Cooper instability

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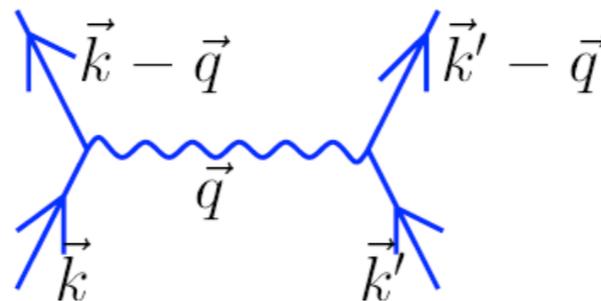
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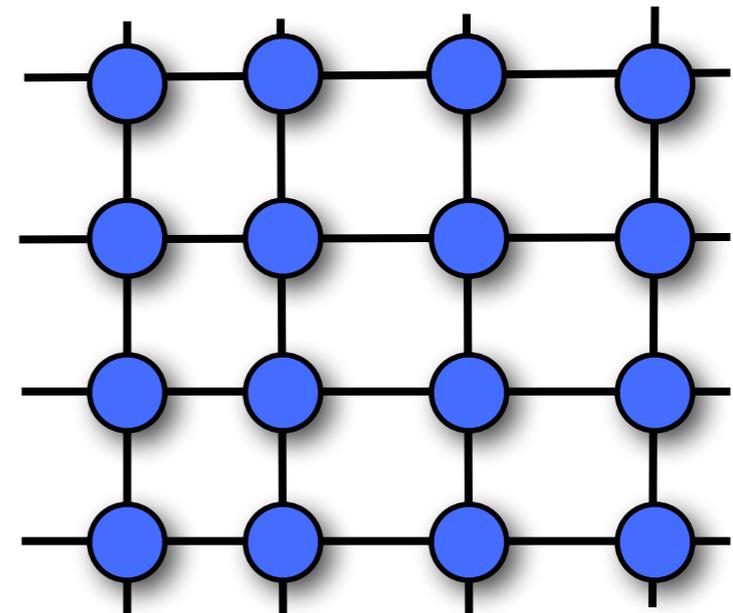
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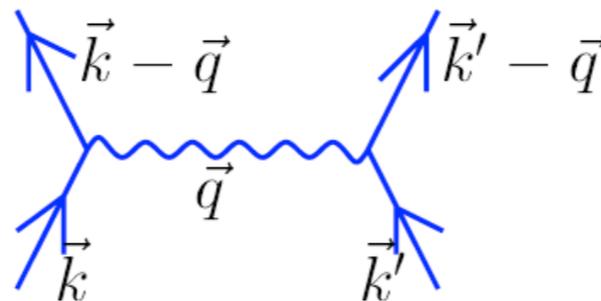
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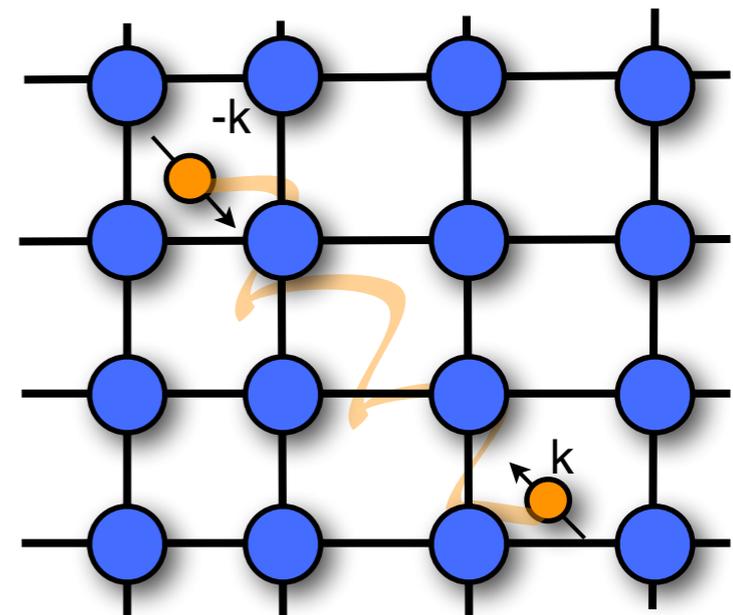
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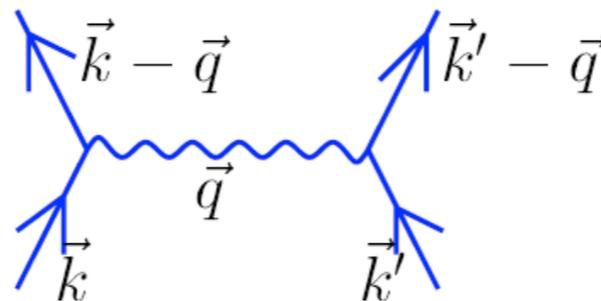
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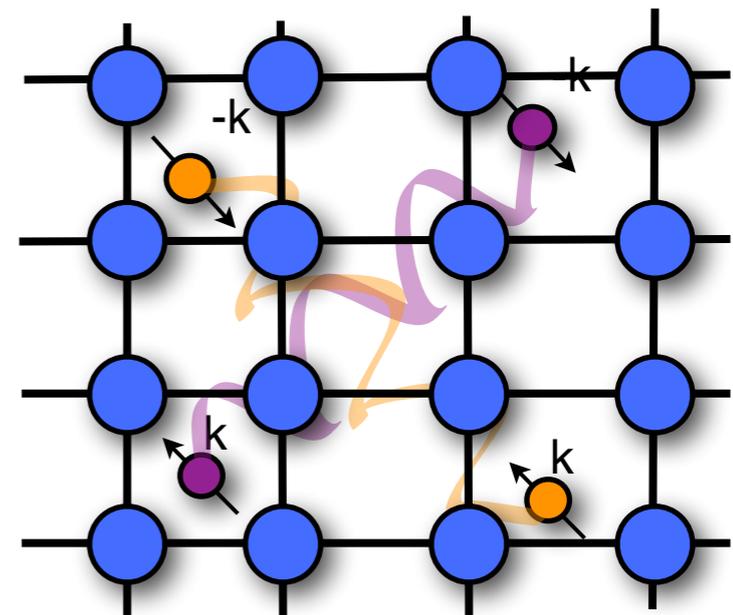
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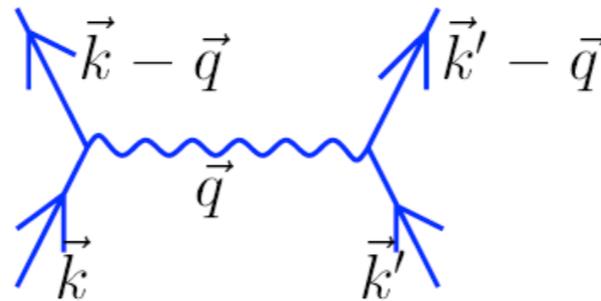
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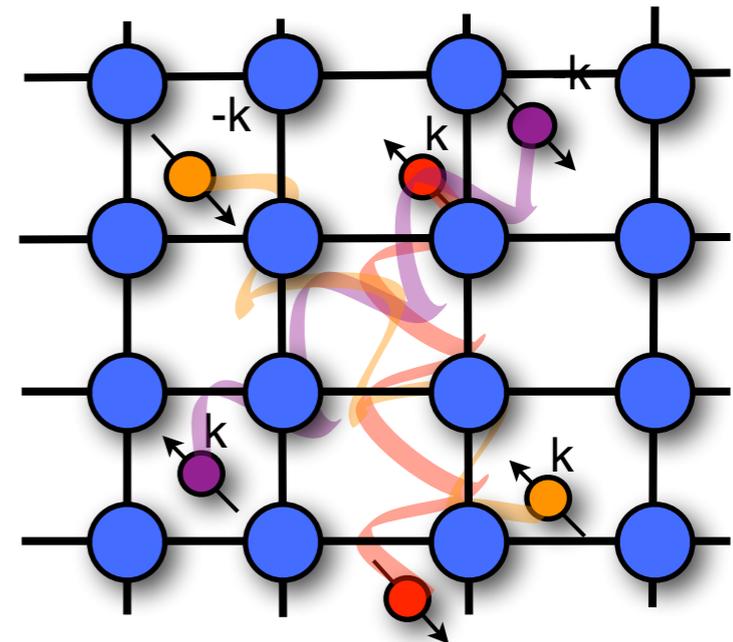
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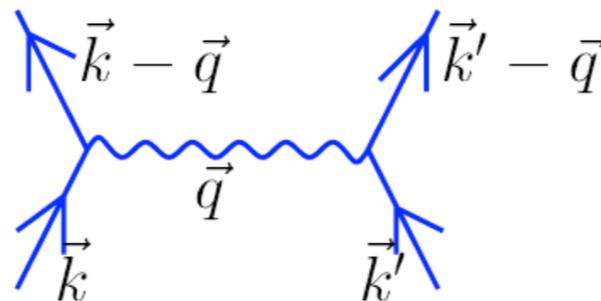
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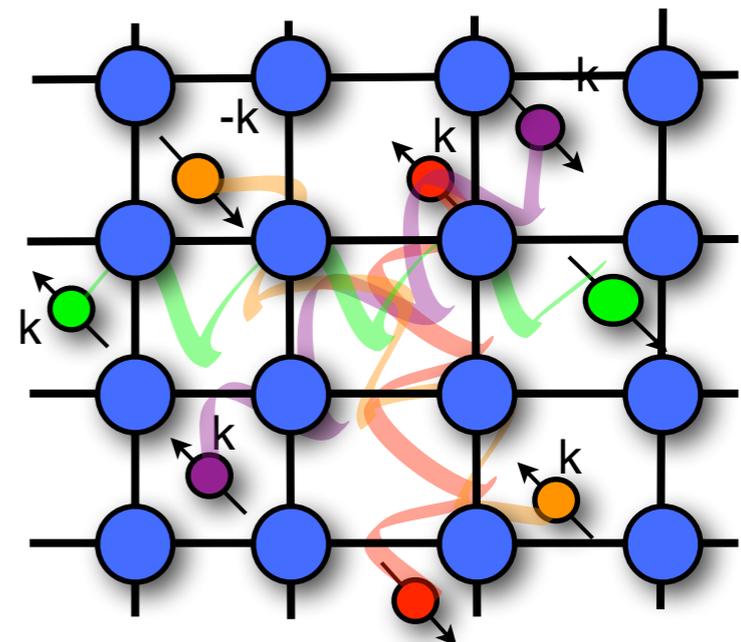
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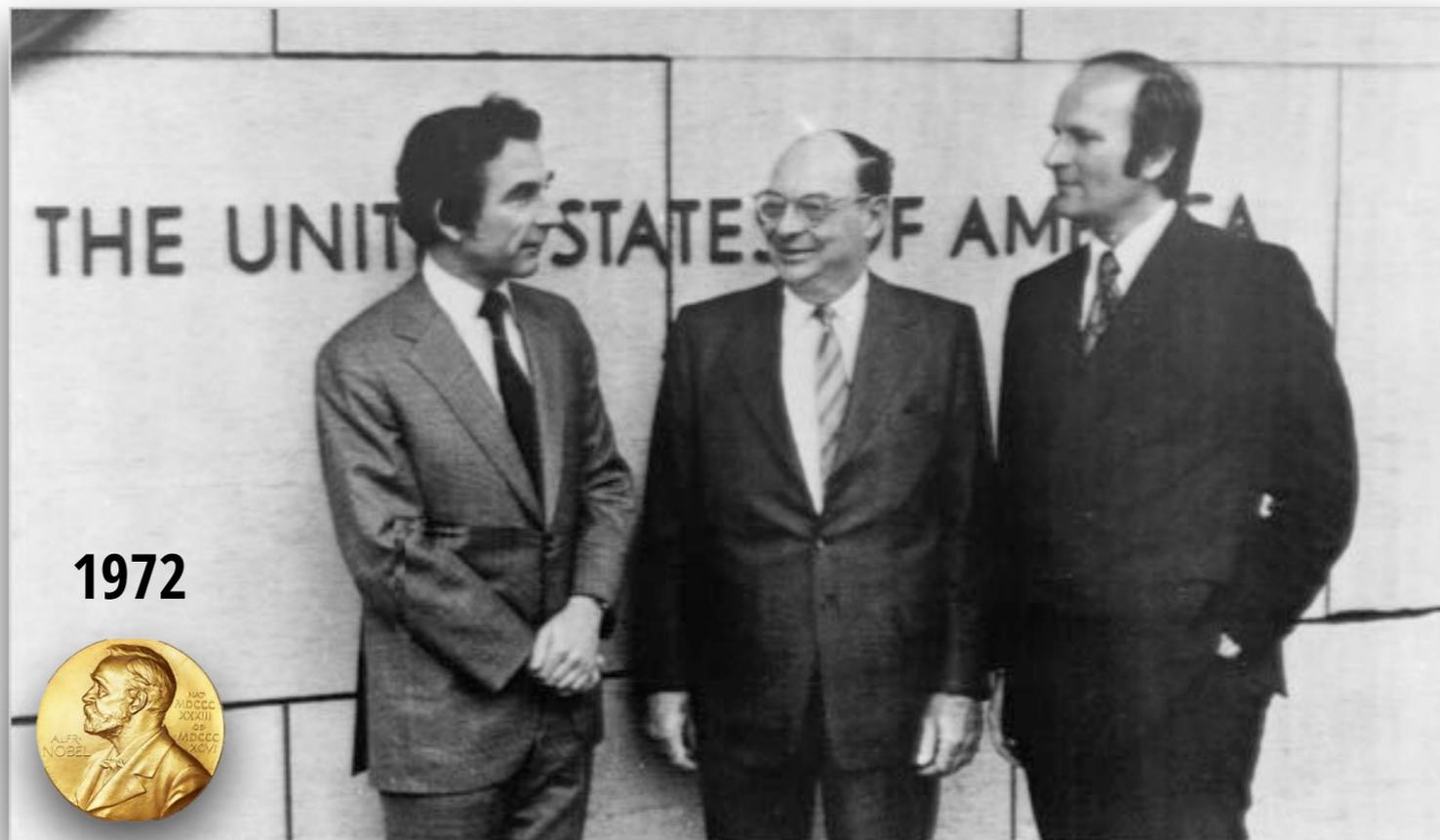
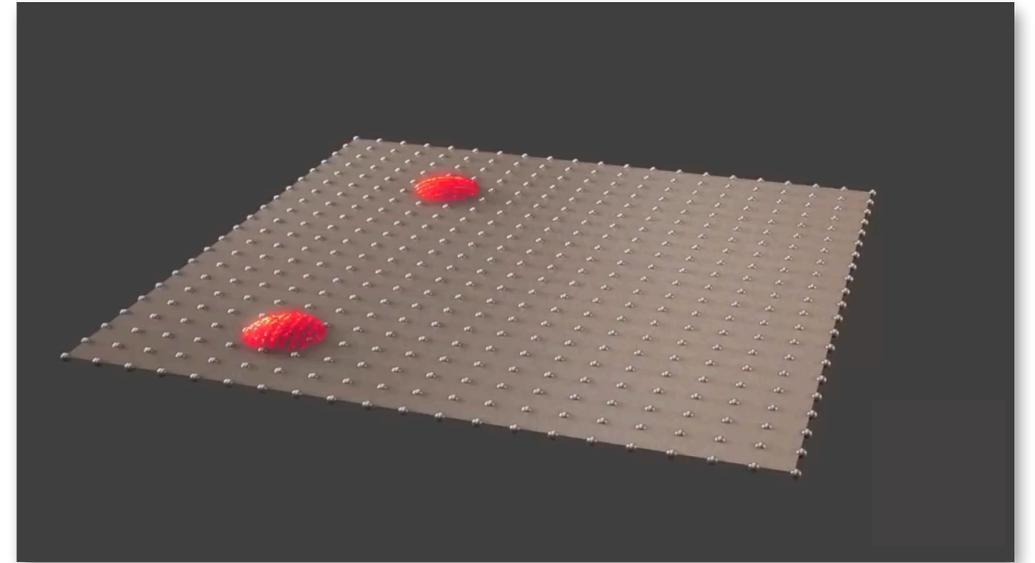


# Coherent (macroscopic) state

$$\Psi_N(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \mathcal{A} \psi(\mathbf{r}_1, \mathbf{r}_2) \psi(\mathbf{r}_3, \mathbf{r}_4) \cdots \psi(\mathbf{r}_{N-1}, \mathbf{r}_N) (1 \uparrow)(2 \downarrow)(3 \uparrow)(4 \downarrow) \cdots (N-1 \uparrow)(N \downarrow).$$

$$|\Psi_{BCS}\rangle = \text{const.} \prod_{\mathbf{k}} \exp(\alpha_{\mathbf{k}} \hat{P}_{\mathbf{k}}^+) |0\rangle$$

$$\Psi = \prod_{\mathbf{k}=\mathbf{k}_1, \dots, \mathbf{k}_{N/2}} \left( u_{\mathbf{k}} + v_{\mathbf{k}} a_{\mathbf{k}\uparrow}^\dagger a_{-\mathbf{k}\downarrow}^\dagger \right) |0\rangle$$

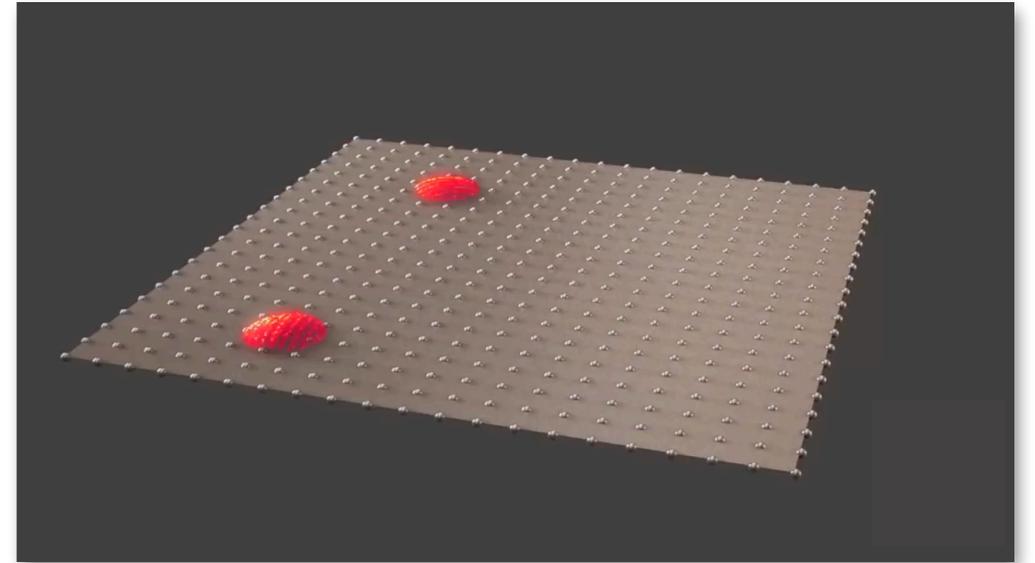


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# BCS predictions

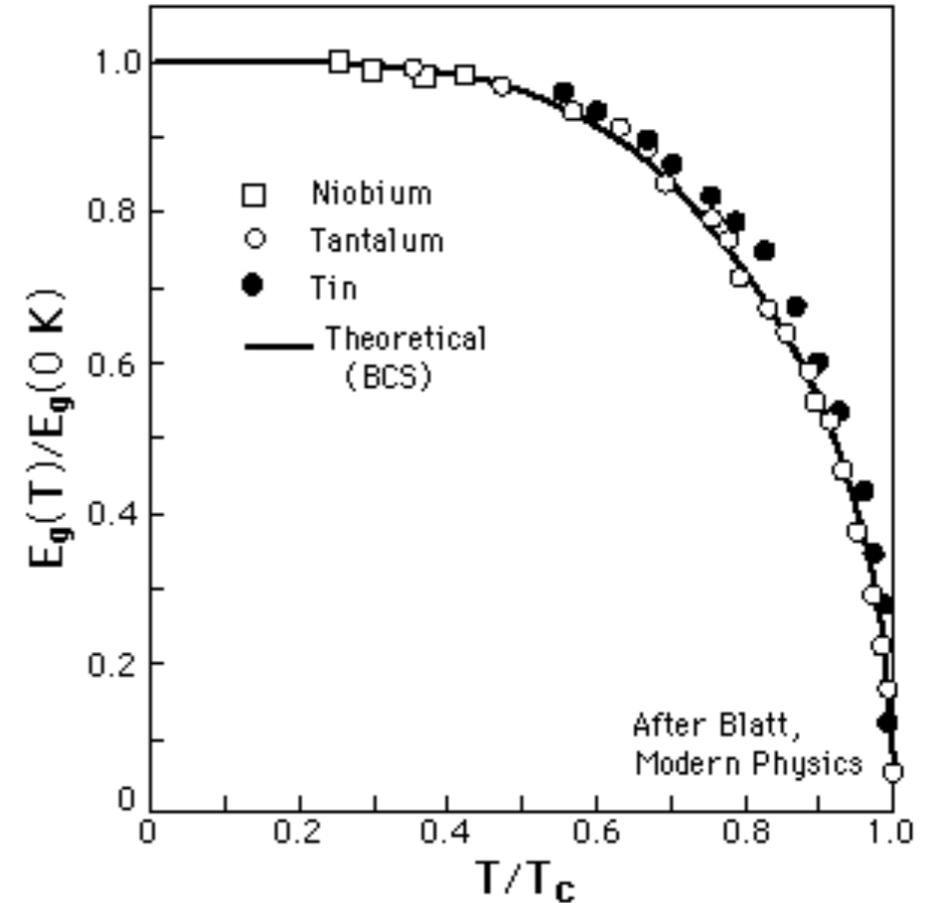
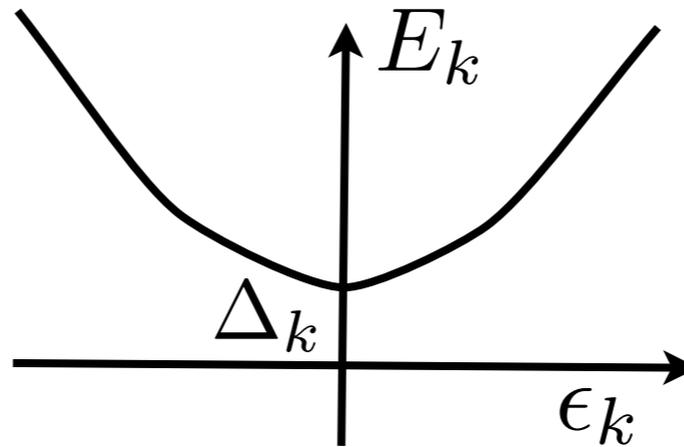
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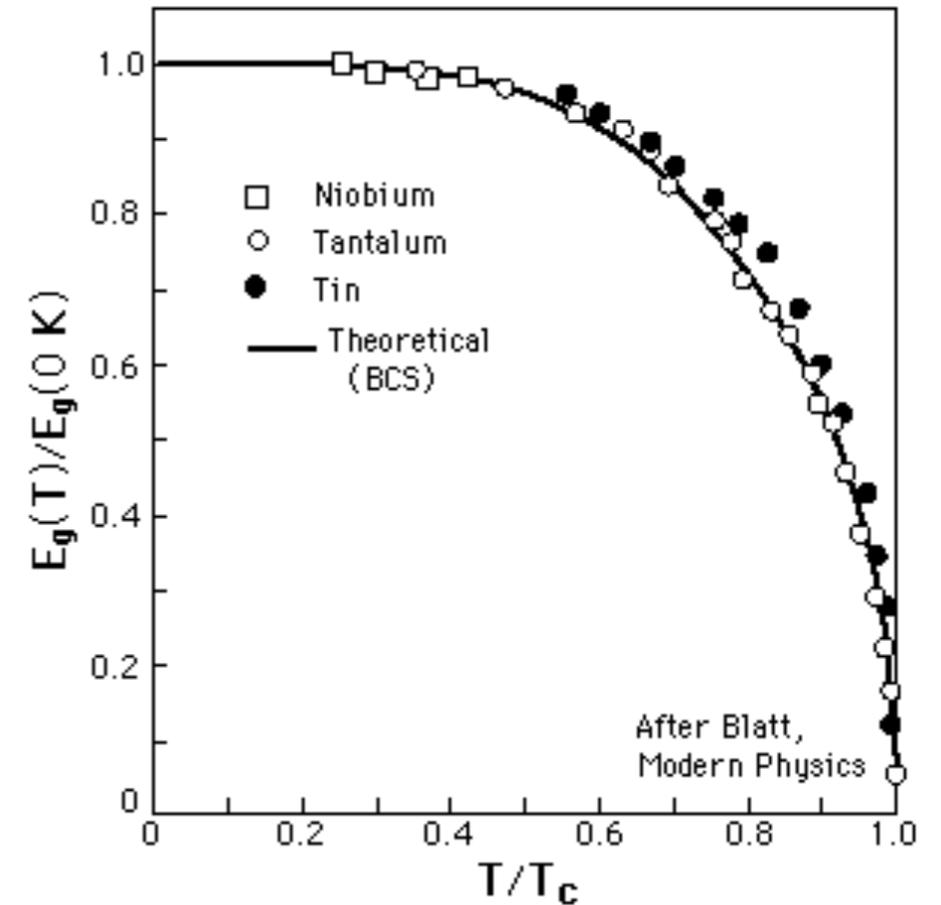
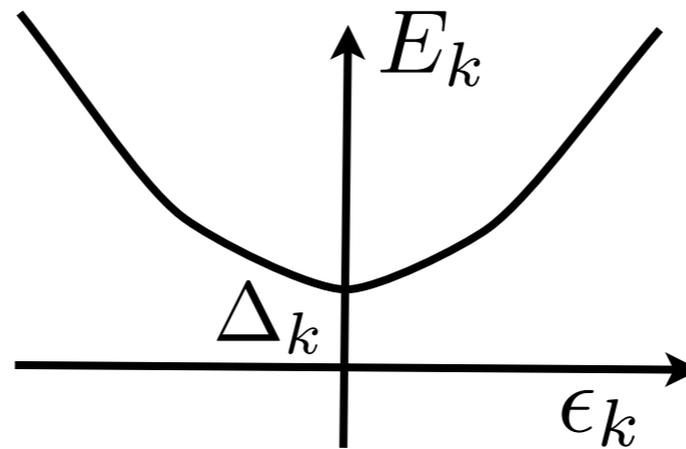
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## Pros

The first microscopic theory of SC

Explains many experimental evidences

The  $T_c$  formula is simple

## Cons

The  $T_c$  formula is wrong

No Coulomb interaction

No retardation effects

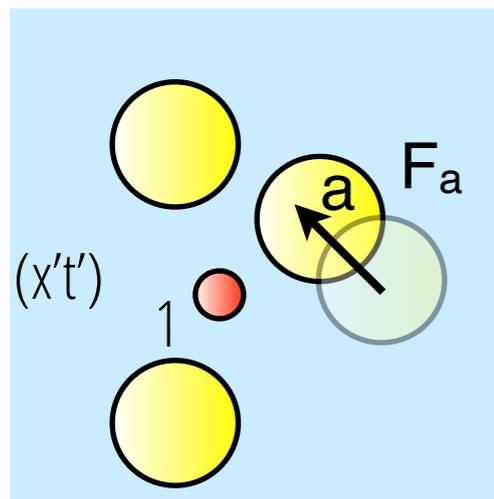
# The Migdal - Eliashberg theory ('58-'60)

The electron-electron interaction mediated by phonons is time-dependent



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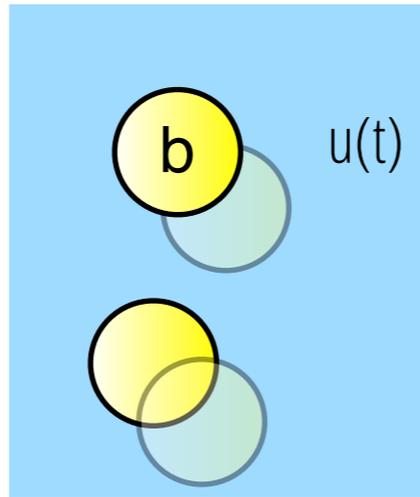
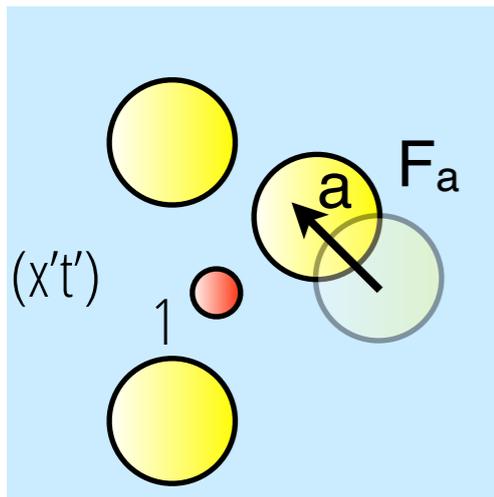


The electron 1 at  $(x't')$   
causes an impulsive  
force  $F$  on a ion  $a$   
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space-time

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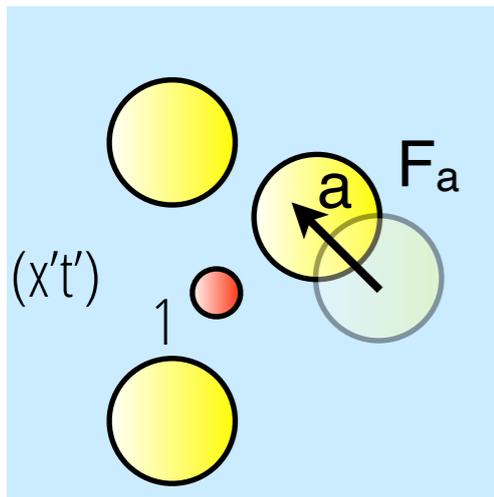
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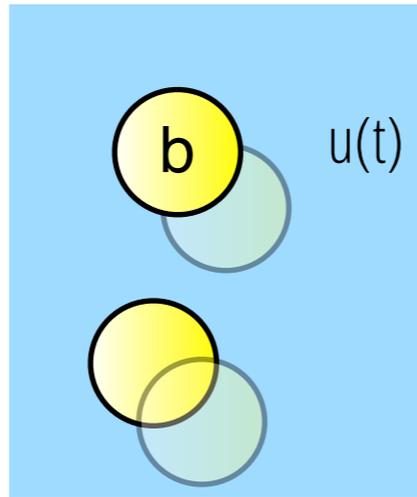
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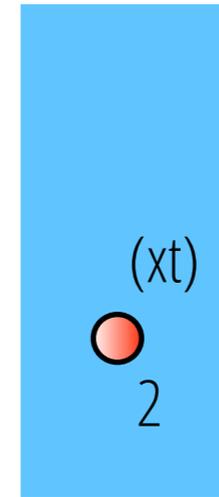
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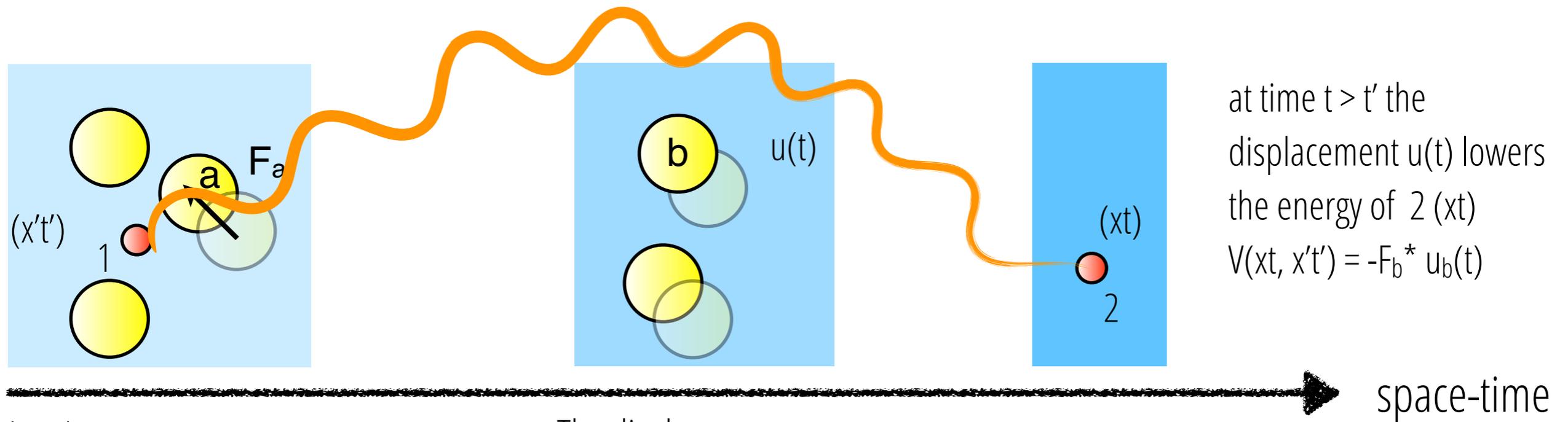


at time  $t > t'$  the  
displacement  $u(t)$  lowers  
the energy of 2  $(x, t)$   
 $V(x, t, x't') = -F_b^* u_b(t)$

space-time

# The Migdal - Eliashberg theory ('58-'60)

The electron-electron interaction mediated by phonons is time-dependent



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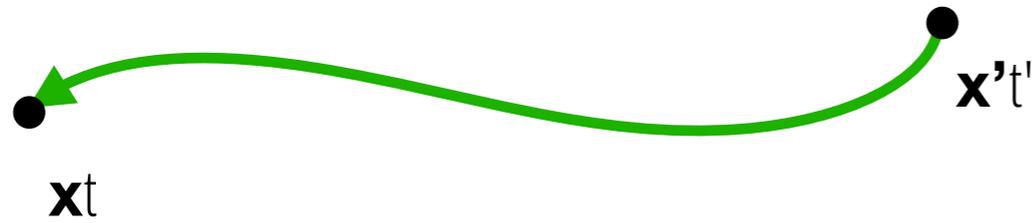
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$$G(\mathbf{x}t, \mathbf{x}'t') = -\frac{i}{\hbar} \langle N | \hat{T} \hat{\psi}(\mathbf{x}t) \hat{\psi}^\dagger(\mathbf{x}'t') | N \rangle$$

**Electrons**

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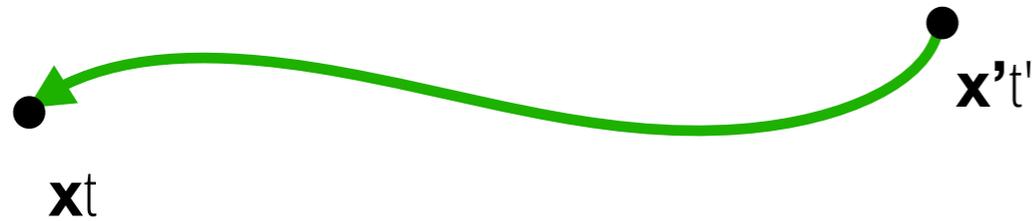
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Dyson equation

$$G^{-1}(\omega, k) = G_0^{-1}(\omega, k) - \Sigma(\omega, k)$$

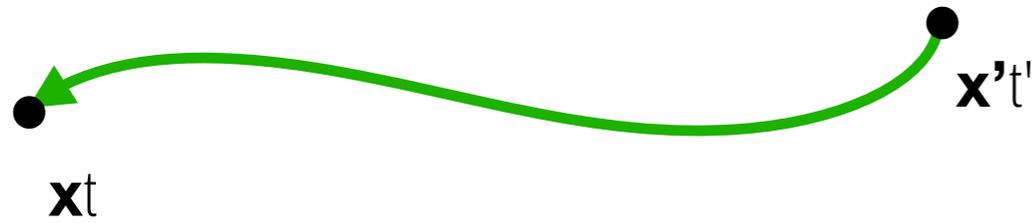
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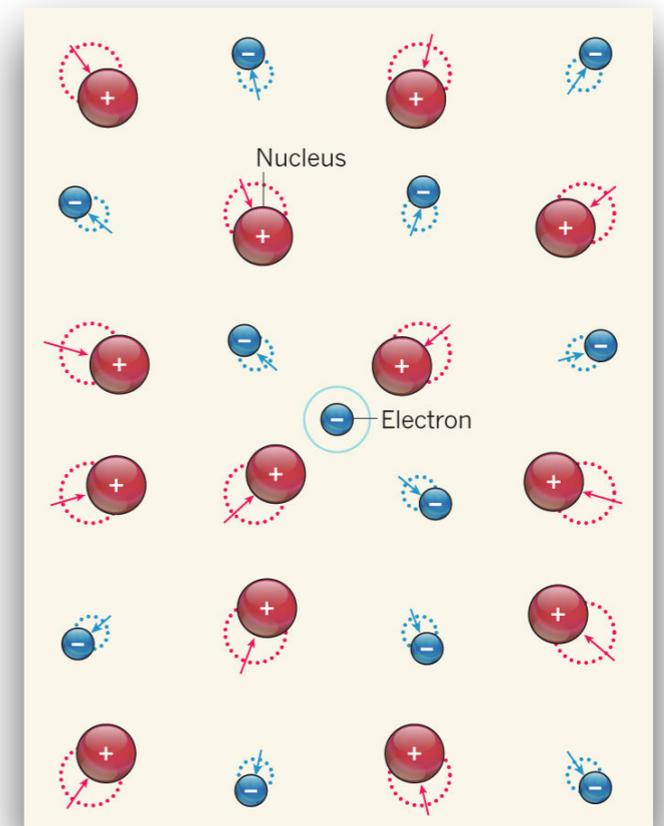
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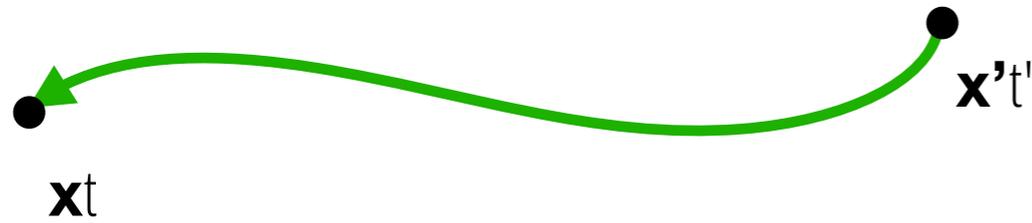
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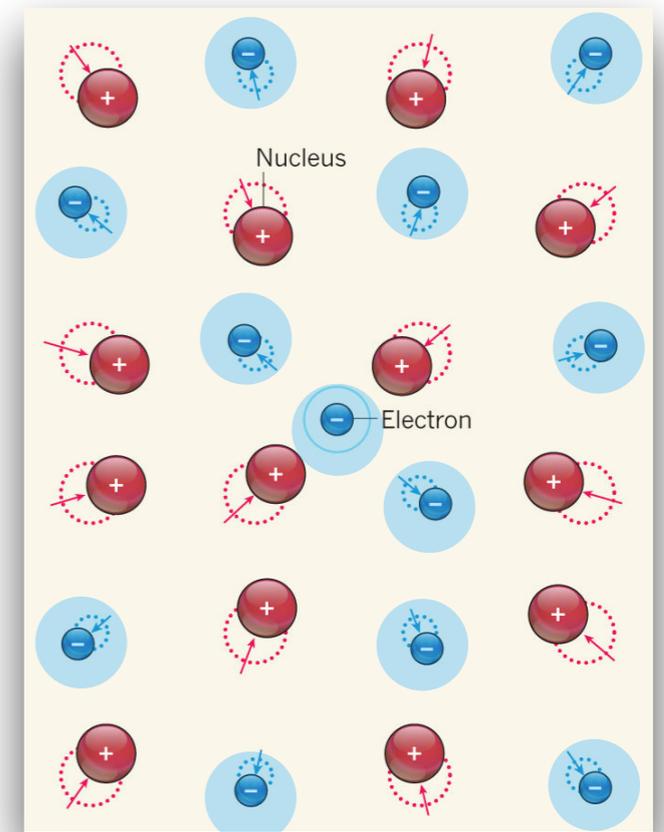
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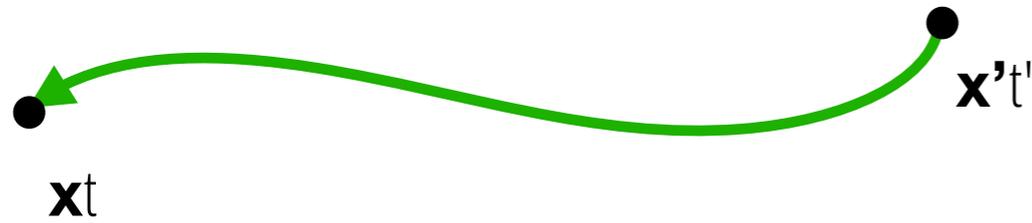
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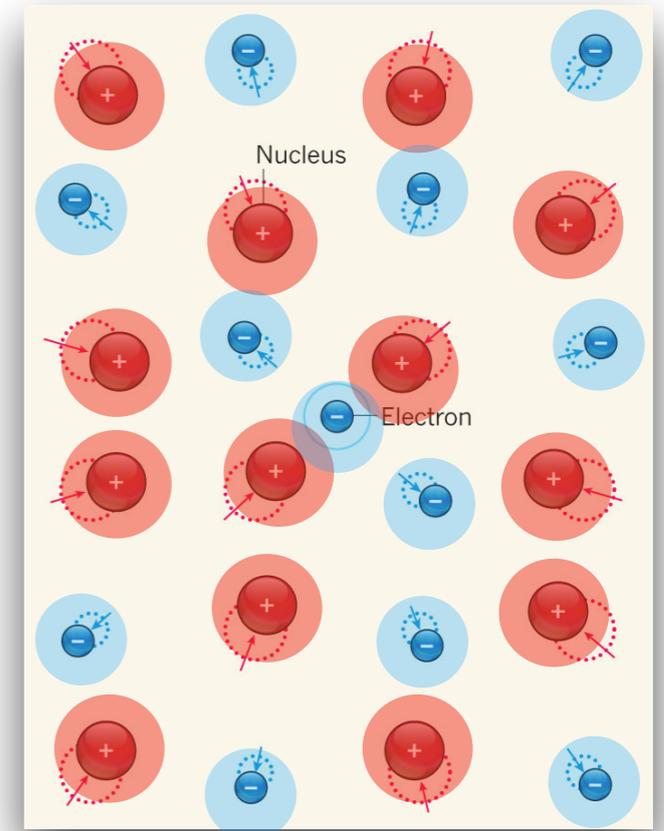
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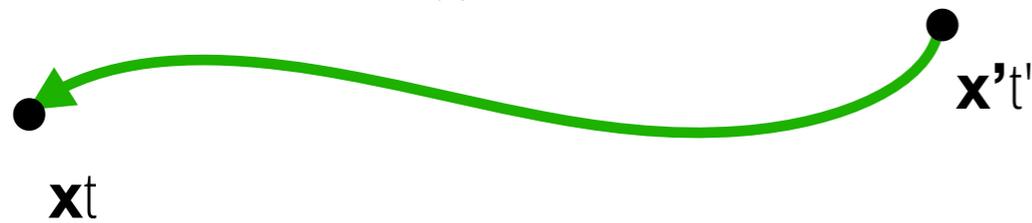
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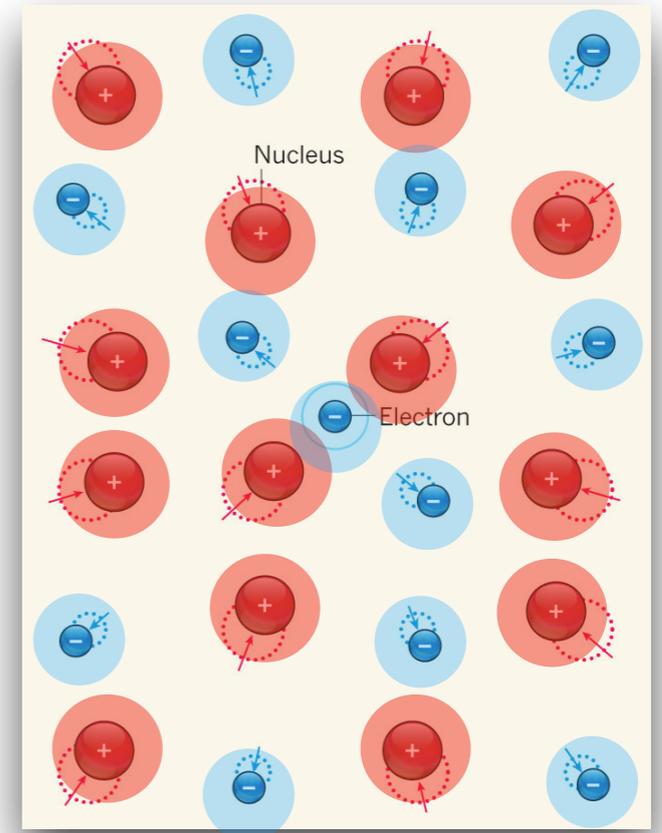


The pairing is included considering the field operator in the Nambu notation

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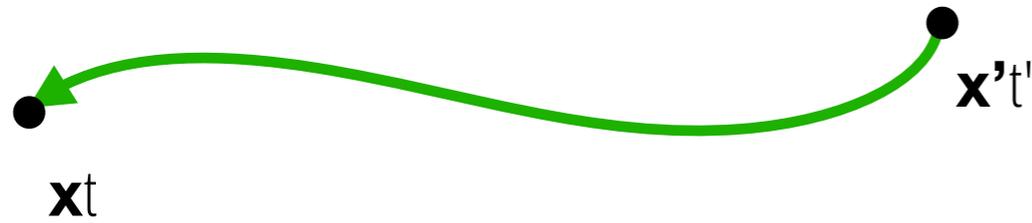
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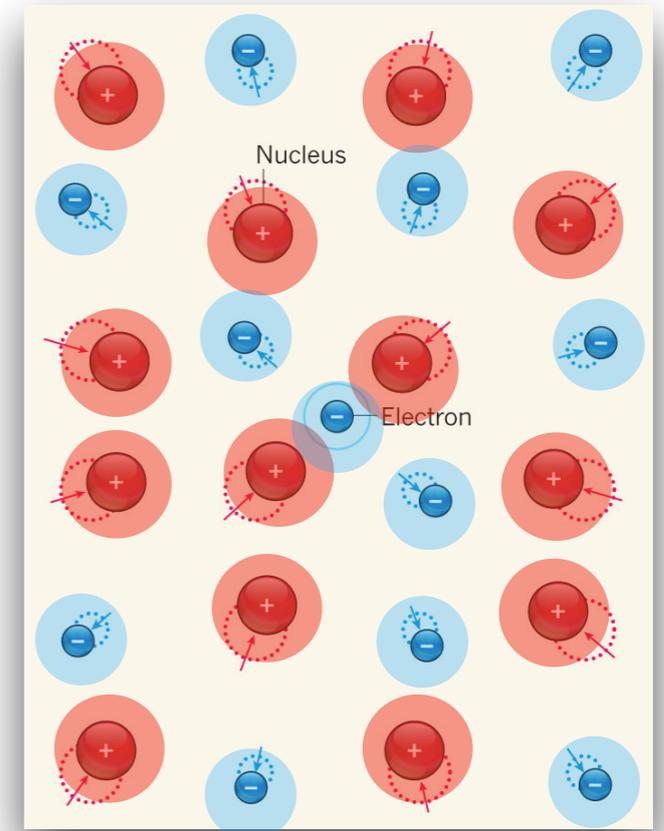
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↑↑
↓↑

↑↓
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**Which is the best superconductor?**

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**Neil's (Ashcroft) answer**

# Which is the best superconductor?

## Neil's (Ashcroft) answer

VOLUME 21, NUMBER 26

PHYSICAL REVIEW LETTERS

23 DECEMBER 1968

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### METALLIC HYDROGEN: A HIGH-TEMPERATURE SUPERCONDUCTOR?

N. W. Ashcroft

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

(Received 3 May 1968)

Application of the BCS theory to the proposed metallic modification of hydrogen suggests that it will be a high-temperature superconductor. This prediction has interesting astrophysical consequences, as well as implications for the possible development of a superconductor for use at elevated temperatures.

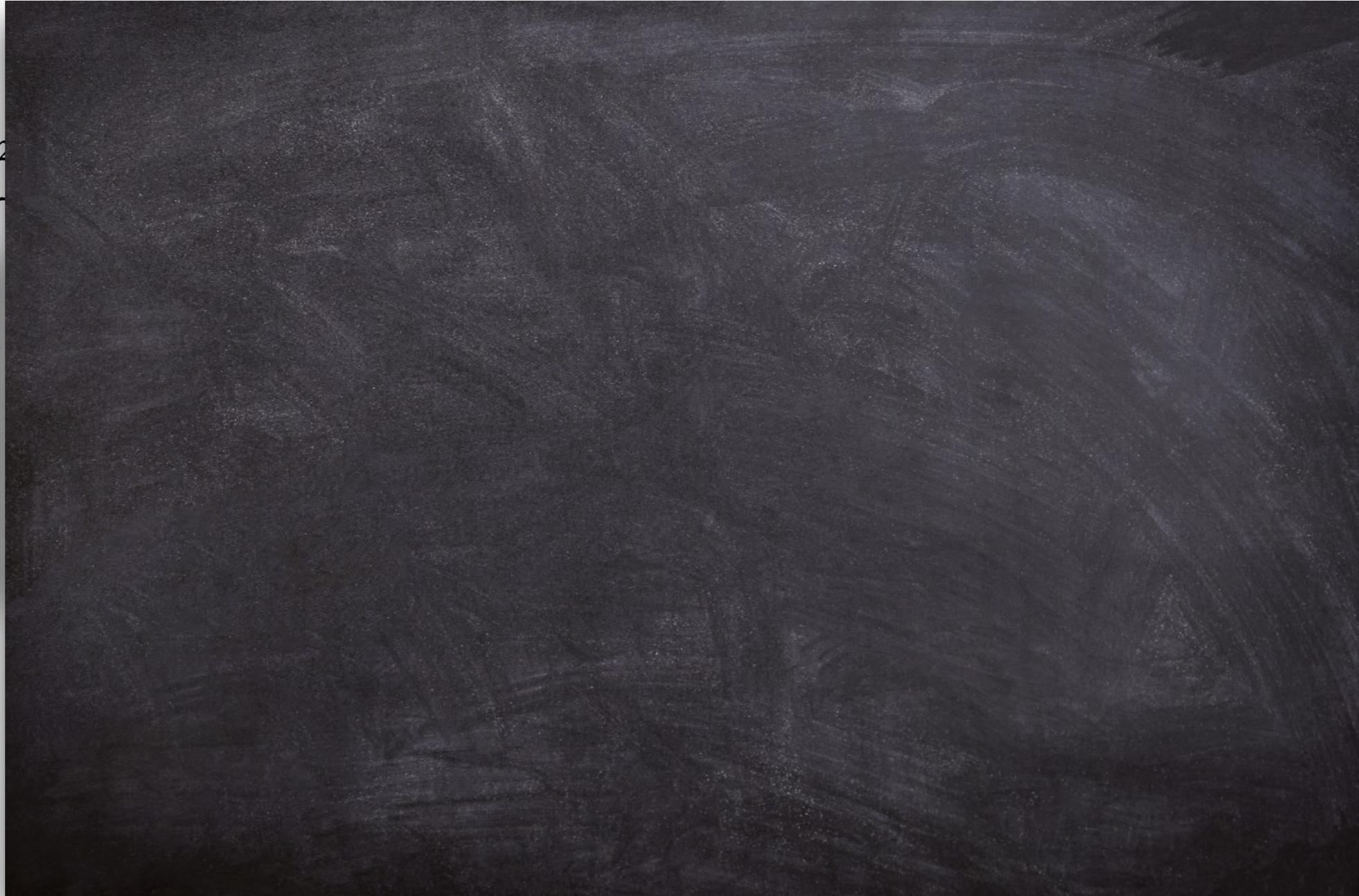


# Which is the best superconductor?

## Neil's (Ashcroft) answer

VOLUME 2

BER 1968



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VOLUME 2

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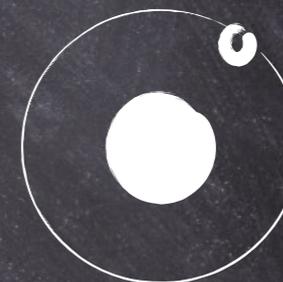
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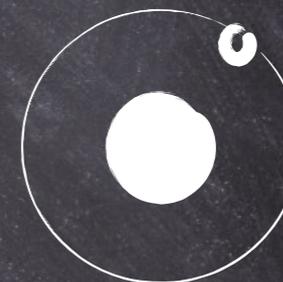
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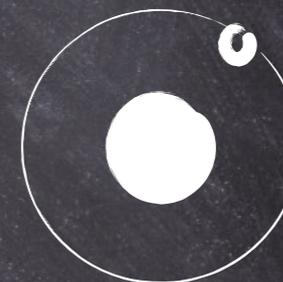
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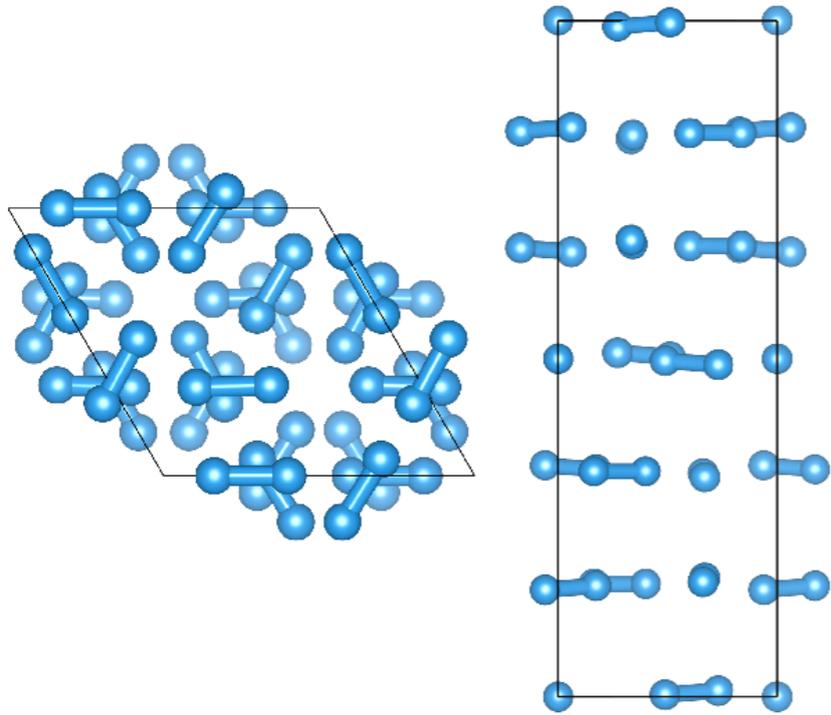
Hydrogen under high pressure is a  
(high-T) superconductor

VOLUME 2

BER 1968



**But hydrogen is (always) an insulator  
where is the metal?**



# ON SUPERCONDUCTIVITY AND SUPERFLUIDITY

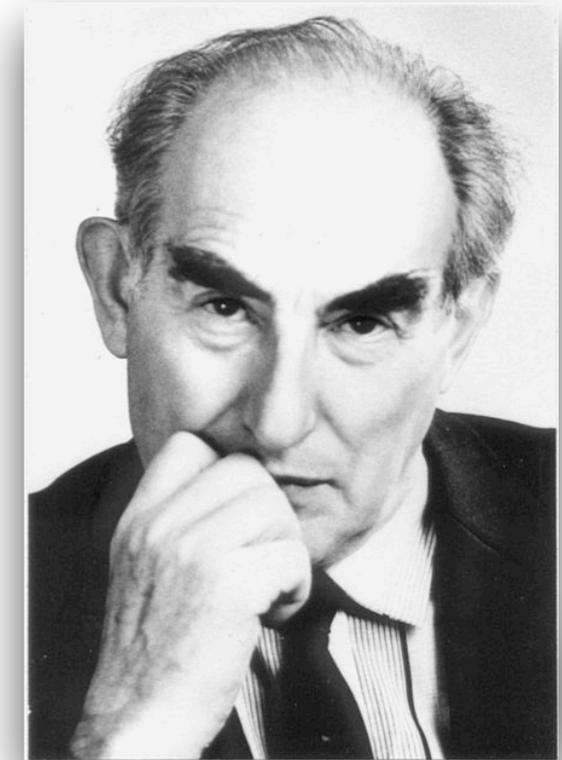
Nobel Lecture, December 8, 2003

by

VITALY L. GINZBURG

P. N. Lebedev Physics Institute, Russian Academy of Sciences, Moscow, Russia.

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2003



“pioneering contributions to the theory of superconductors and superfluids”

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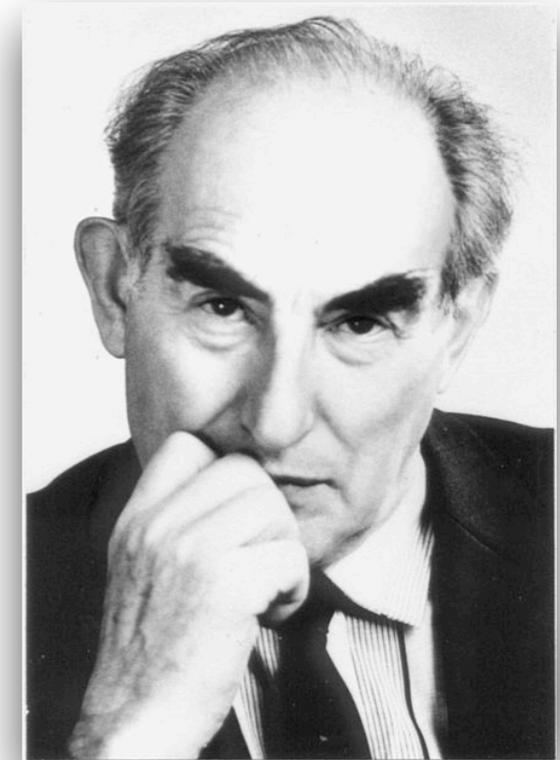
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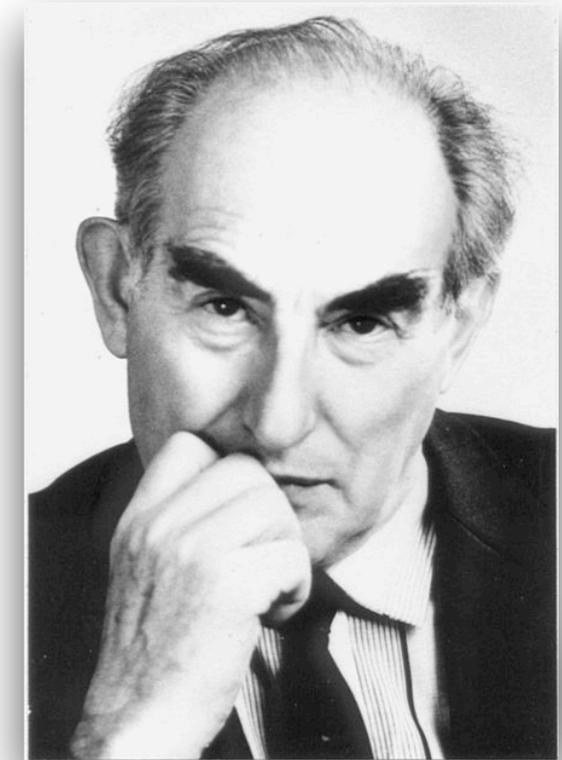
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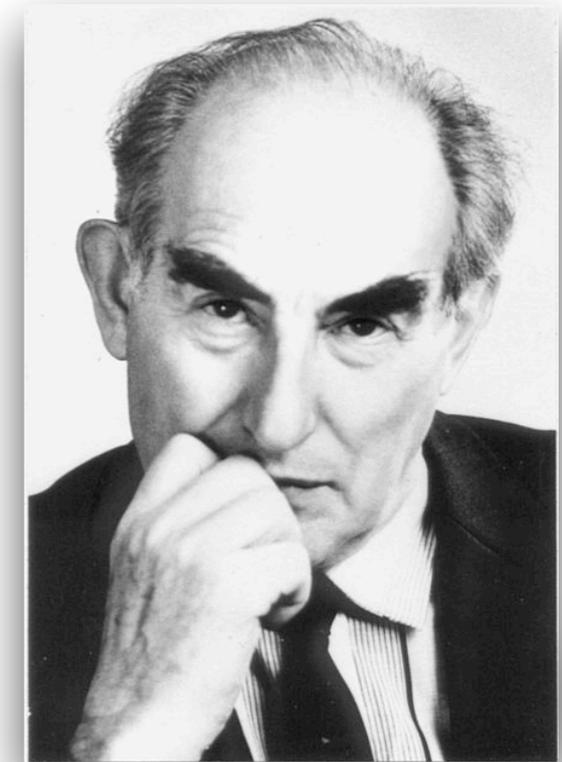
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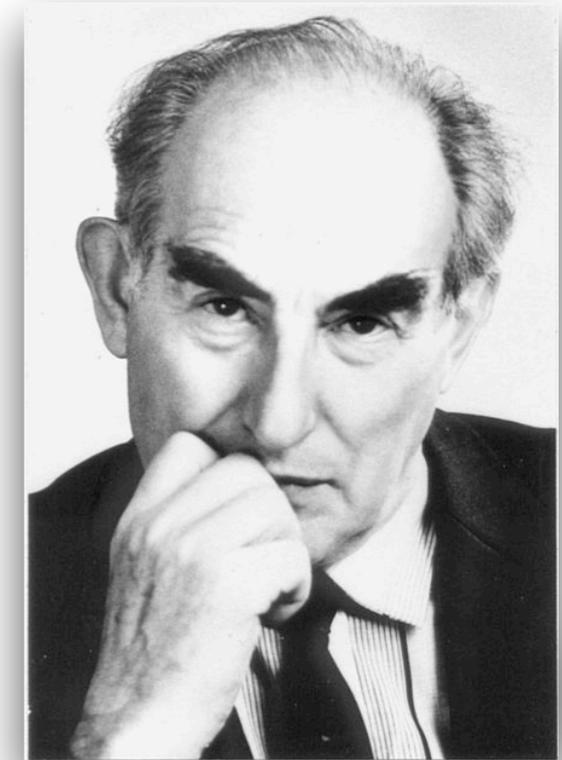
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# **Metallic hydrogen: The holy grail of physics**

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# Metallic hydrogen: The holy grail of physics



1899: Dewar produces solid hydrogen



1935: Wigner predicted metallic hydrogen at 25 GPa

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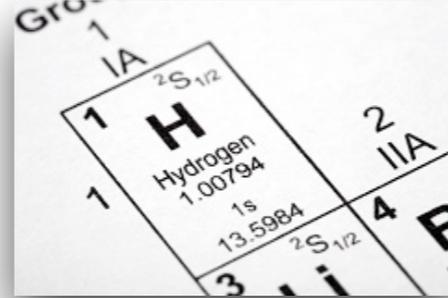


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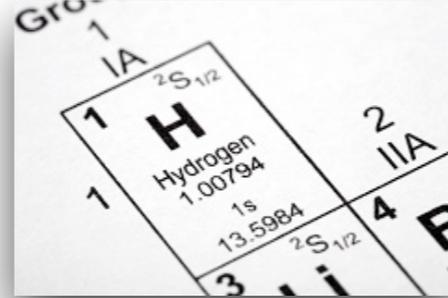


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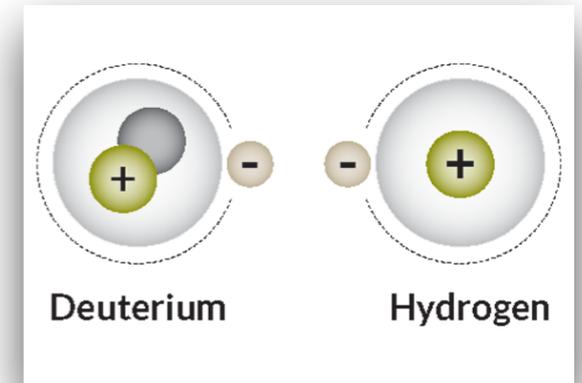


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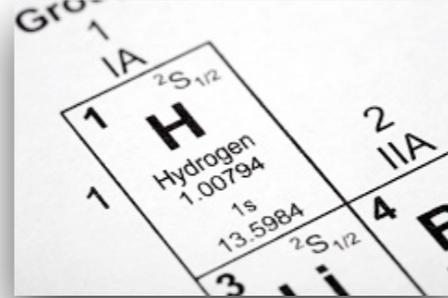


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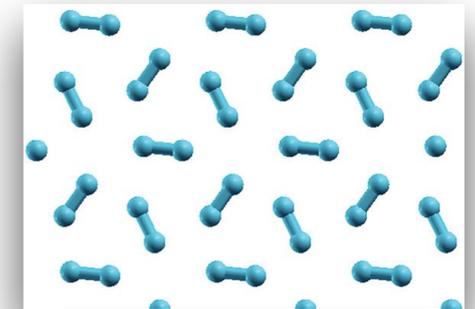
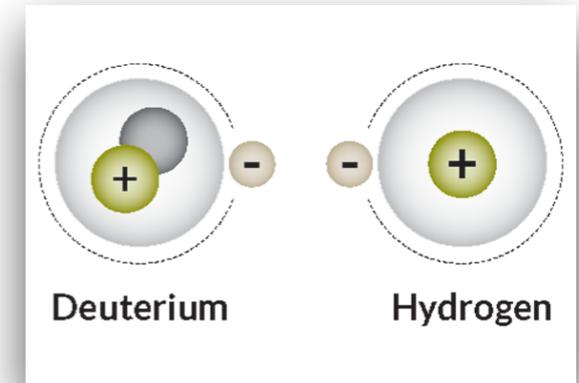


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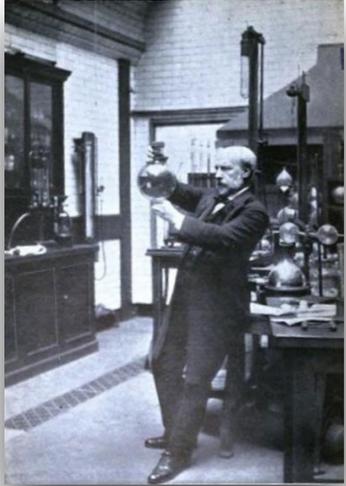


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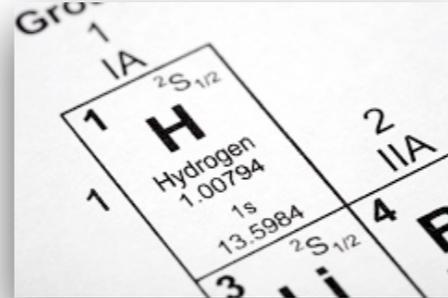


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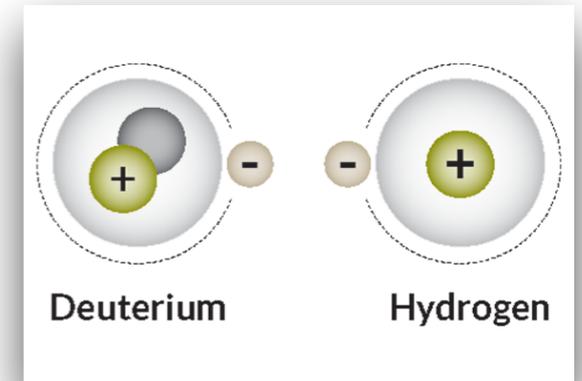


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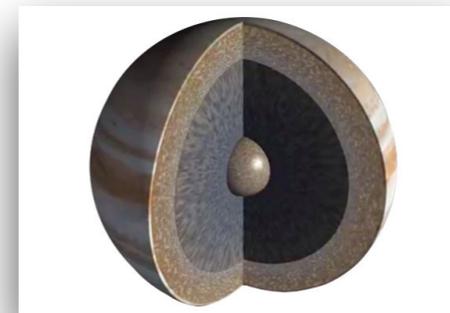
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# Metallic hydrogen: The holy grail of physics

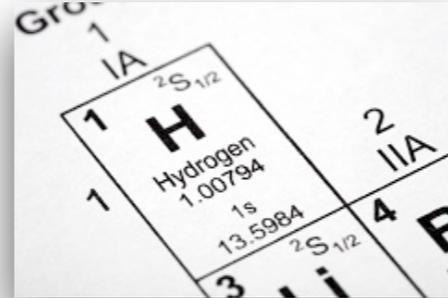


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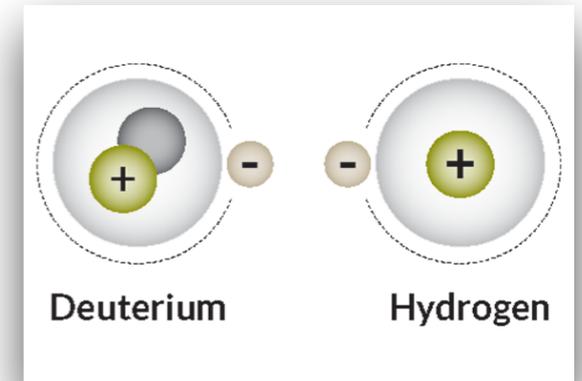


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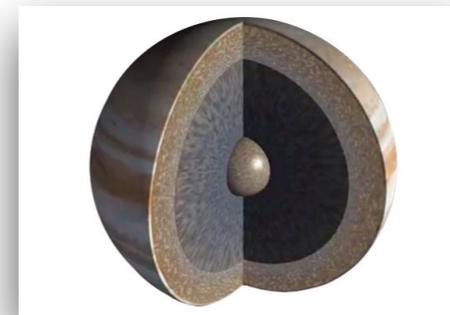


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The pressure is too high, experiments too complicated,  
.....theoretical predictions are welcome for both  
normal and superconducting properties.



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# The Density Functional Theory

PHYSICAL REVIEW

VOLUME 136, NUMBER 3B

9 NOVEMBER 1964

## Inhomogeneous Electron Gas\*

P. HOHENBERG†

*École Normale Supérieure, Paris, France*

AND

W. KOHN‡

*École Normale Supérieure, Paris, France and Faculté des Sciences, Orsay, France*

and

*University of California at San Diego, La Jolla, California*

(Received 18 June 1964)

This paper deals with the ground state of an interacting electron gas in an external potential  $v(\mathbf{r})$ . It is proved that there exists a universal functional of the density,  $F[n(\mathbf{r})]$ , independent of  $v(\mathbf{r})$ , such that the expression  $E \equiv \int v(\mathbf{r})n(\mathbf{r})d\mathbf{r} + F[n(\mathbf{r})]$  has as its minimum value the correct ground-state energy associated with  $v(\mathbf{r})$ . The functional  $F[n(\mathbf{r})]$  is then discussed for two situations: (1)  $n(\mathbf{r}) = n_0 + \tilde{n}(\mathbf{r})$ ,  $\tilde{n}/n_0 \ll 1$ , and (2)  $n(\mathbf{r}) = \varphi(\mathbf{r}/r_0)$  with  $\varphi$  arbitrary and  $r_0 \rightarrow \infty$ . In both cases  $F$  can be expressed entirely in terms of the correlation energy and linear and higher order electronic polarizabilities of a uniform electron gas. This approach also sheds some light on generalized Thomas-Fermi methods and their limitations. Some new extensions of these methods are presented.

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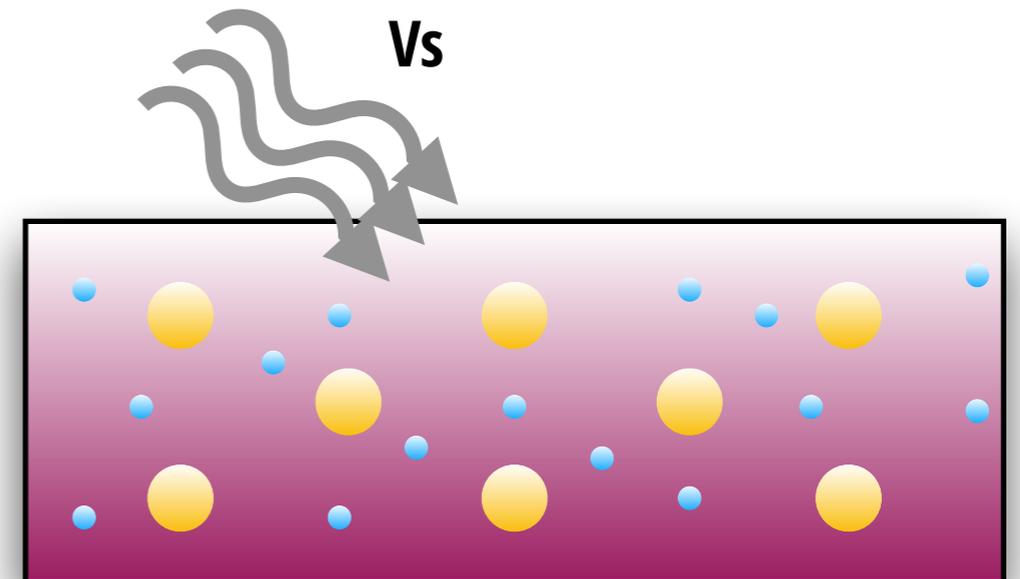
$$n \leftrightarrow V_{ext}$$

$$E[n_0] \leq E[n] = T[n] + U[n] + V[n]$$

$$\left[ \frac{\hbar^2 \nabla^2}{2m} + v_s(r) \right] \phi_i(r) = \epsilon_i \phi_i(r)$$

$$v_s(r) = v(r) + v_H(r) + v_{xc}(r)$$

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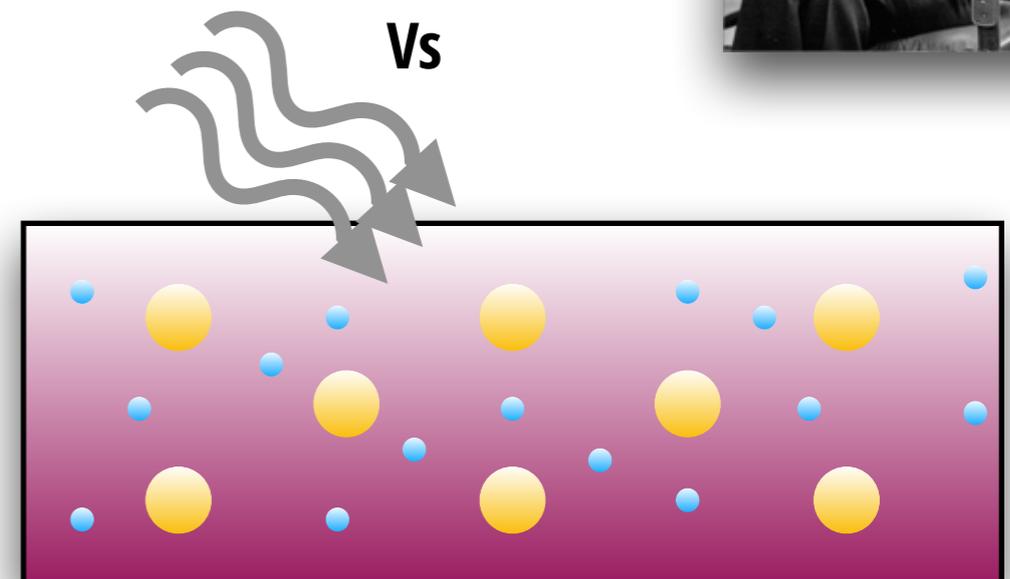
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1998



# Super Conducting Density Functional Theory (2005)

Kohn-Sham system for superconductor

$$H = H_e + H_{en} + H_n + H_{ext}$$

$$\rho(\mathbf{r}) = \text{Tr} \left[ \varrho_0 \sum_{\sigma} \psi_{\sigma}^{\dagger}(\mathbf{r}) \psi_{\sigma}(\mathbf{r}) \right]$$

$$\chi(\mathbf{r}, \mathbf{r}') = \text{Tr} [\varrho_0 \psi_{\uparrow}(\mathbf{r}) \psi_{\downarrow}(\mathbf{r}')] ]$$

$$\Gamma(\{\mathbf{R}_i\}) = \text{Tr} \left[ \varrho_0 \prod_j \Phi^{\dagger}(\mathbf{R}_j) \Phi(\mathbf{R}_j) \right]$$



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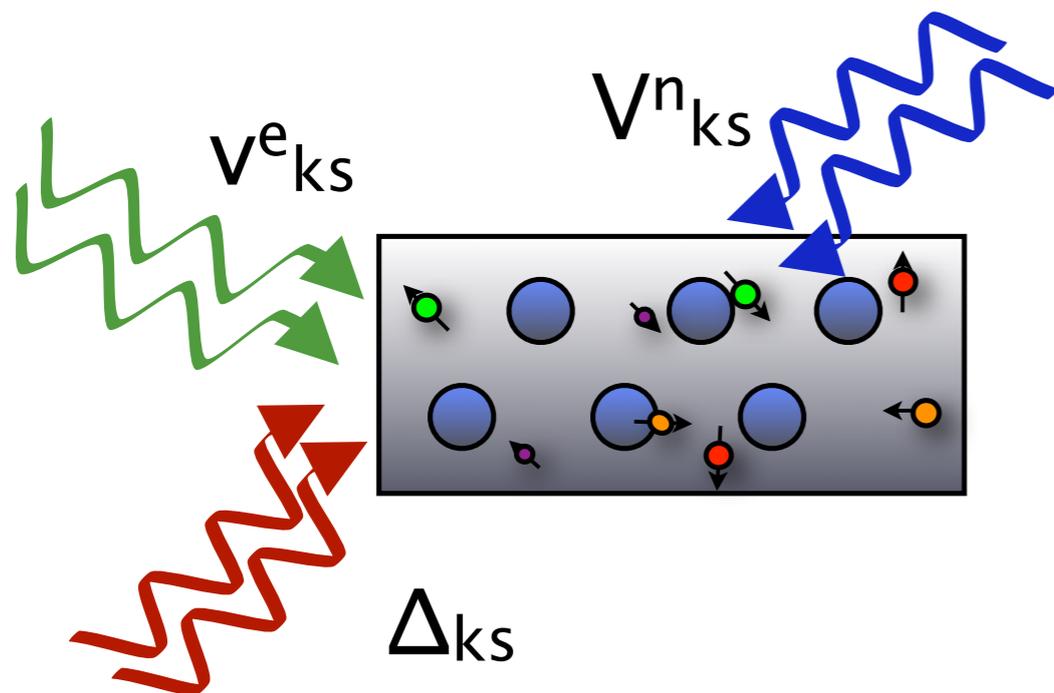


PHYSICAL REVIEW B 72, 024545 (2005)

## *Ab initio* theory of superconductivity. I. Density functional formalism and approximate functionals

M. Lüders,<sup>1,2</sup> M. A. L. Marques,<sup>2,3</sup> N. N. Lathiotakis,<sup>2,3</sup> A. Floris,<sup>3,4</sup> G. Profeta,<sup>5</sup> L. Fast,<sup>2,6</sup> A. Continenza,<sup>5</sup> S. Massidda,<sup>4,\*</sup> and E. K. U. Gross<sup>2,3</sup>

An approach to the description of superconductors in thermal equilibrium is developed within a formally exact density functional framework. The theory is formulated in terms of three “densities:” the ordinary electron density, the superconducting order parameter, and the diagonal of the nuclear  $N$ -body density matrix. The electron density and the order parameter are determined by Kohn-Sham equations that resemble the Bogoliubov–de Gennes equations. The nuclear density matrix follows from a Schrödinger equation with an effective  $N$ -body interaction. These equations are coupled to each other via exchange-correlation potentials which are universal functionals of the three densities. Approximations of these exchange-correlation functionals are derived using the diagrammatic techniques of many-body perturbation theory. The bare Coulomb repulsion between the electrons and the electron-phonon interaction enter this perturbative treatment on the same footing. In this way, a truly *ab initio* description is achieved which does not contain any empirical parameters.



# Super Conducting Density Functional Theory (2005)

$$\Sigma \approx \text{[Diagram: wavy line labeled } W \text{ on a horizontal line]} + \text{[Diagram: semi-circular dashed line labeled } \Lambda^{\text{Ph}} \text{ above a horizontal line labeled } \bar{G}^{\text{KS}} \text{]}.$$

Interactions from first-principles

$$\bar{\Sigma}_k(\omega_n) \approx \sum_m \sum_{k'} W_{kk'}(\omega_n - \omega_m) \bar{G}_{k'}^{\text{KS}}(\omega_m) + \sum_m \sum_{k'} \Lambda_{kk'}^{\text{Ph}}(\omega_n - \omega_m) \bar{G}_{k'}^{\text{KS}}(\omega_m),$$

$$\Lambda_{kk'}^{\text{Ph}}(\omega_n) = -\frac{1}{\pi} \int_0^\infty d\omega \frac{2\omega}{\omega_n^2 + \omega^2} \Im[\Lambda_{kk'}^{\text{Ph}}(\omega)]$$

$$\Im[\Lambda_{kk'}^{\text{Ph}}(\omega)] = -\pi \sum_{\lambda \mathbf{q}} |g_{\lambda \mathbf{q}}^{kk'}|^2 \delta(\omega - \Omega_{\lambda \mathbf{q}}).$$

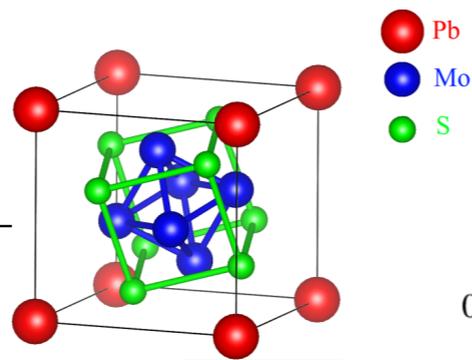
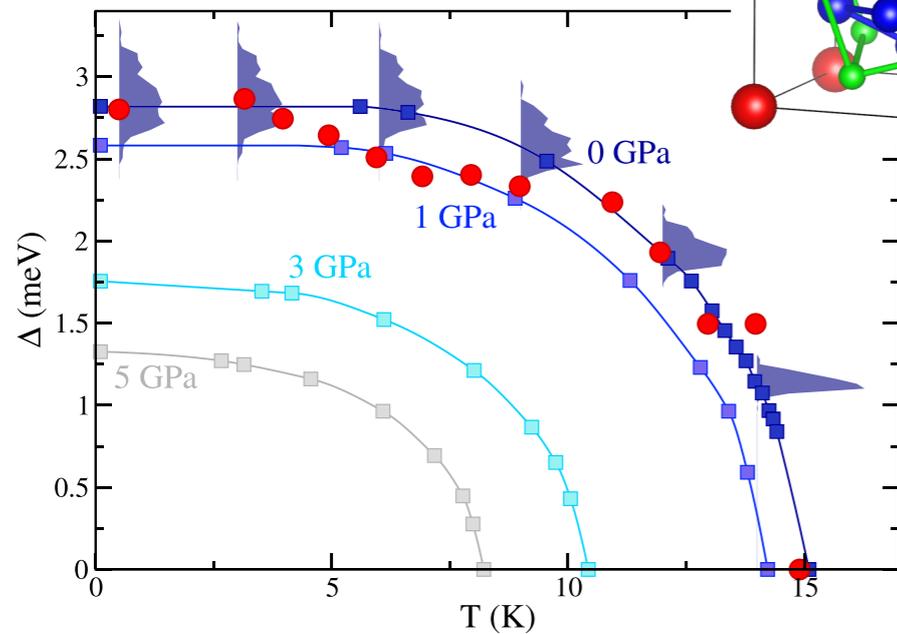
$$W_{k_1 k_2}(\omega_n) = \sum_{k'} \epsilon_{k_1 k'}^{-1}(\omega_n) v_{k' k_2}$$

$$\bar{G}_k(\omega_n) = \tau^z \begin{pmatrix} G_k(\omega_n) & F_k(\omega_n) \\ F_k^\dagger(\omega_n) & G_k^\dagger(\omega_n) \end{pmatrix}$$

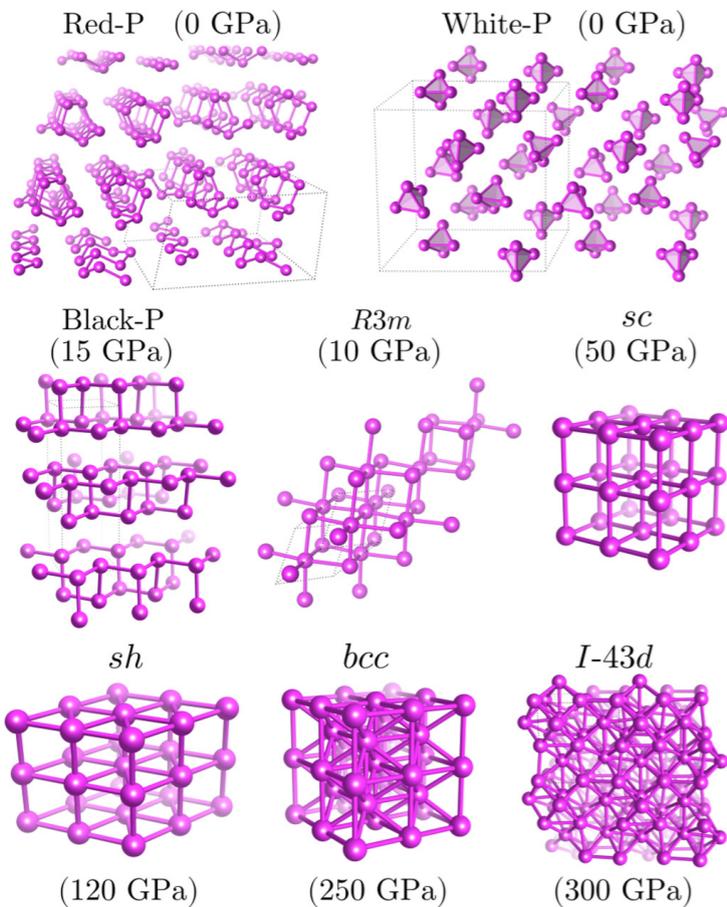
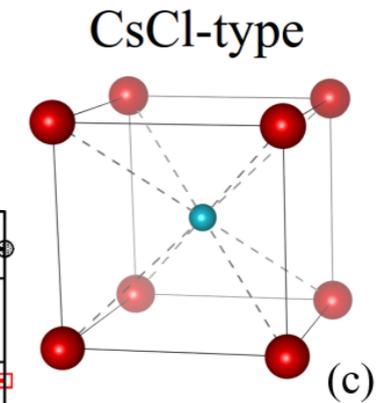
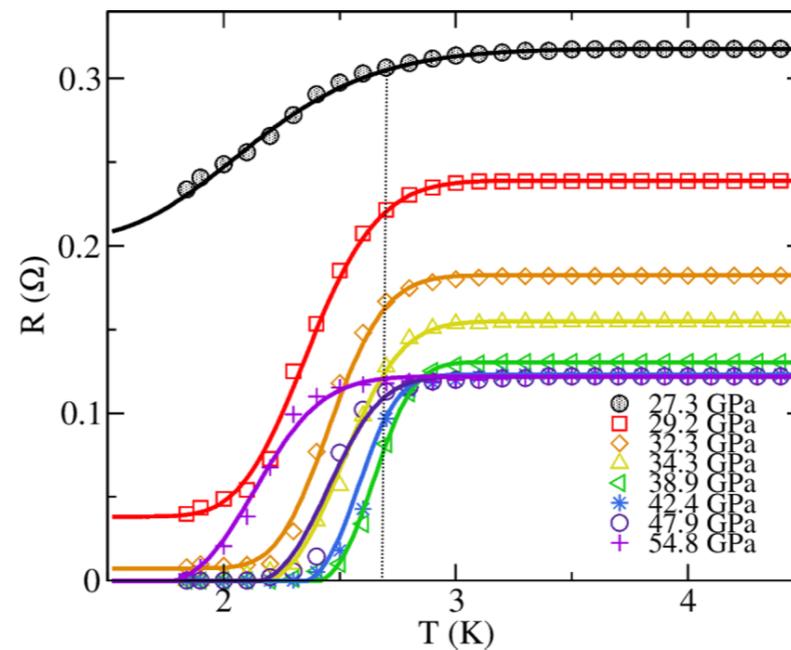
$$\Delta_k^{\text{xc}} = -\Delta_k^{\text{xc}} \mathcal{Z}_k^{\text{D}} - \sum_{k'} \mathcal{K}_{kk'}^{\text{C}} \frac{\tanh\left(\frac{\beta E_{k'}}{2}\right)}{2E_{k'}} \Delta_{k'}^{\text{xc}}$$

# It works

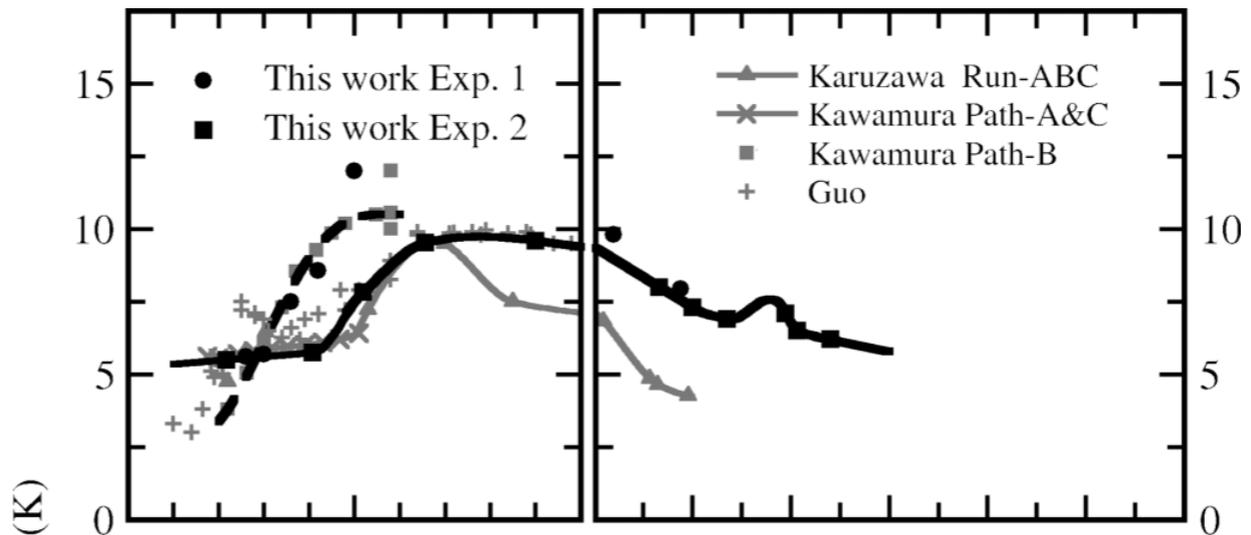
Chevrel phases,  $\text{PbMo}_6\text{S}_8$



Tin-selenide,  $\text{SnSe}$

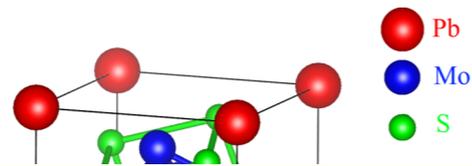


The many phases of phosphorus under pressure

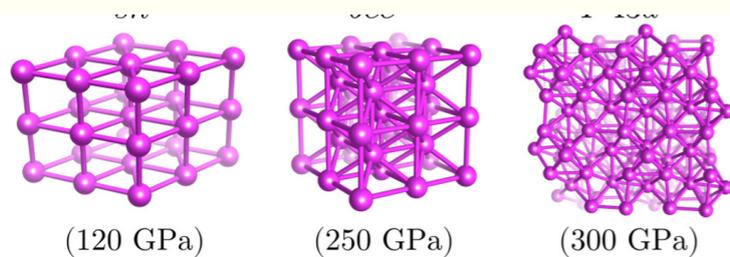
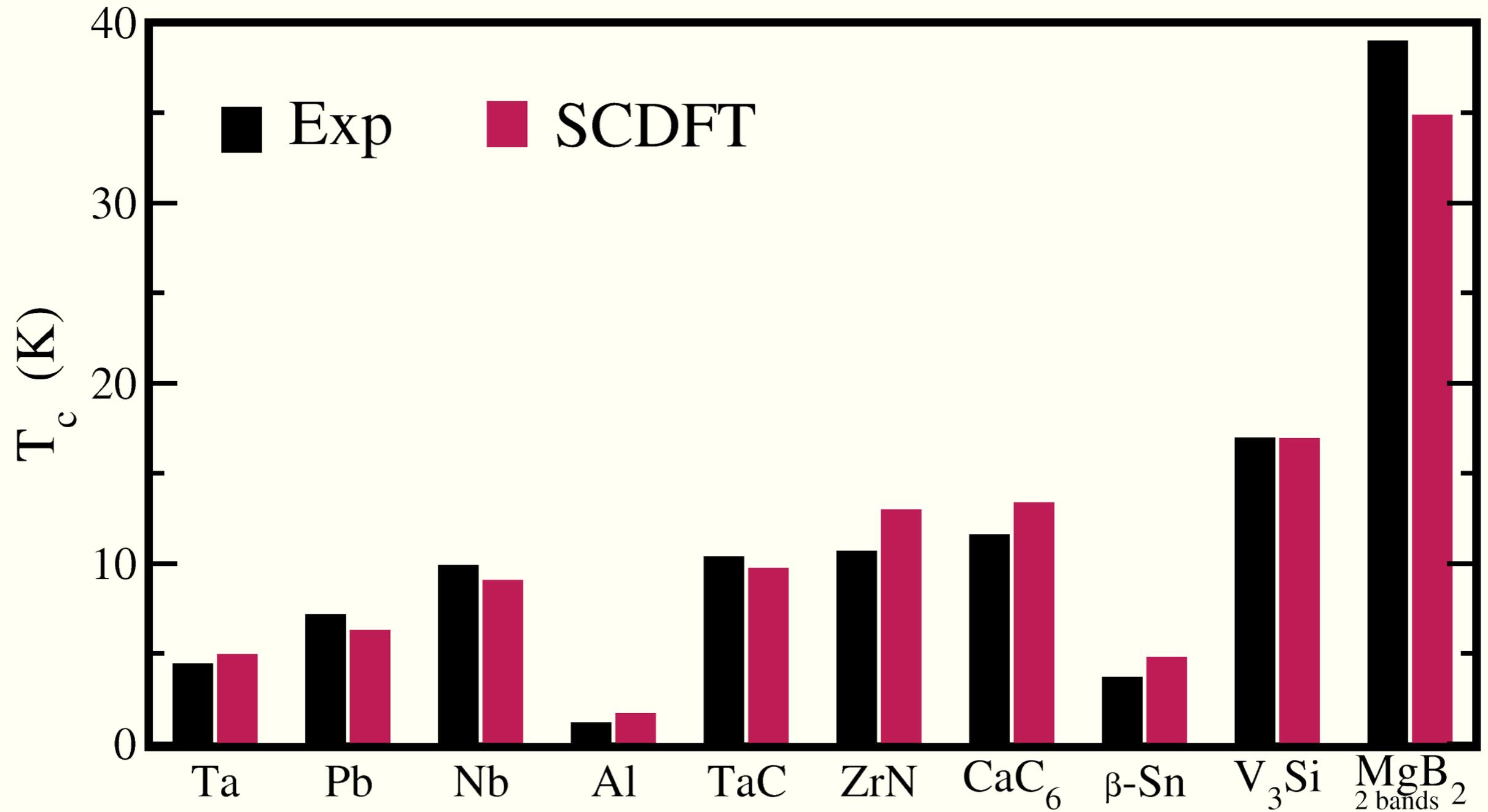
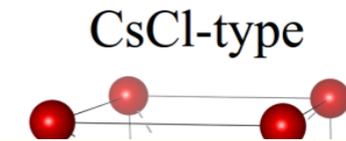


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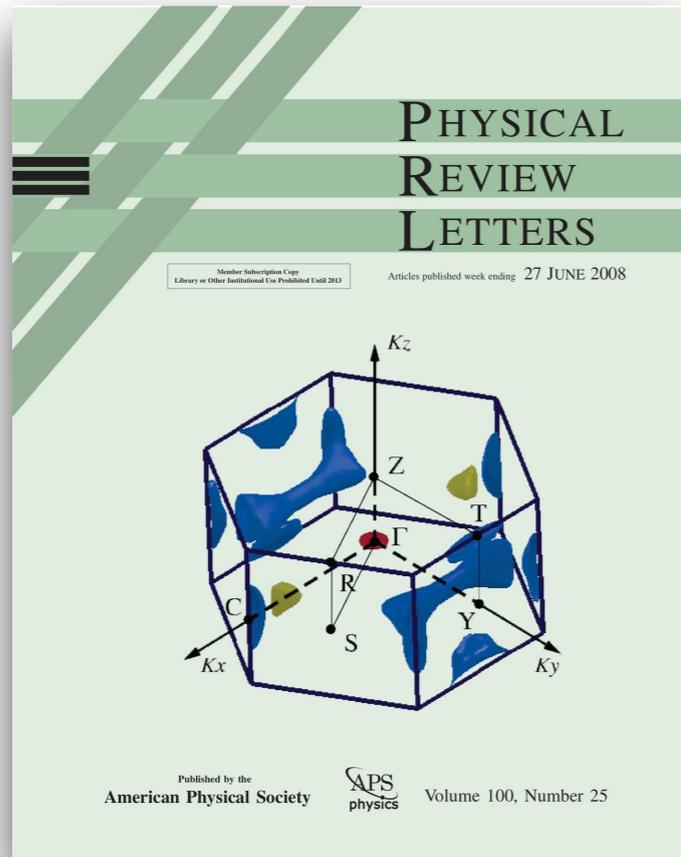
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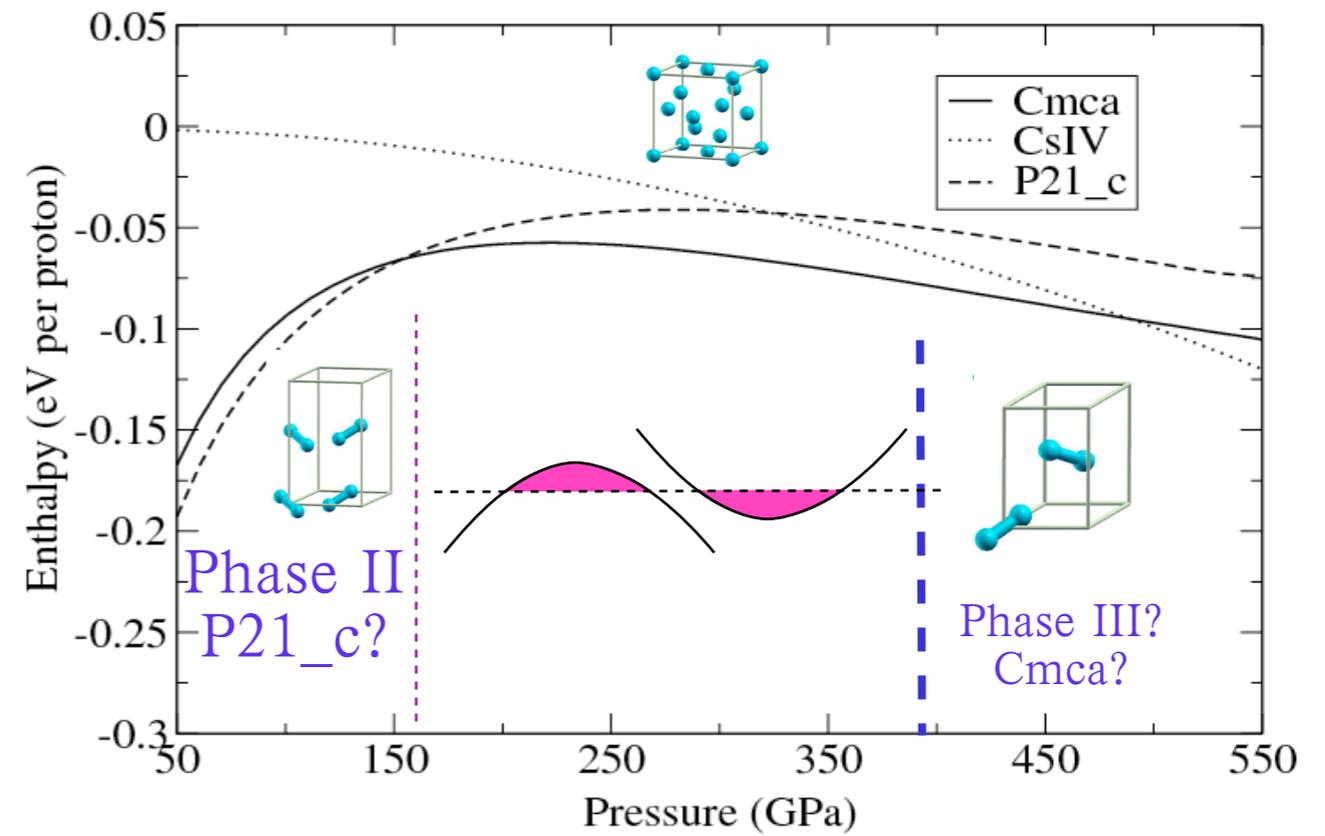
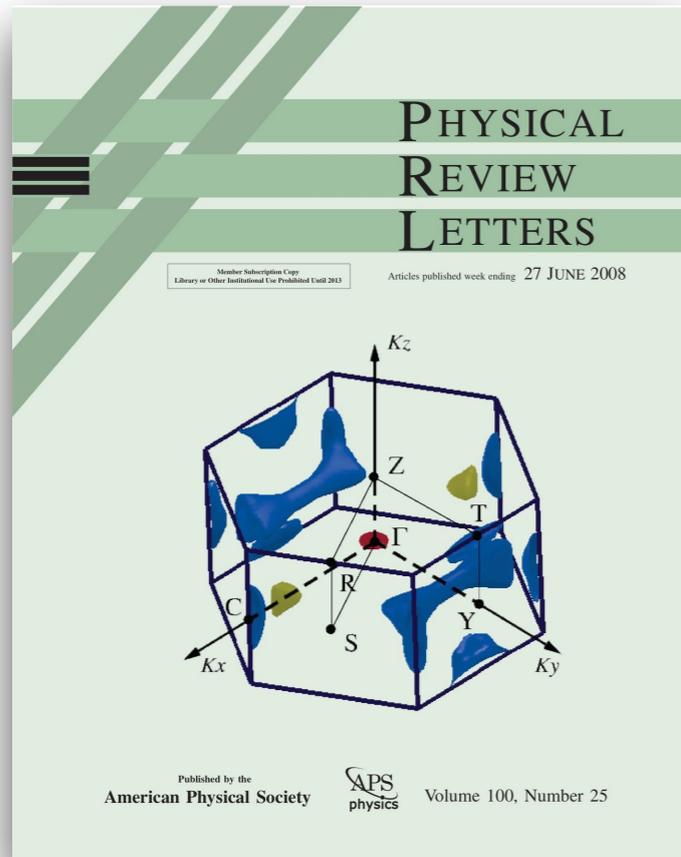
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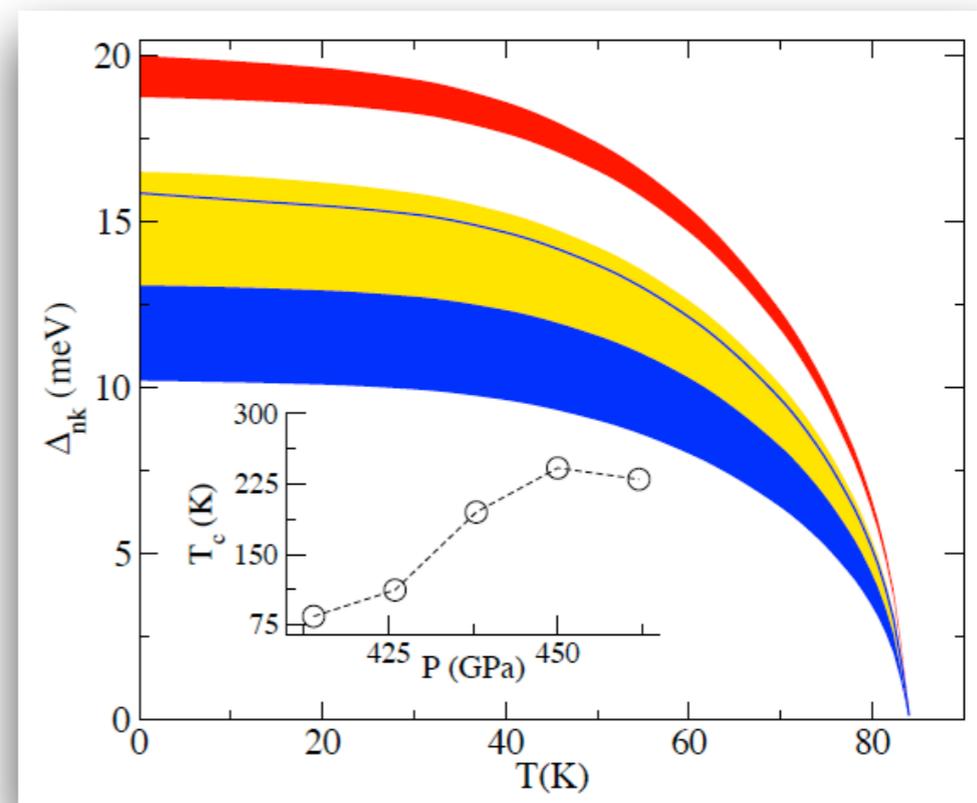
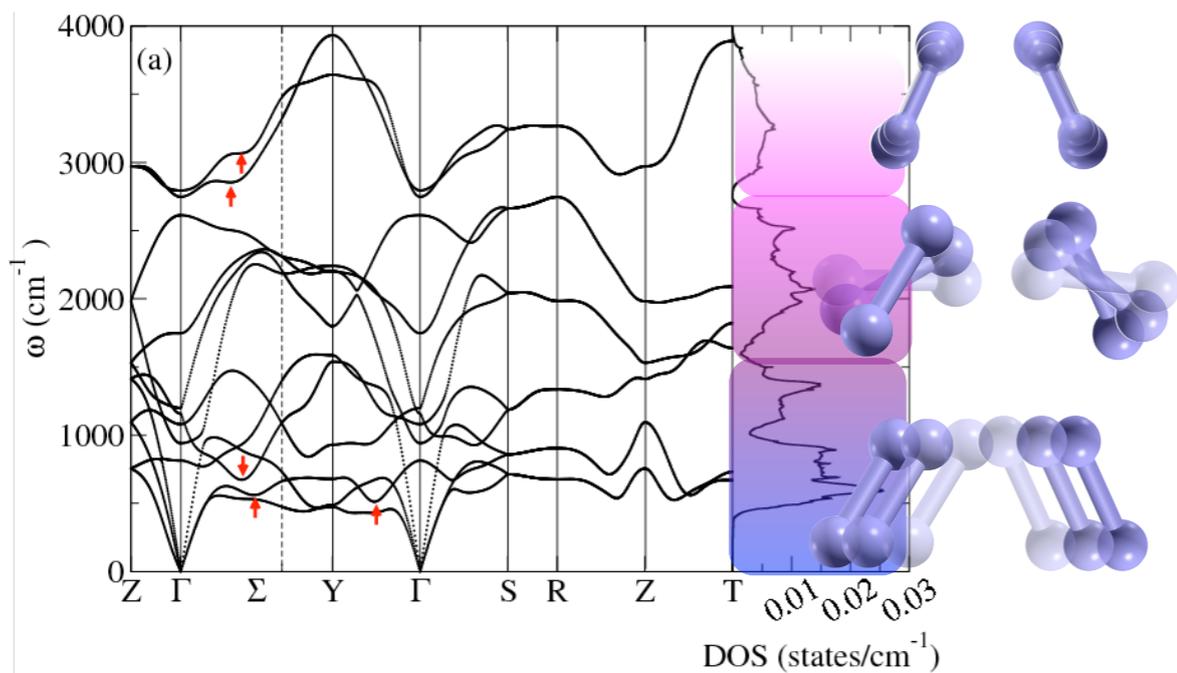
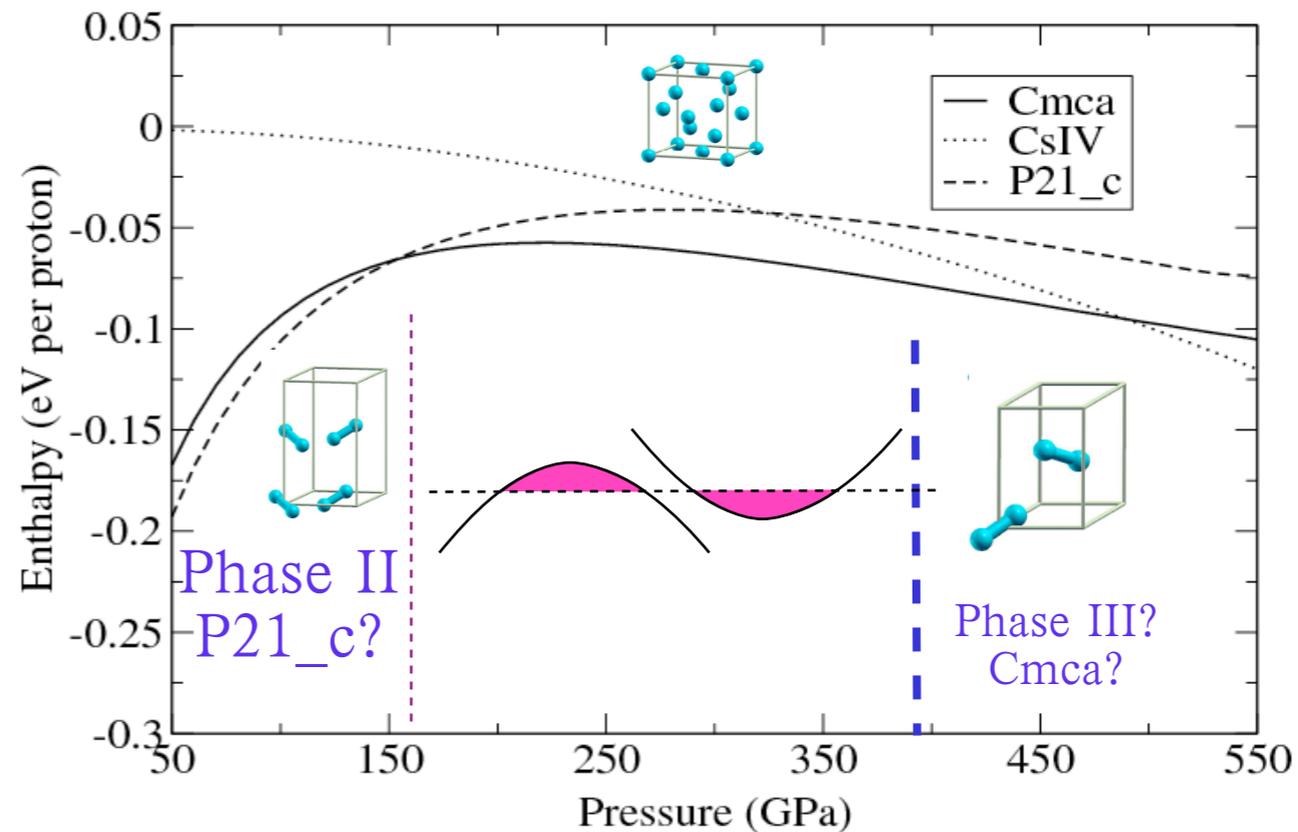
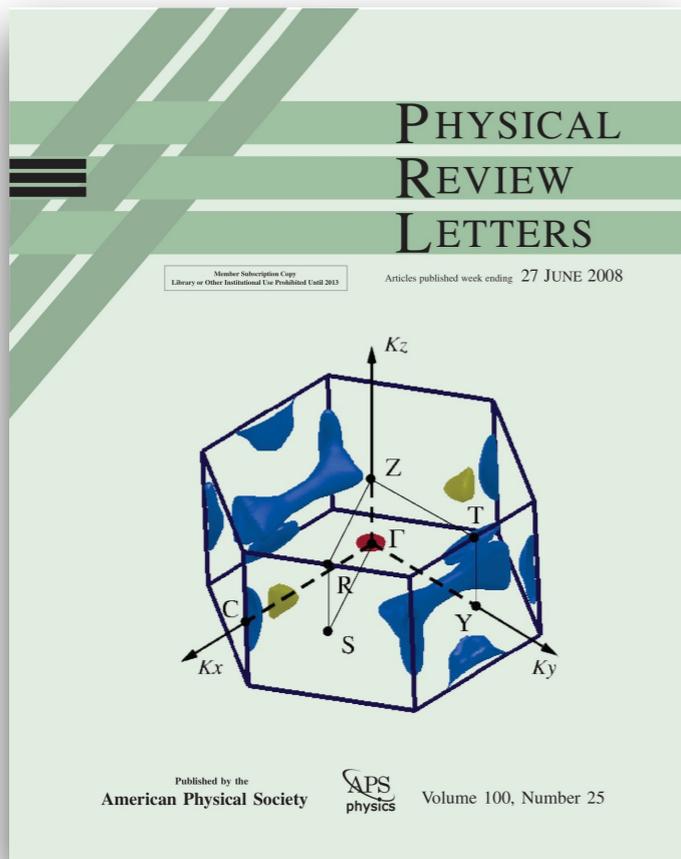
# High-temperature superconducting phase in hydrogen phase III (2008)



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# Metallic hydrogen: The holy grail of physics

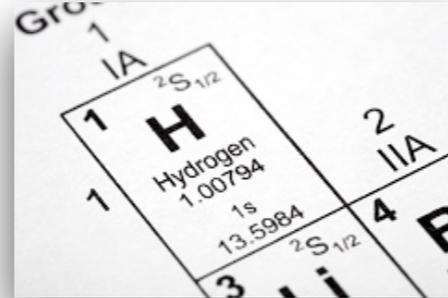


1899: Dewar produces solid hydrogen

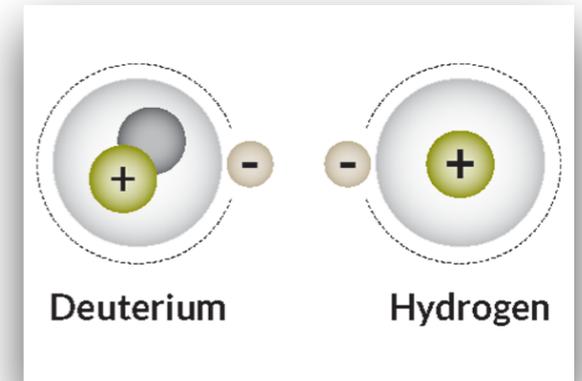


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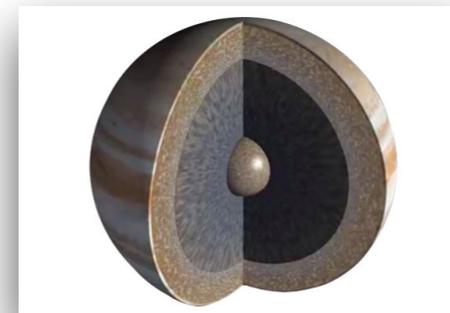
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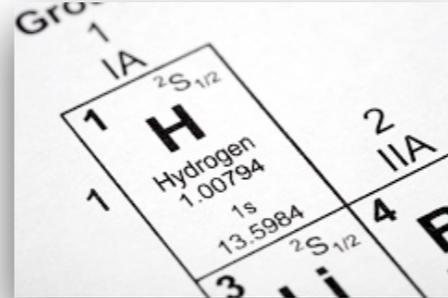


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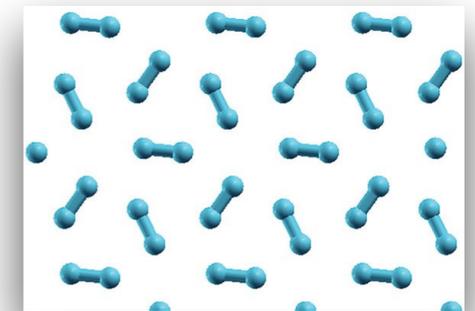
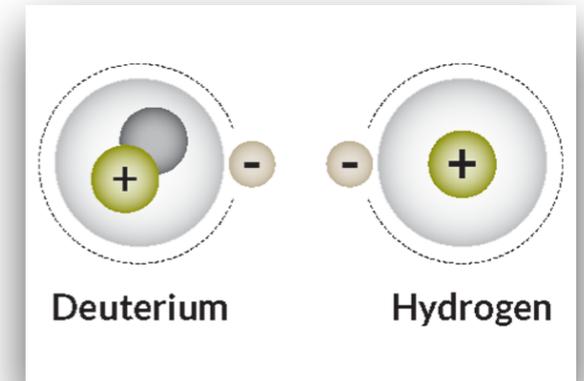


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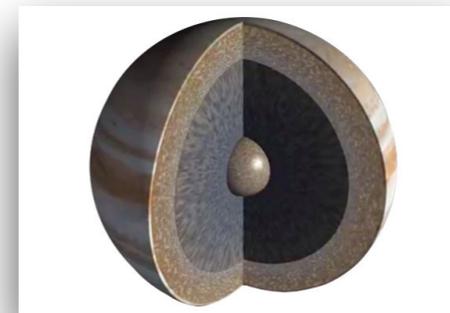
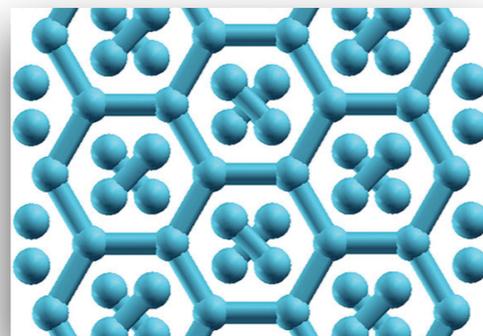


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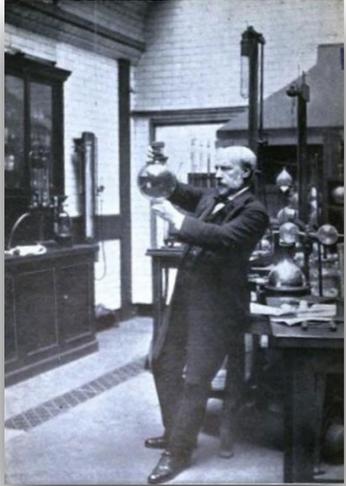
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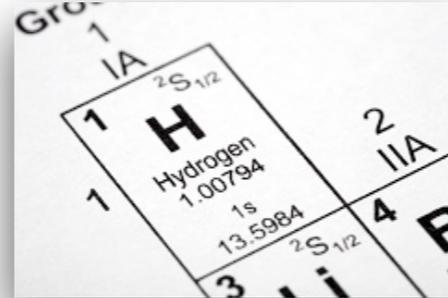


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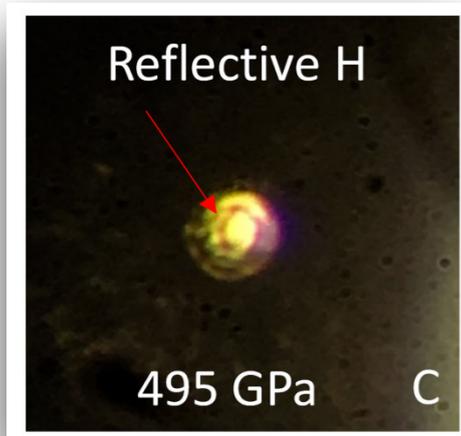
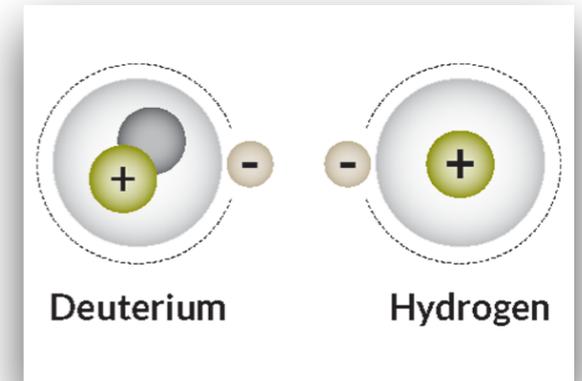


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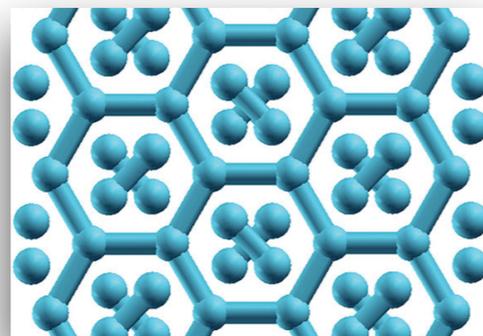


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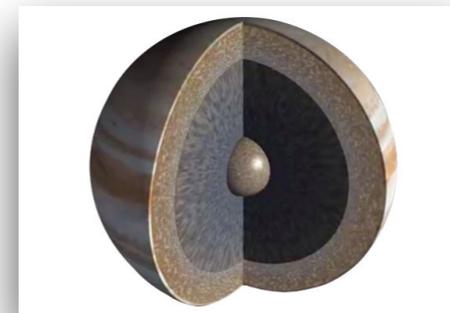


2017: Metallic hydrogen discovered by Silveira at 500 GPa (?)

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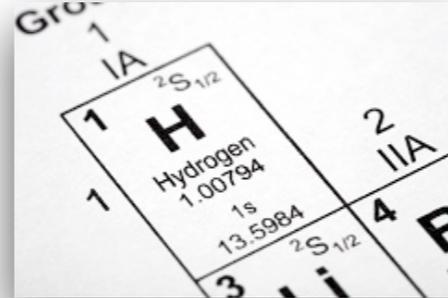


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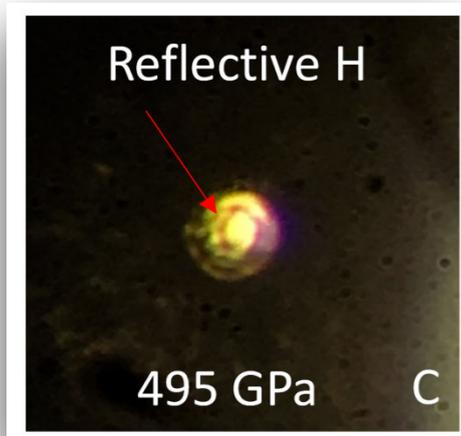
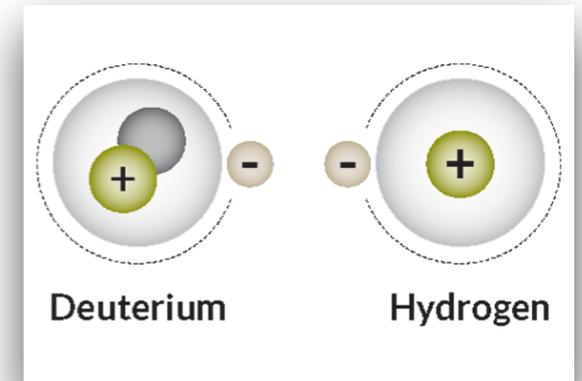
1968: Ashcroft's proposal



2020: Loubeyre: (Probable) Metallic hydrogen

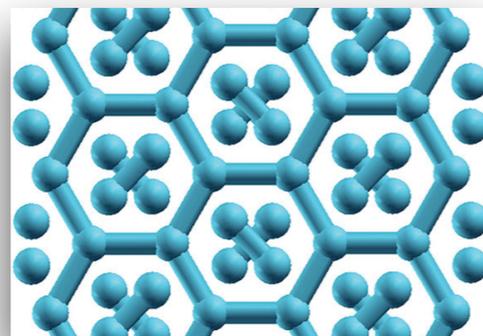


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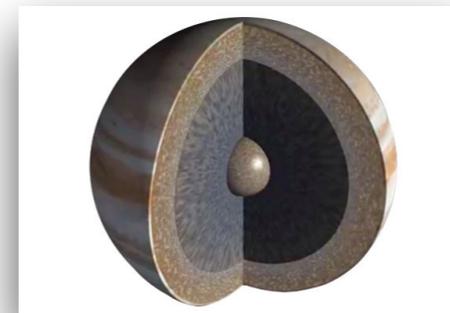


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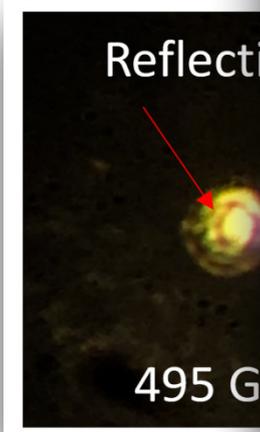


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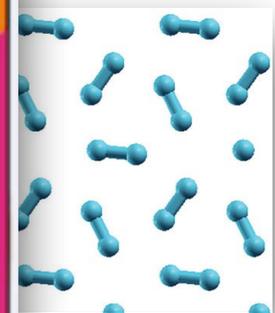
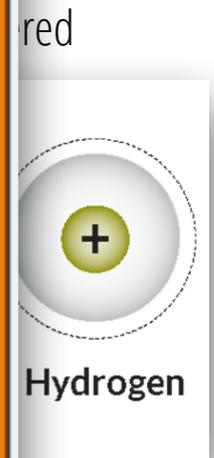
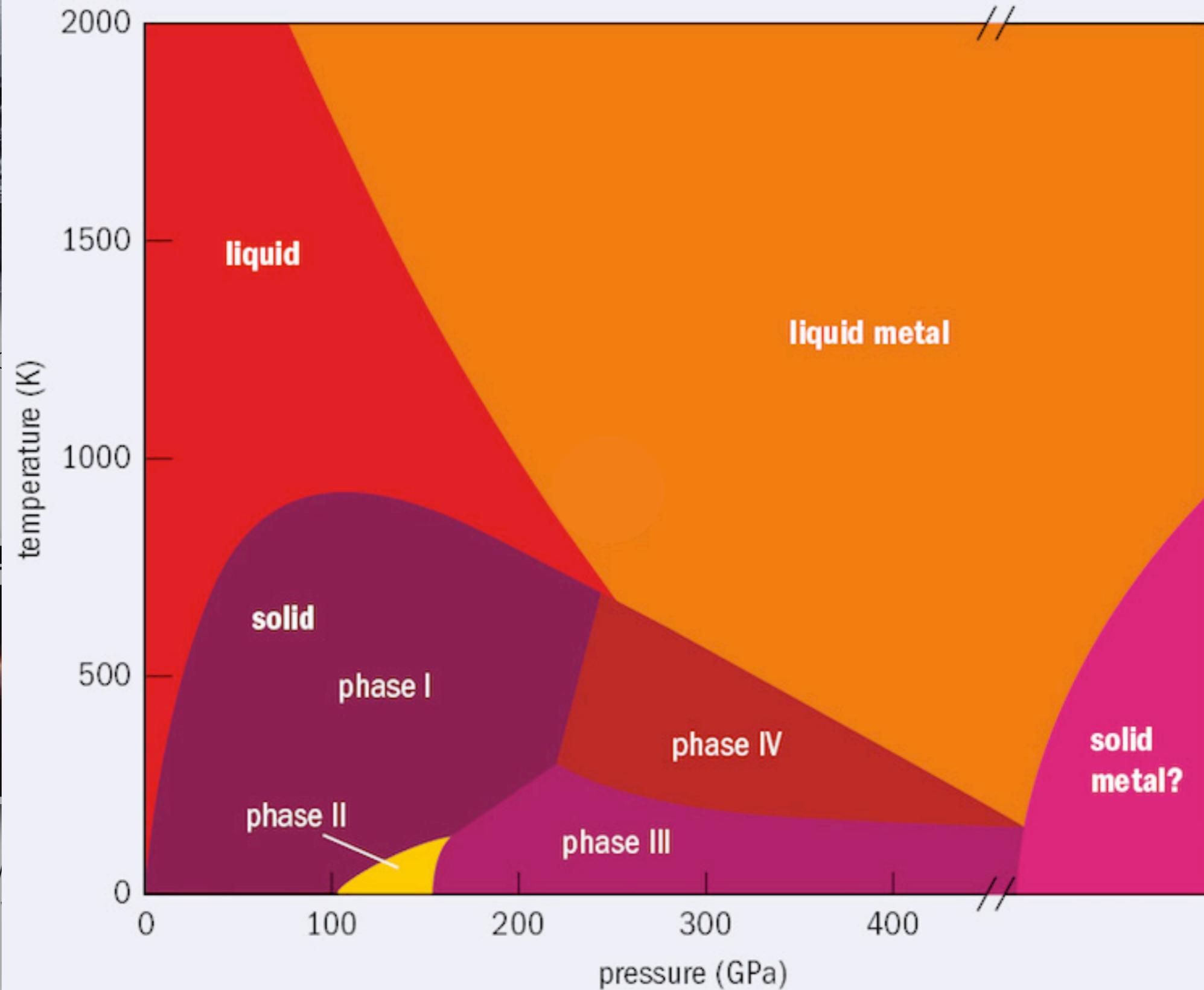
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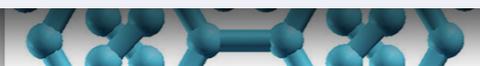
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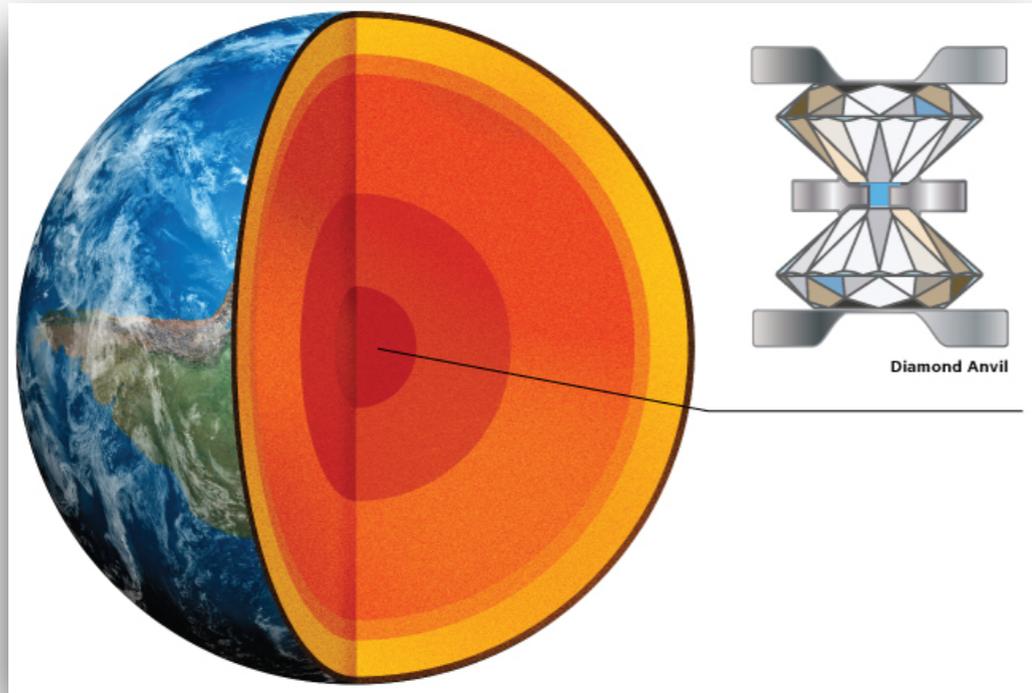


phase III



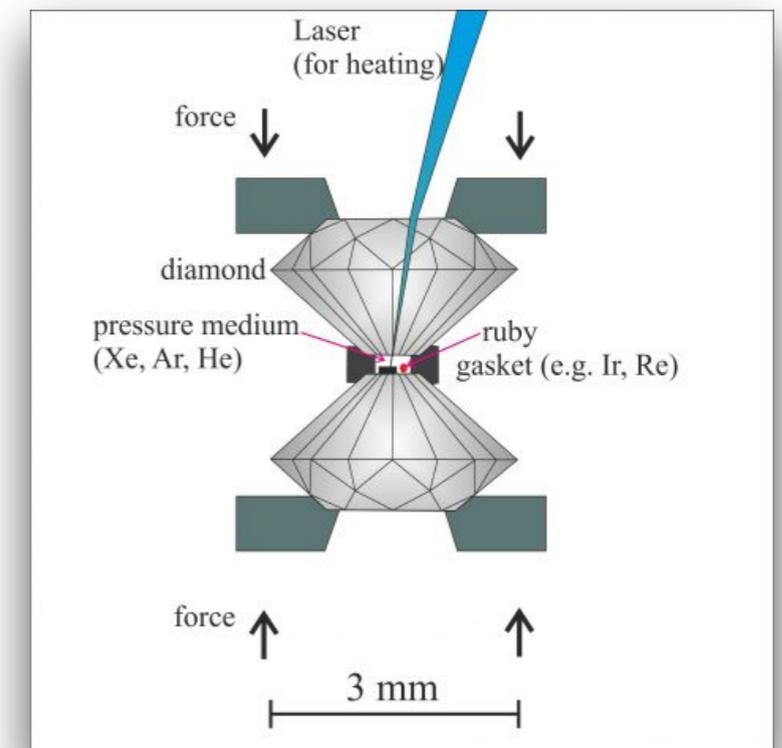
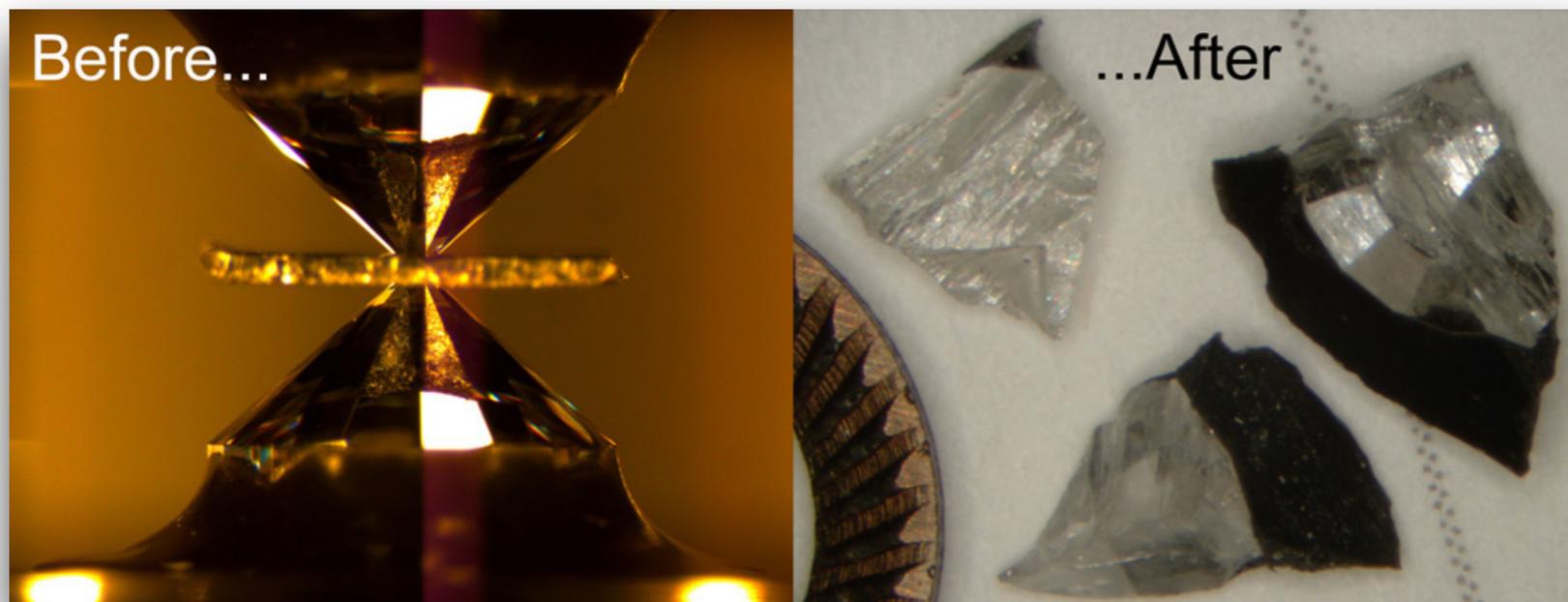
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# Hydrogen metallization: $P > 450$ GPa



Modern anvil cell reaches 350 GPa

Pressure in the inner core of earth is about 330 GPa!



# Better call Ashcroft

VOLUME 92, NUMBER 18

PHYSICAL REVIEW LETTERS

week ending  
7 MAY 2004

## Hydrogen Dominant Metallic Alloys: High Temperature Superconductors?

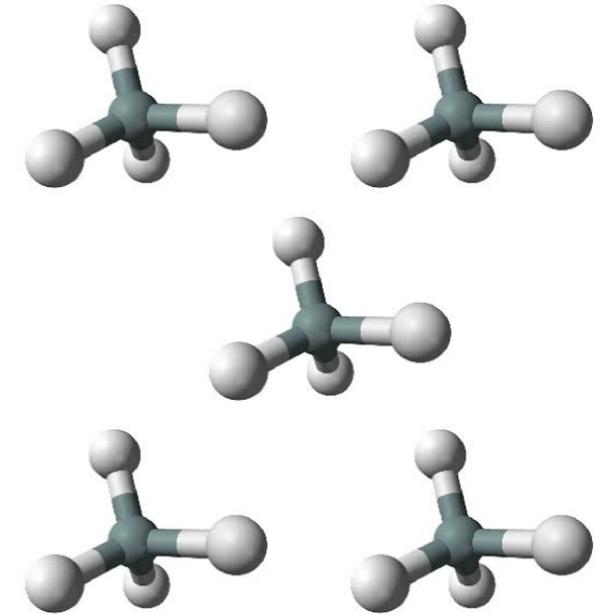
N.W. Ashcroft

*Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501, USA*

*Donostia International Physics Center, San Sebastian, Spain*

(Received 29 December 2003; published 6 May 2004)

The arguments suggesting that metallic hydrogen, either as a monatomic or paired metal, should be a candidate for high temperature superconductivity are shown to apply with comparable weight to alloys of metallic hydrogen where hydrogen is a dominant constituent, for example, in the dense group IVa hydrides. The attainment of metallic states should be well within current capabilities of diamond anvil cells, but at pressures considerably lower than may be necessary for hydrogen.



# Better call Ashcroft

VOLUME 92, NUMBER 18

PHYSICAL REVIEW LETTERS

week ending  
7 MAY 2004

## Hydrogen Dominant Metallic Alloys: High Temperature Superconductors?

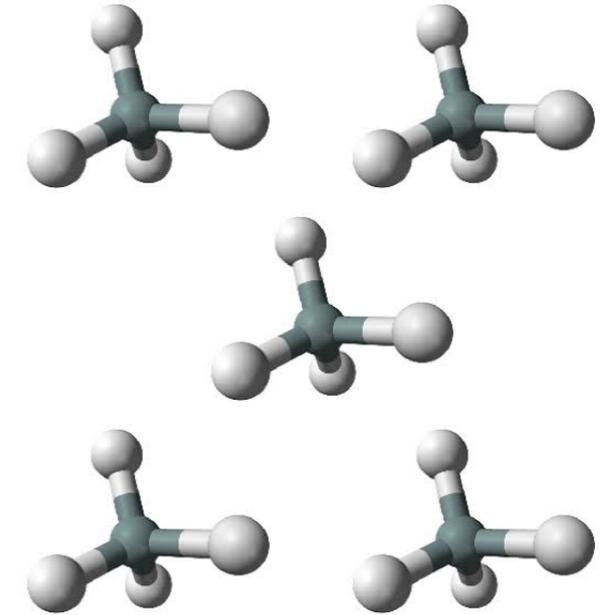
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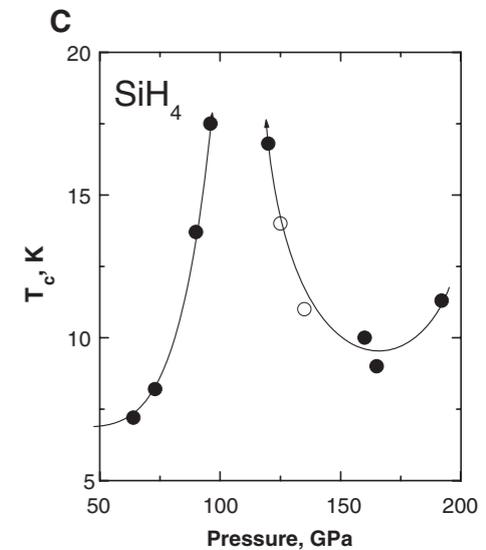
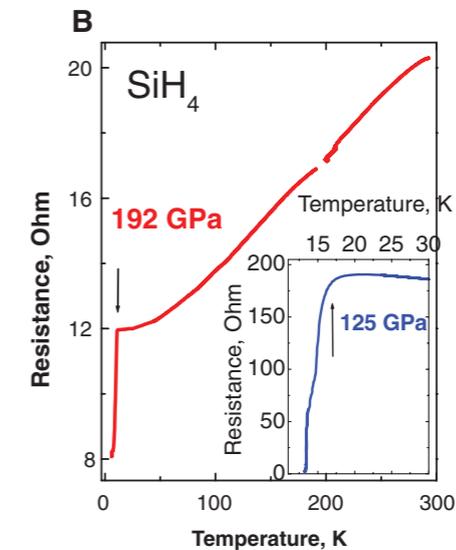
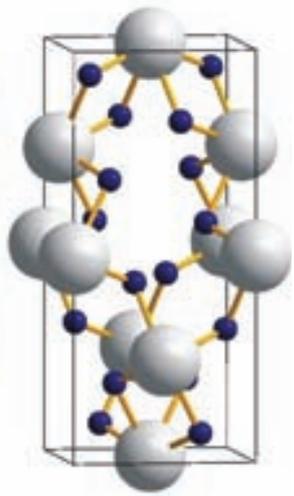


## 2008

## Superconductivity in Hydrogen Dominant Materials: Silane

M. I. Eremets,<sup>1\*</sup> I. A. Trojan,<sup>1†</sup> S. A. Medvedev,<sup>1</sup> J. S. Tse,<sup>2</sup> Y. Yao<sup>2</sup>

The metallization of hydrogen directly would require pressure in excess of 400 gigapascals (GPa), out of the reach of present experimental techniques. The dense group IVa hydrides attract considerable attention because hydrogen in these compounds is chemically precompressed and a metallic state is expected to be achievable at experimentally accessible pressures. We report the transformation of insulating molecular silane to a metal at 50 GPa, becoming superconducting at a transition temperature of  $T_c = 17$  kelvin at 96 and 120 GPa. The metallic phase has a hexagonal close-packed structure with a high density of atomic hydrogen, creating a three-dimensional conducting network. These experimental findings support the idea of modeling metallic hydrogen with hydrogen-rich alloy.



**17 K is much lower than the predicted Tc**

# Hydrides at high pressure: search by computers

NATURE VOL. 335 15 SEPTEMBER 1988

NEWS AND VIEWS

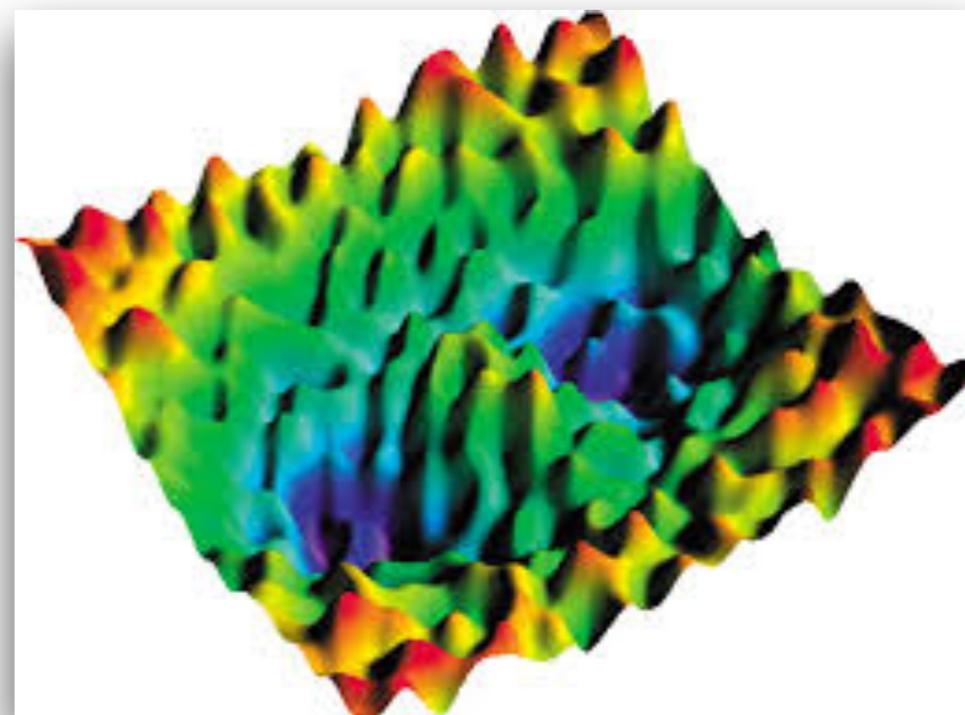
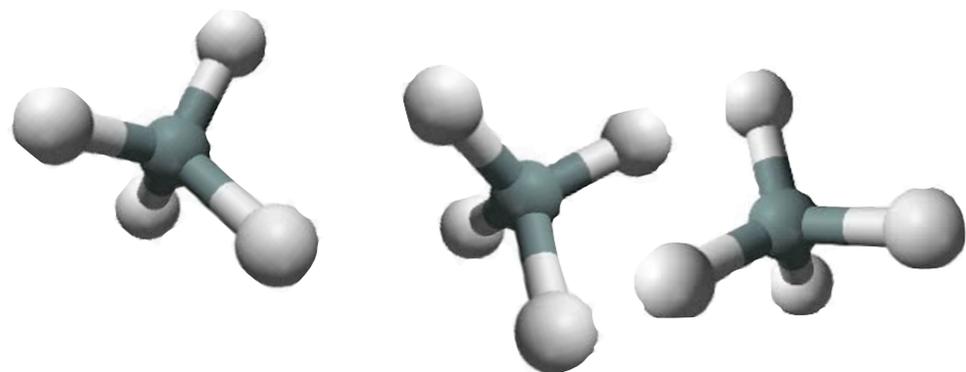
201

## Crystals from first principles

by J. Maddox

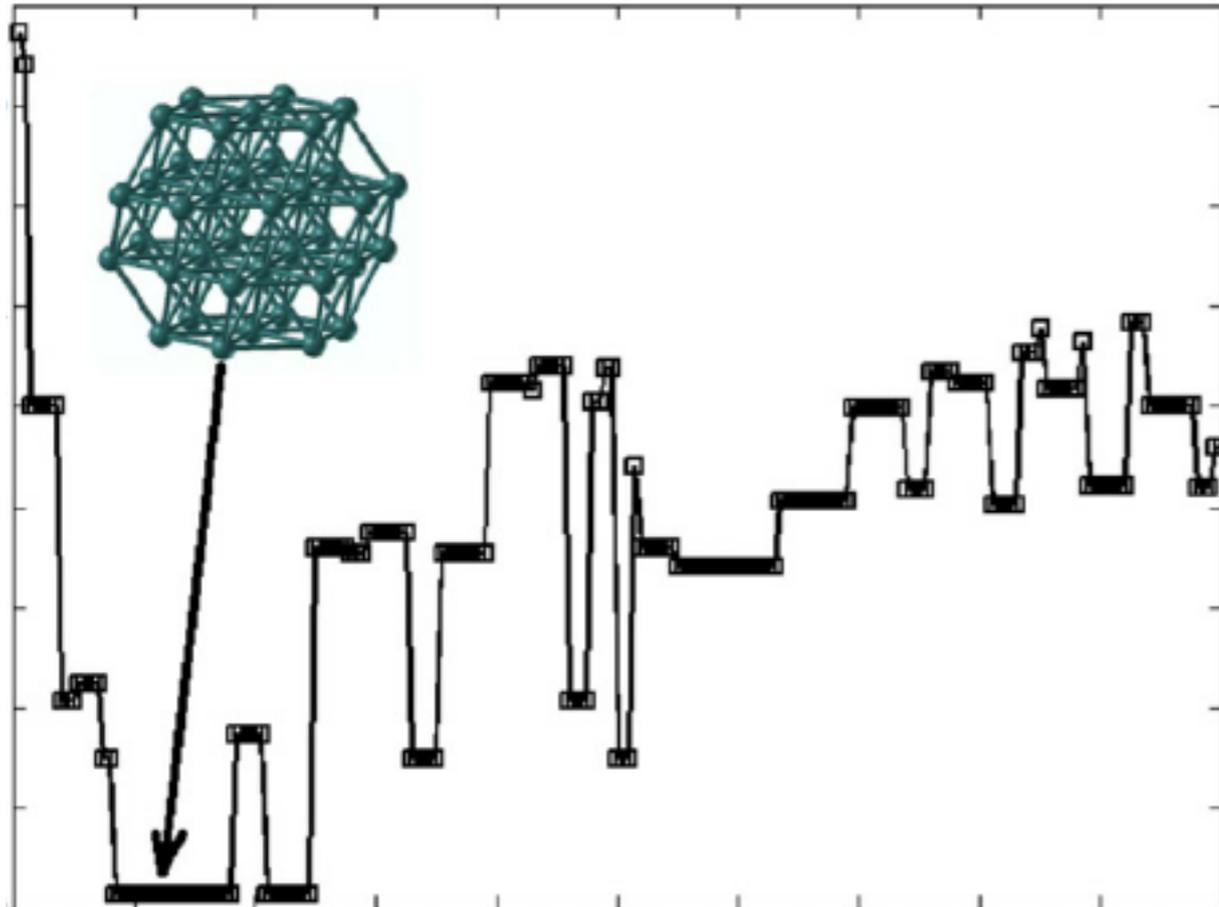
ONE of the continuing scandals in the physical sciences is that it remains in general impossible to predict the structure of even the simplest crystalline solids from a knowledge of their chemical composition. Who, for example, would guess that graphite, not diamond, is the thermodynamically stable allotrope of carbon at ordinary temperature and pressure? Solids such as crystalline water (ice) are still thought to lie beyond mortals' ken.

Yet one would have thought that, by now, it should be possible to equip a sufficiently large computer with a sufficiently large program, type in the formula of the chemical and obtain, as output, the atomic coordinates of the atoms in a unit cell.

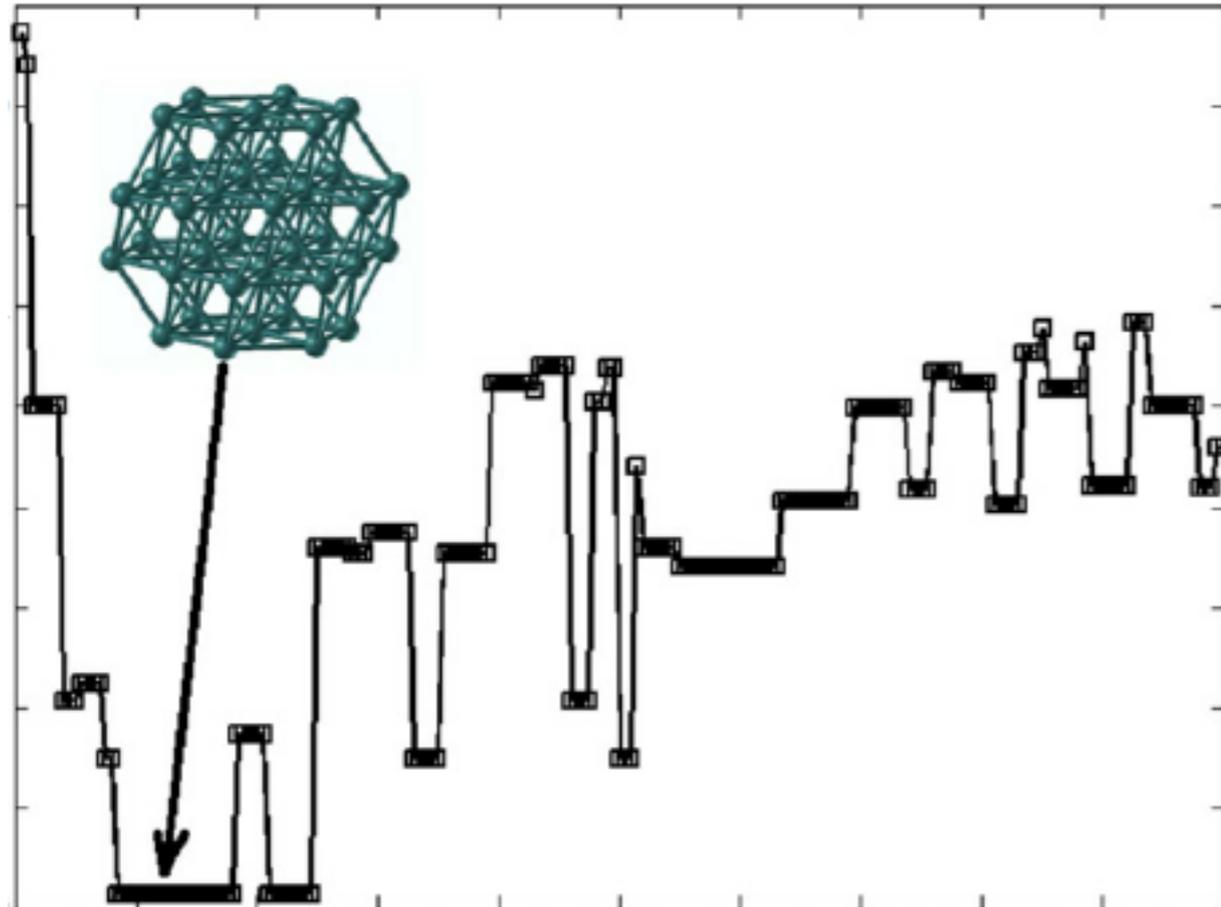


# Genetic Algorithms

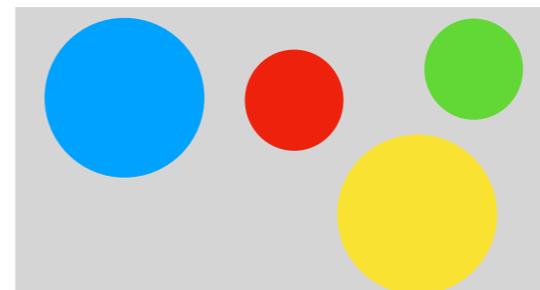
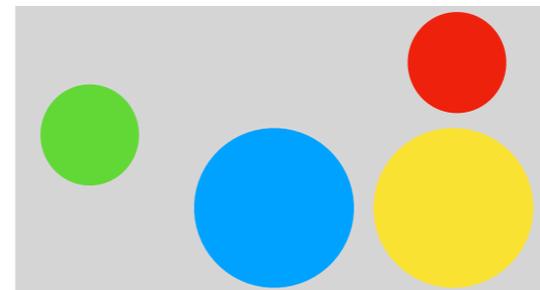
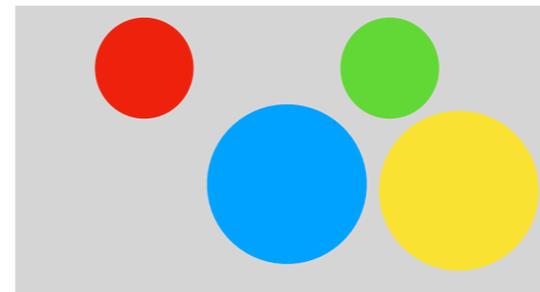
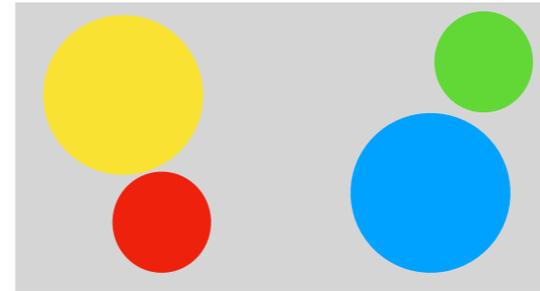
## Ab-initio random structure searching (The Columbus egg)



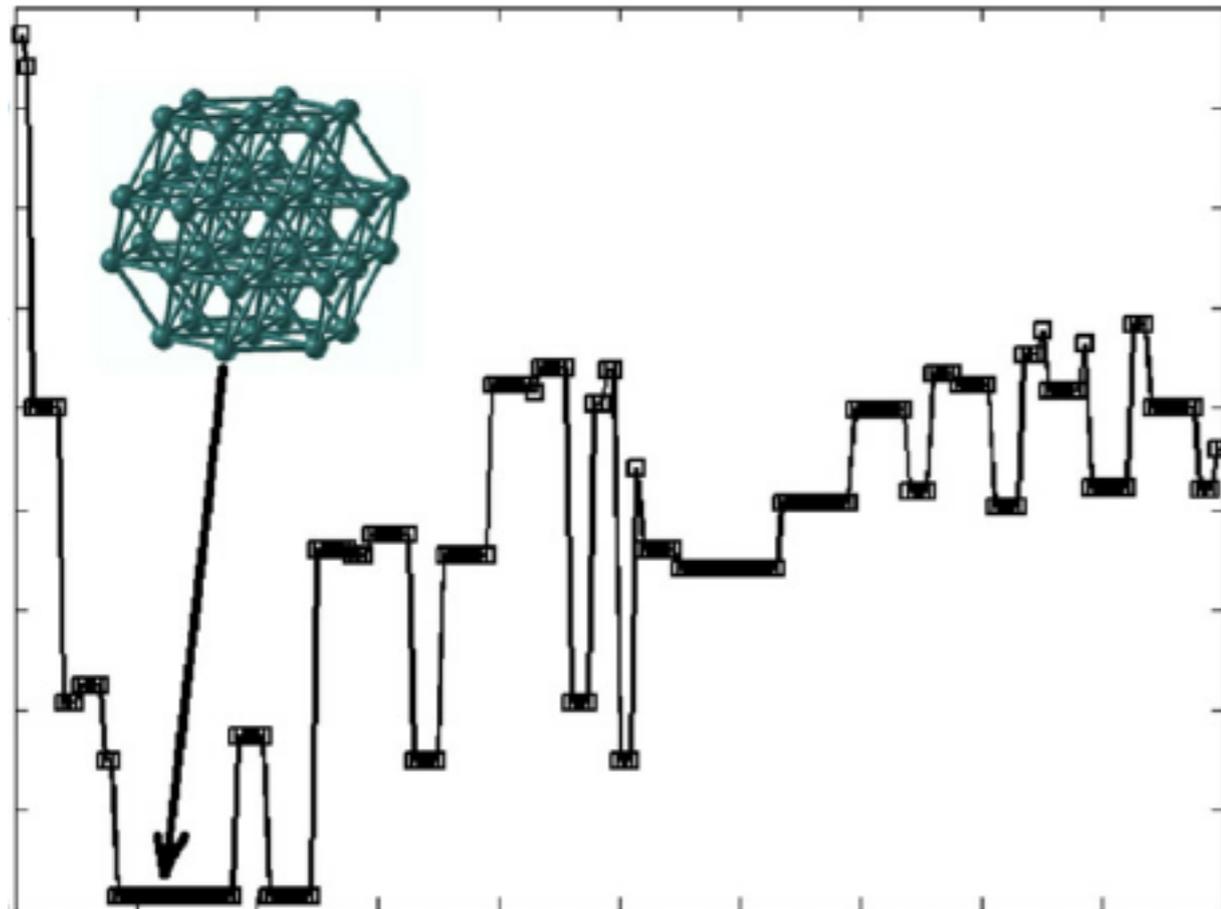
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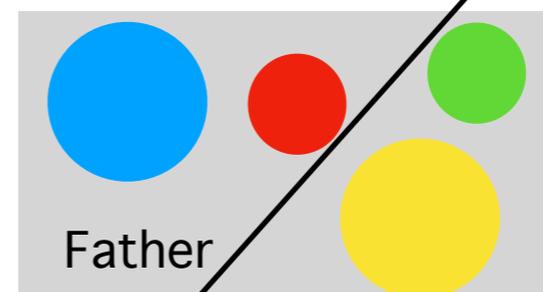
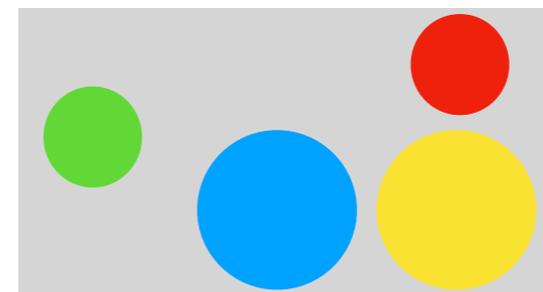
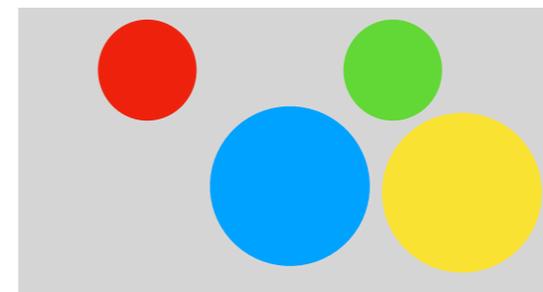
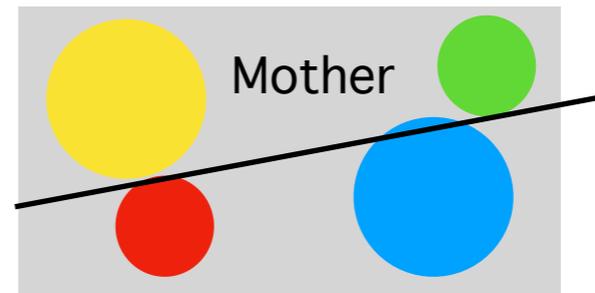
## Genetic Algorithms



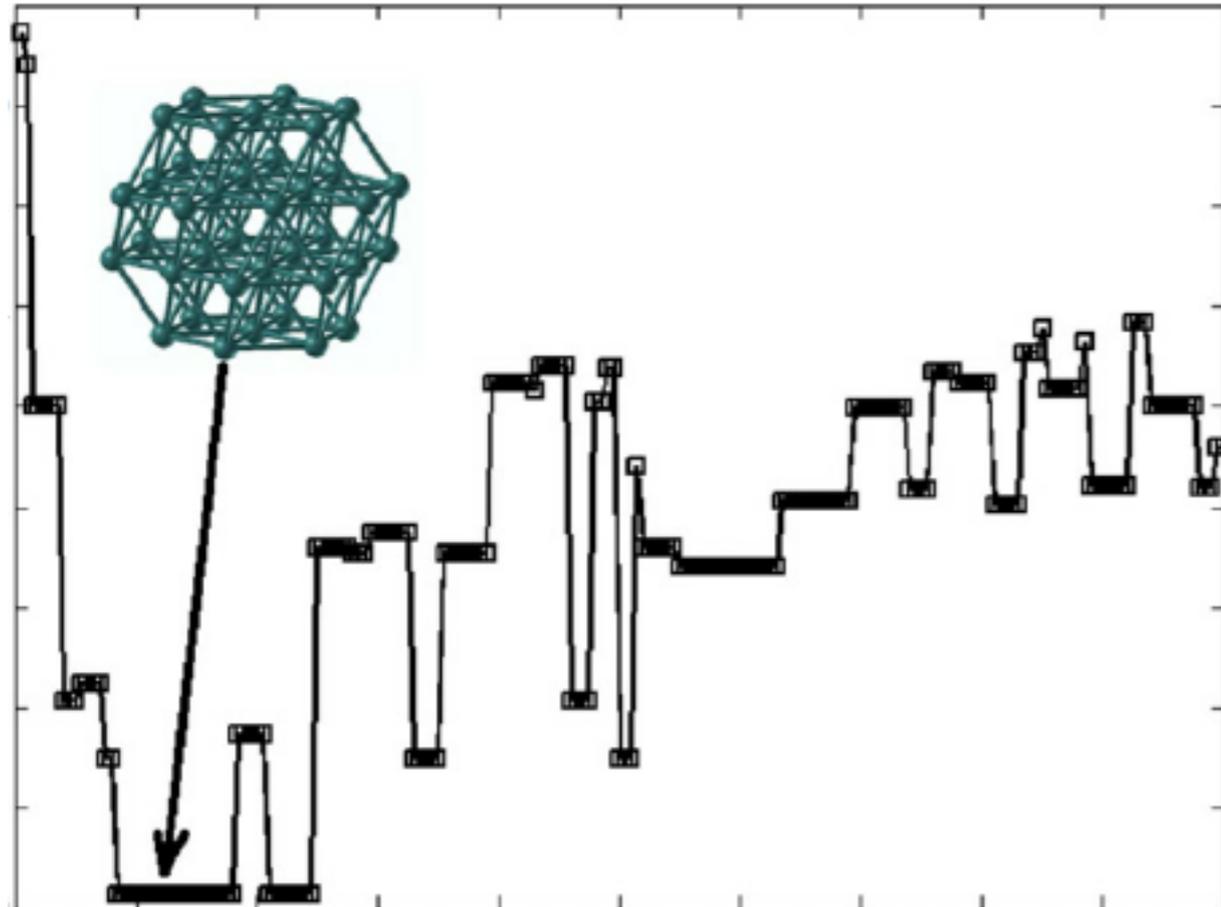
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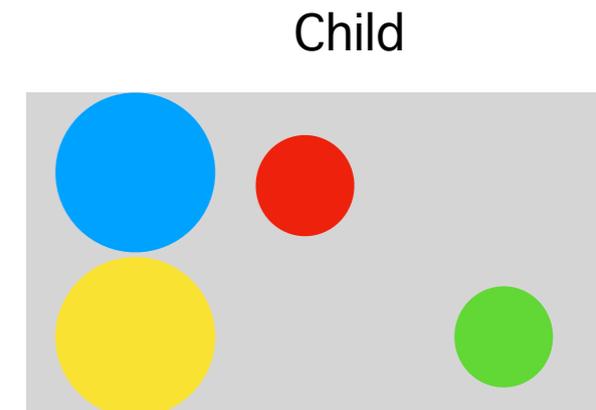
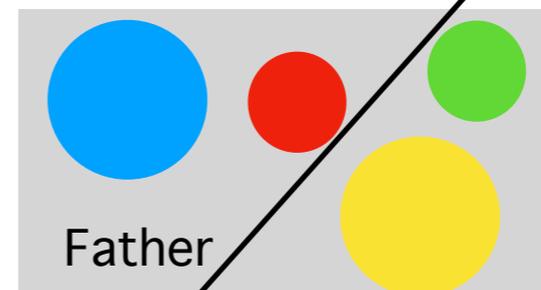
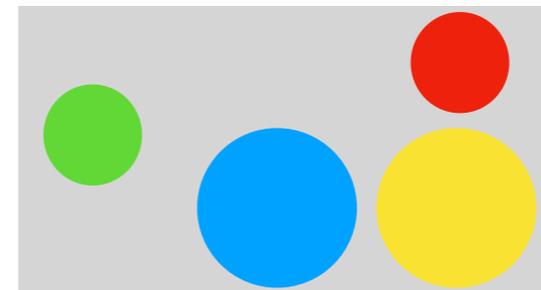
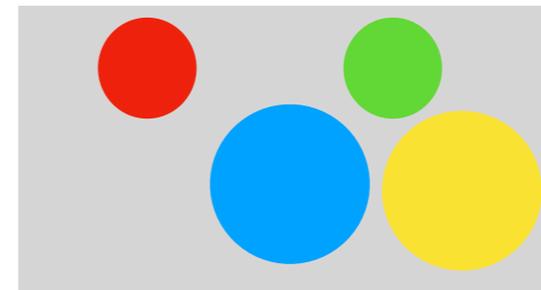
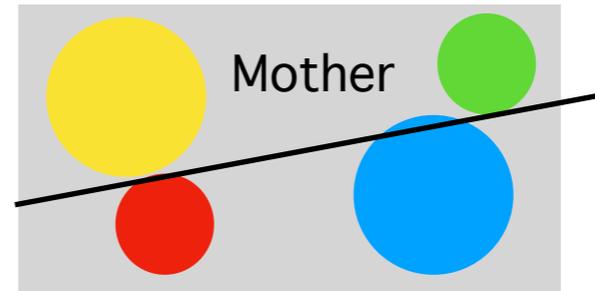
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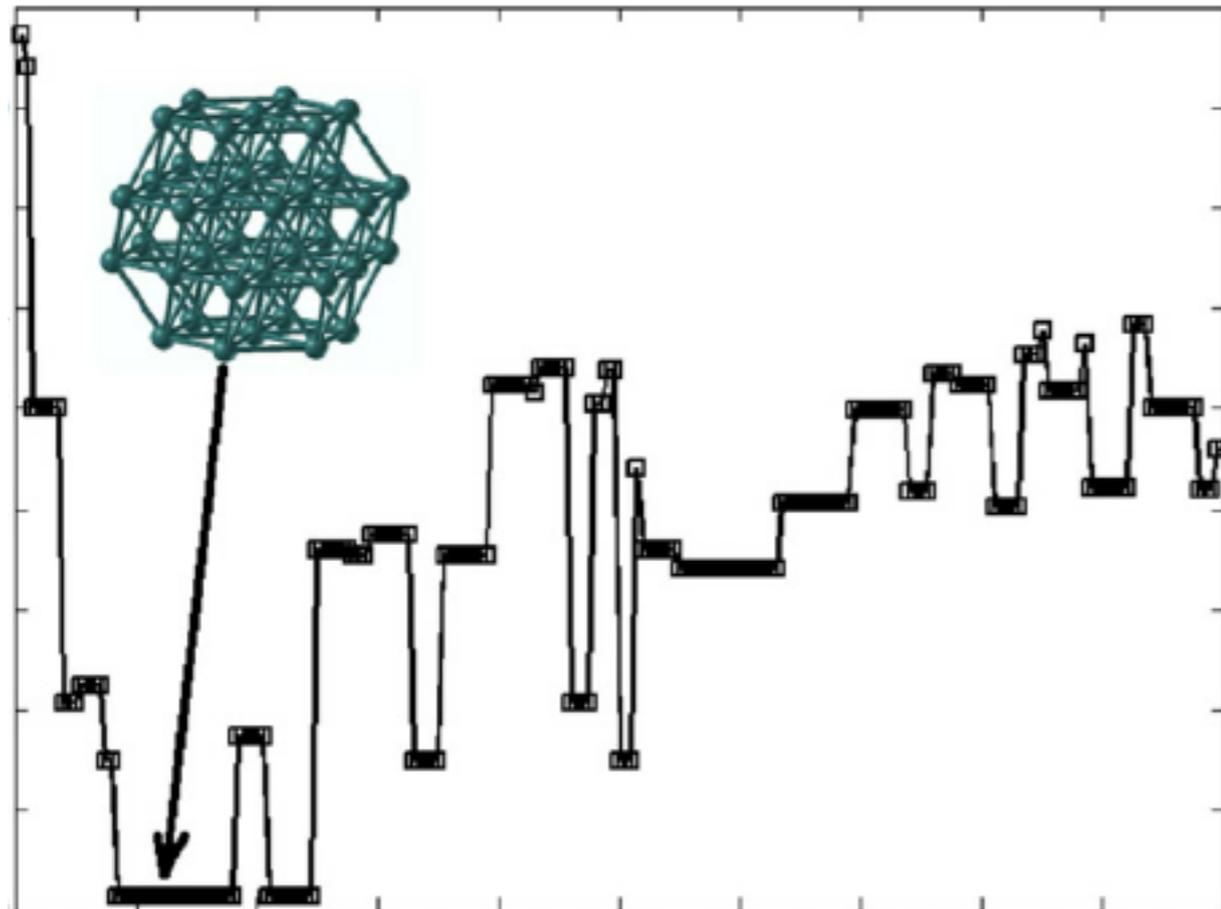
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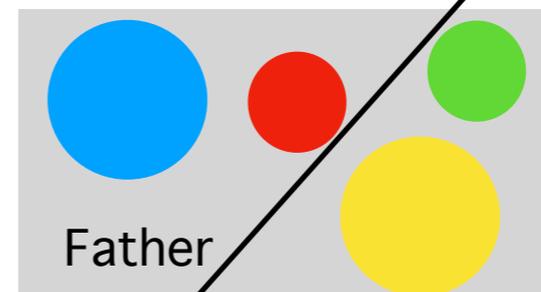
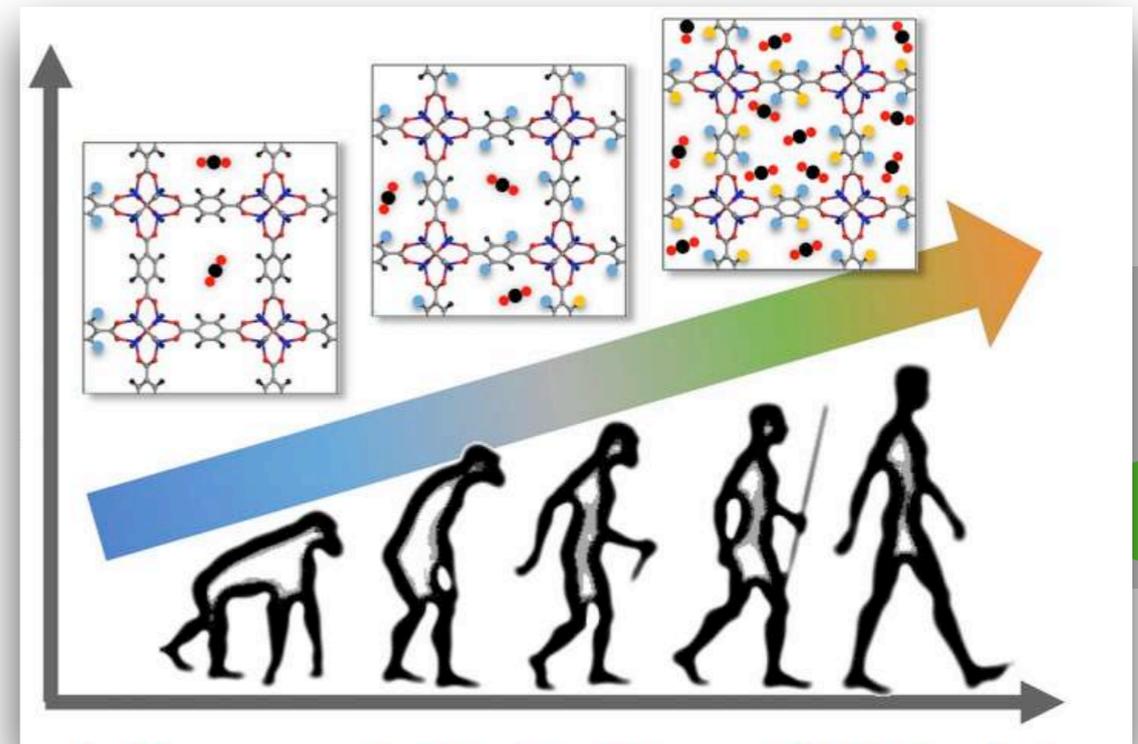
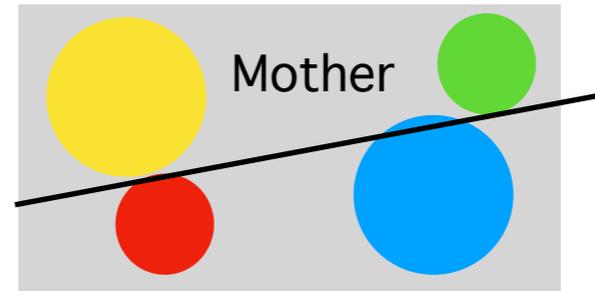
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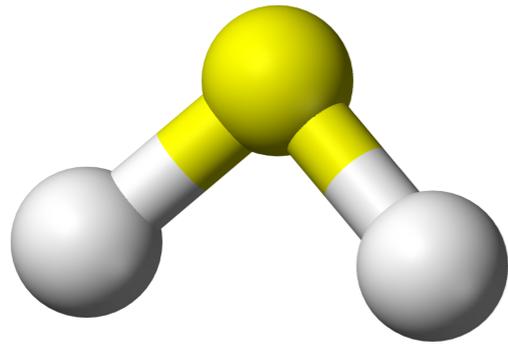


# Genetic Algorithms





# Hydrogen sulfide: the chemistry changes



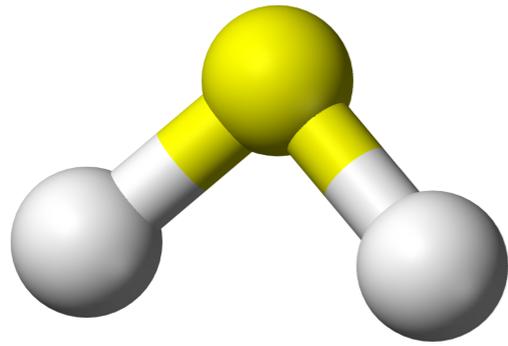
Discovered in 1777

It is a colorless gas with the characteristic foul odor of rotten eggs.

It is very poisonous, corrosive, and flammable, explosive



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THE JOURNAL OF CHEMICAL PHYSICS **140**, 174712 (2014)



## The metallization and superconductivity of dense hydrogen sulfide

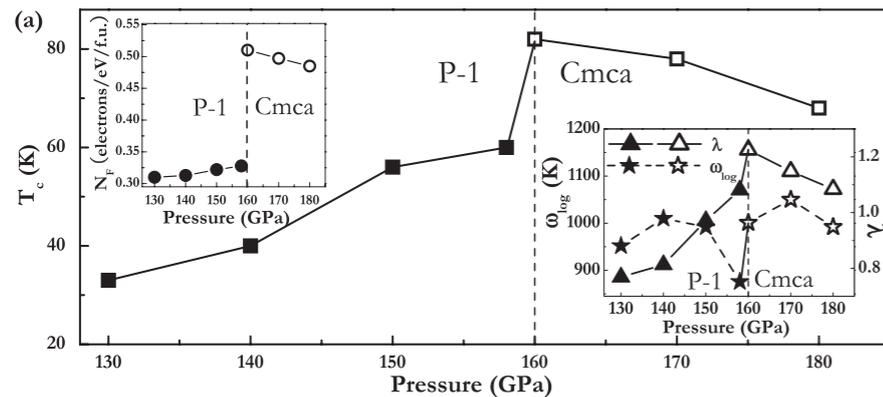
Yinwei Li,<sup>1,a)</sup> Jian Hao,<sup>1</sup> Hanyu Liu,<sup>2</sup> Yanling Li,<sup>1</sup> and Yanming Ma<sup>3,b)</sup>

<sup>1</sup>School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou 221116, People's Republic of China

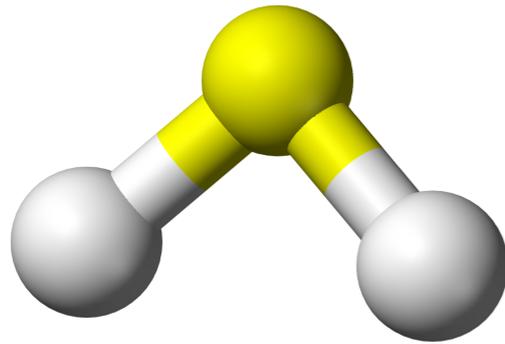
<sup>2</sup>Department of Physics and Engineering Physics, University of Saskatchewan, Saskatchewan S7N 5E2, Canada

<sup>3</sup>State Key Laboratory of Superhard Materials, Jilin University, Changchun 130012, People's Republic of China

(Received 20 March 2014; accepted 18 April 2014; published online 7 May 2014)



# Hydrogen sulfide: the chemistry changes



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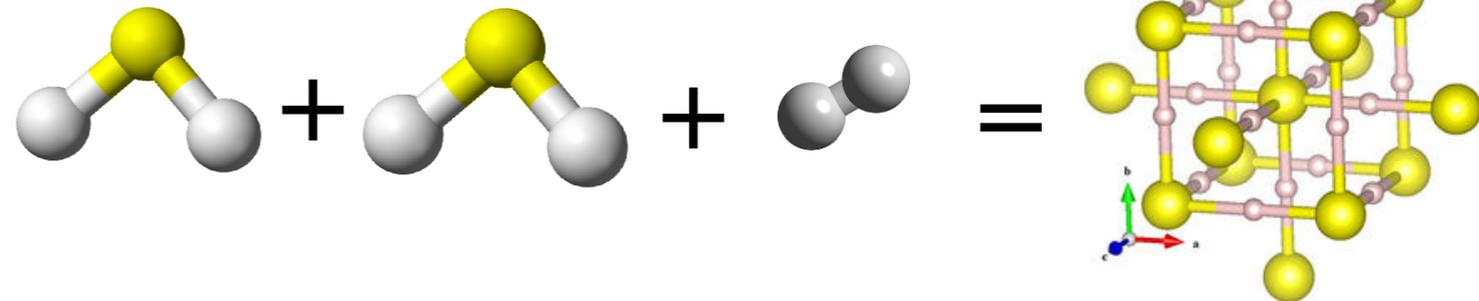
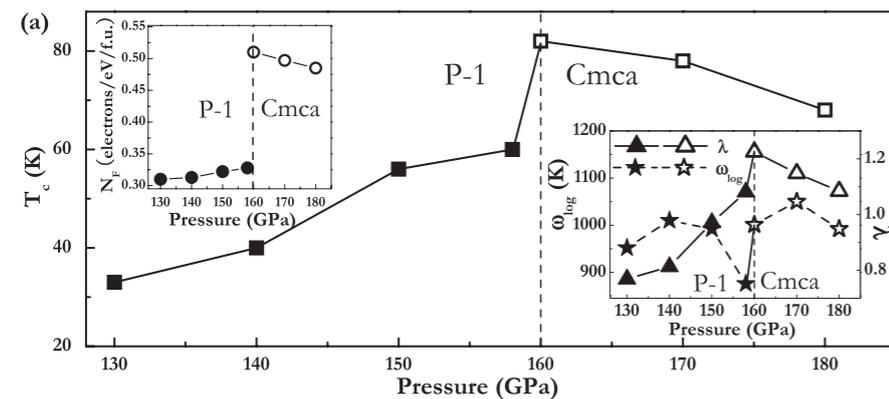
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<sup>3</sup>State Key Laboratory of Superhard Materials, Jilin University, Changchun 130012, People's Republic of China

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SCIENTIFIC  
REPORTS



OPEN

## Pressure-induced metallization of dense $(\text{H}_2\text{S})_2\text{H}_2$ with high- $T_c$ superconductivity

SUBJECT AREAS:  
THEORY AND  
COMPUTATION  
CONDENSED-MATTER PHYSICS

Defang Duan<sup>1,2</sup>, Yunxian Liu<sup>1</sup>, Fubo Tian<sup>1</sup>, Da Li<sup>1</sup>, Xiaoli Huang<sup>1</sup>, Zhonglong Zhao<sup>1</sup>, Hongyu Yu<sup>1</sup>, Bingbing Liu<sup>1</sup>, Wenjing Tian<sup>2</sup> & Tian Cui<sup>1</sup>

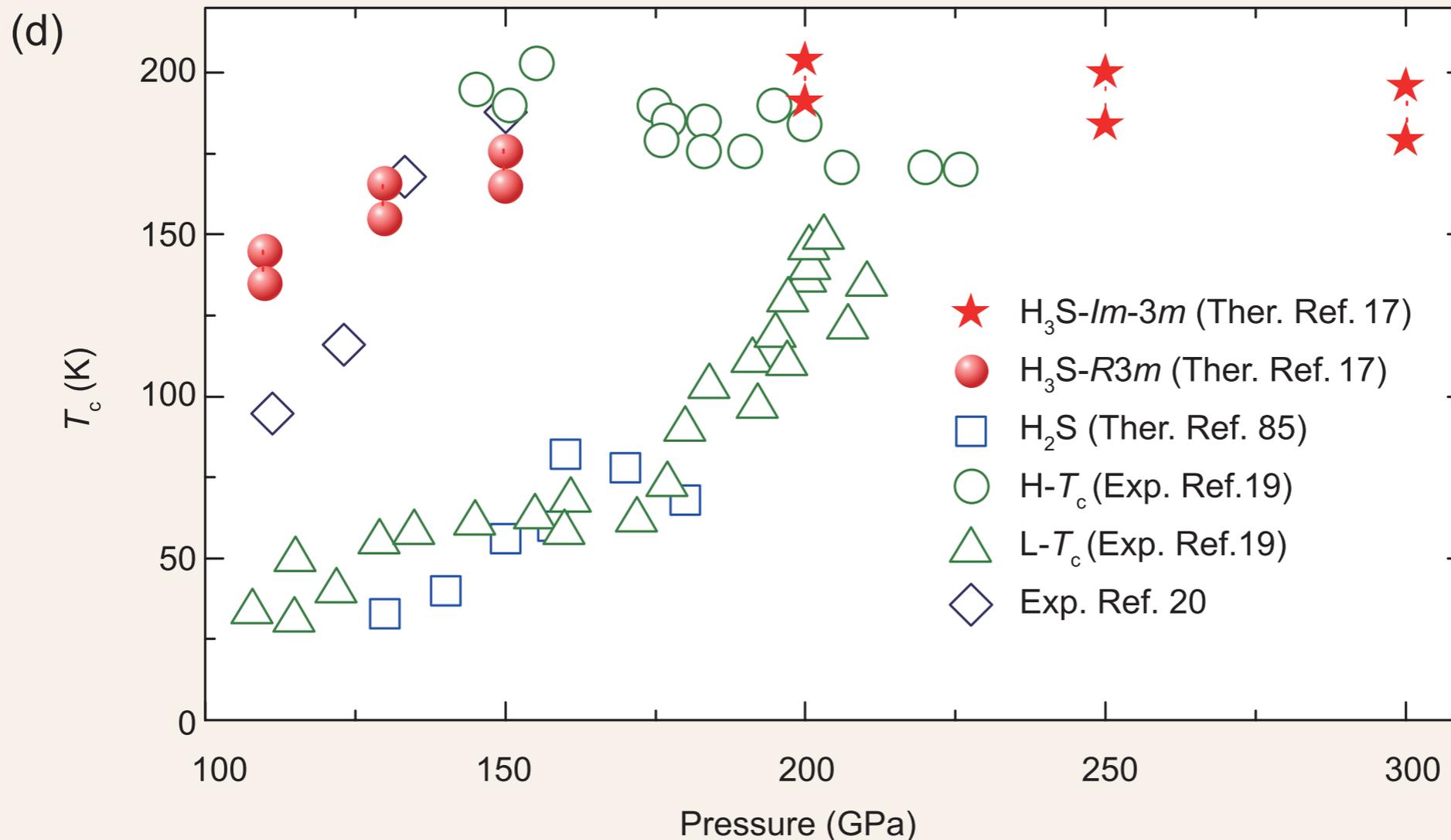
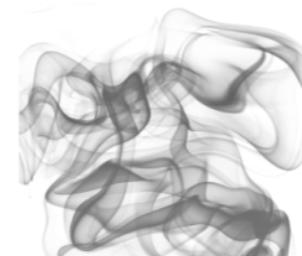
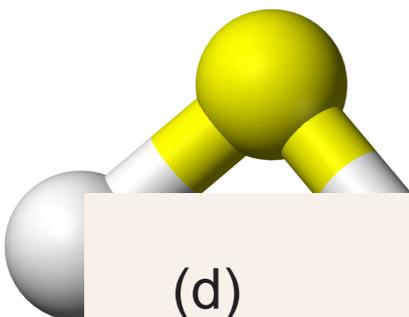
Received  
7 July 2014

<sup>1</sup>State Key Laboratory of Superhard Materials, College of physics, Jilin University, Changchun, 130012, P. R. China, <sup>2</sup>State Key Laboratory of Supramolecular Structure and Materials, Jilin University, Changchun, 130012, P. R. China.

# Hydrogen sulfide: the chemistry changes

Discovered in 1777

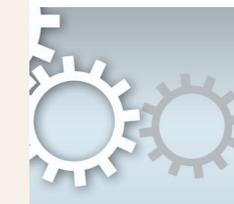
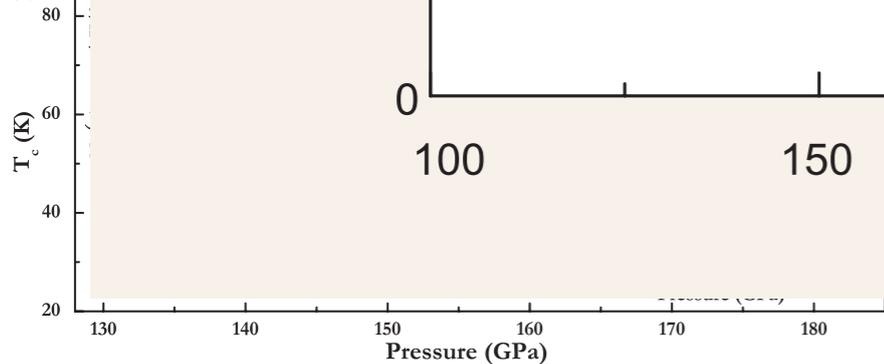
It is a colorless gas with the characteristic foul odor of



## The meta

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<sup>3</sup>State  
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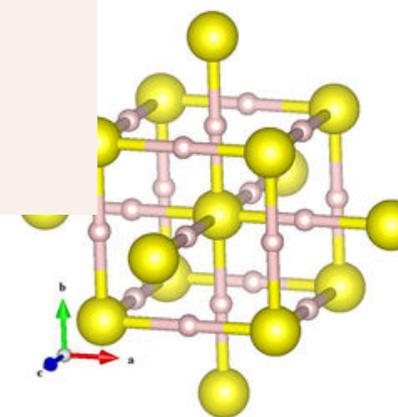
(a)



dense  
 ivity

Hongyu Yu<sup>1</sup>,

<sup>1</sup>P. R. China, <sup>2</sup>State Key



# Eremets's experiment

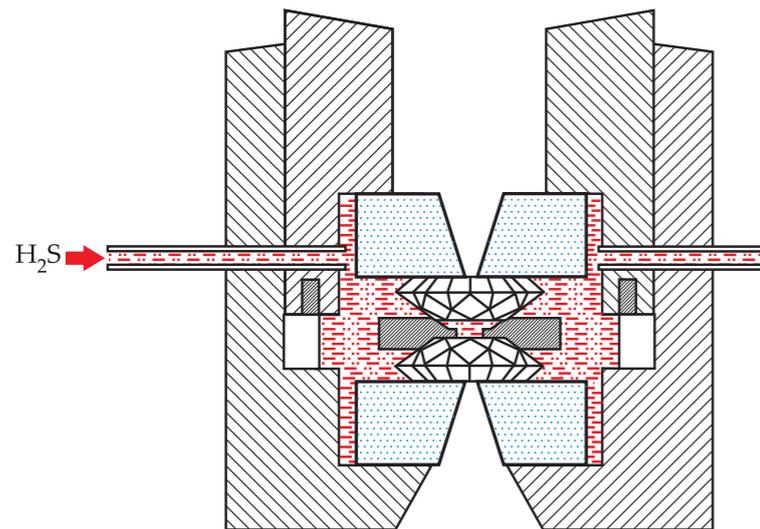
Max-Planck-Institut für Chemie (Mainz), Germany

## LETTER

doi:10.1038/nature14964

### Conventional superconductivity at 203 kelvin at high pressures in the sulfur hydride system

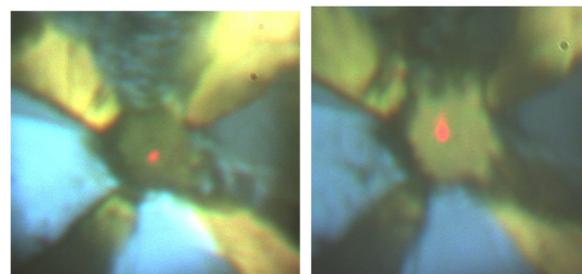
A. P. Drozdov<sup>1\*</sup>, M. I. Eremets<sup>1\*</sup>, I. A. Troyan<sup>1</sup>, V. Ksenofontov<sup>2</sup> & S. I. Shylin<sup>2</sup>



9 GPa

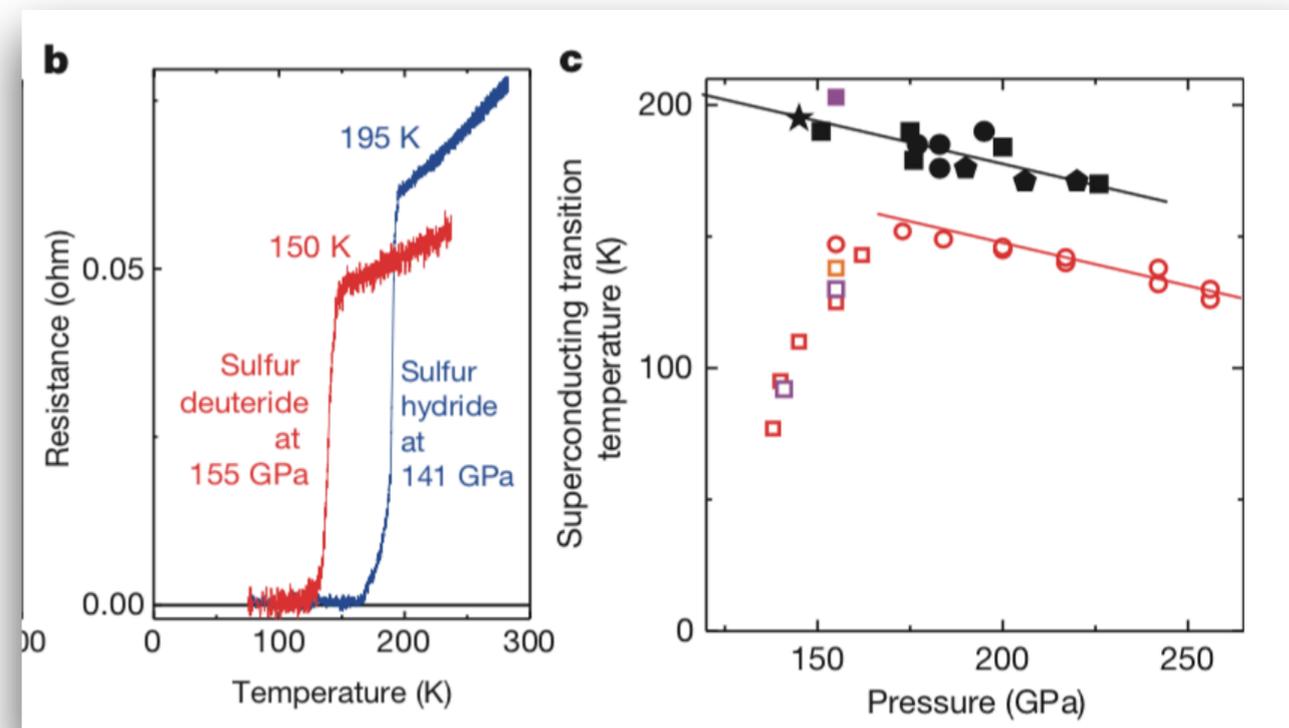
11 GPa

79 GPa



92 GPa

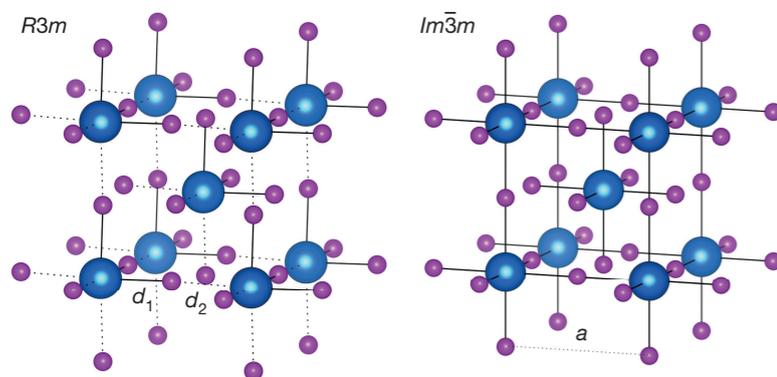
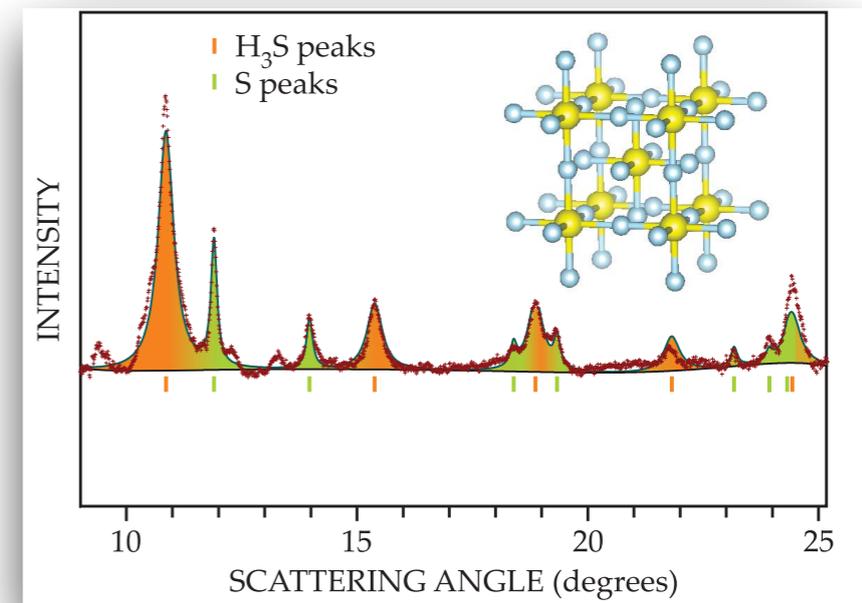
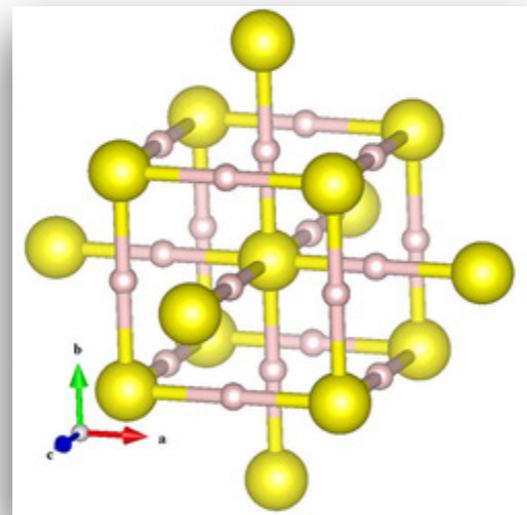
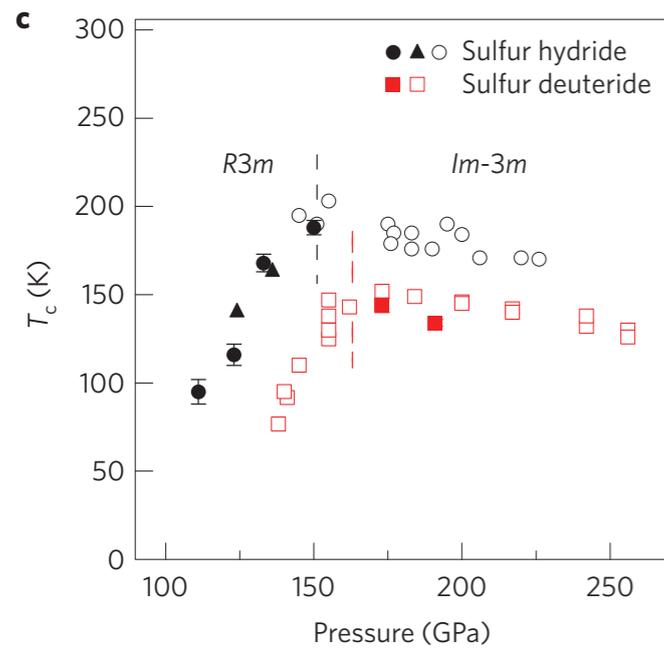
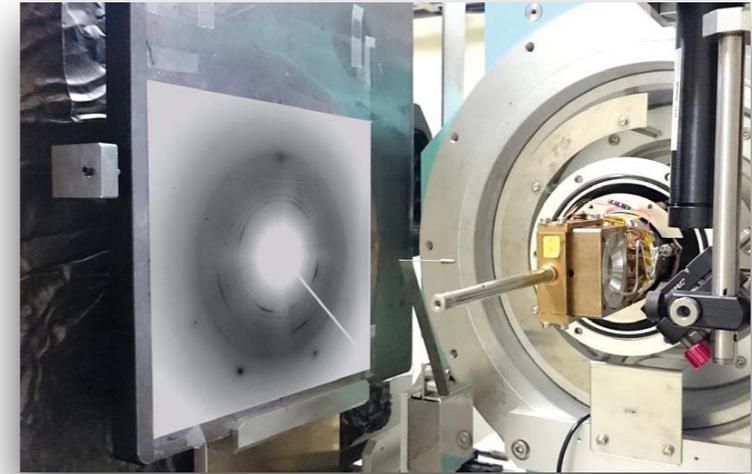
154 GPa



# It is true

## Crystal structure of the superconducting phase of sulfur hydride

Mari Einaga<sup>1\*</sup>, Masafumi Sakata<sup>1</sup>, Takahiro Ishikawa<sup>1</sup>, Katsuya Shimizu<sup>1†</sup>, Mikhail I. Erements<sup>2†</sup>, Alexander P. Drozdov<sup>2</sup>, Ivan A. Troyan<sup>2</sup>, Naohisa Hirao<sup>3</sup> and Yasuo Ohishi<sup>3</sup>



# It is true

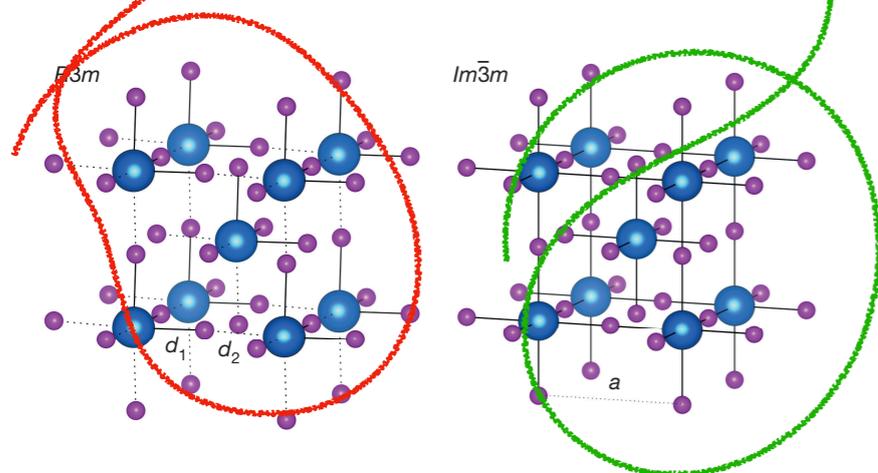
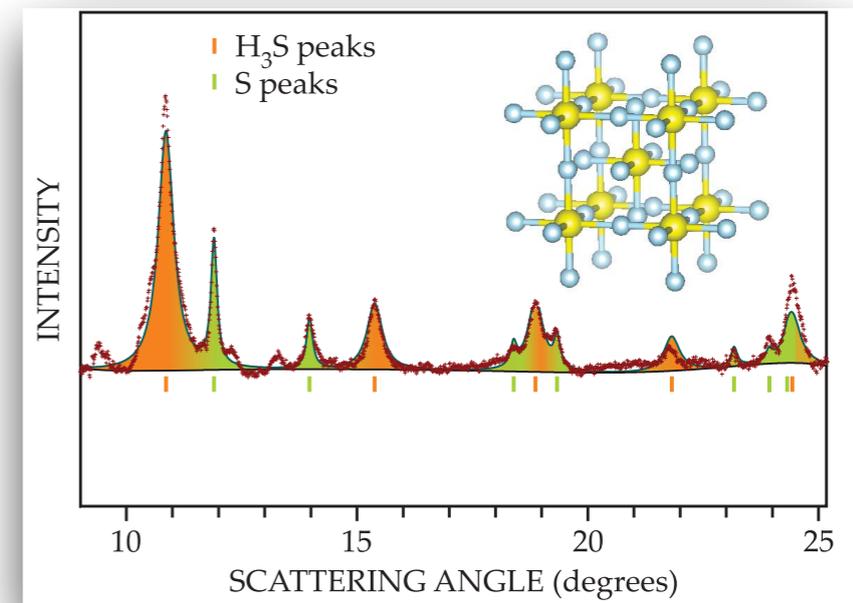
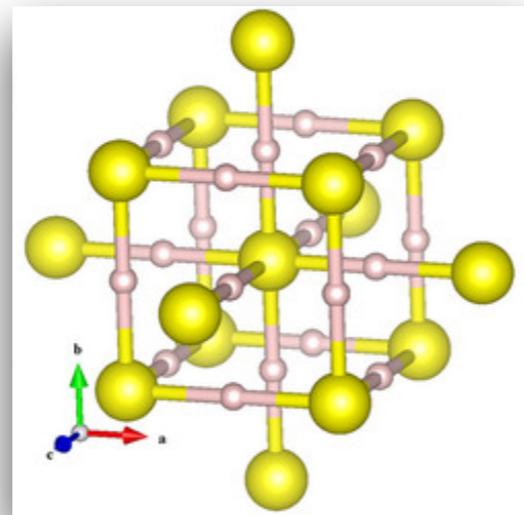
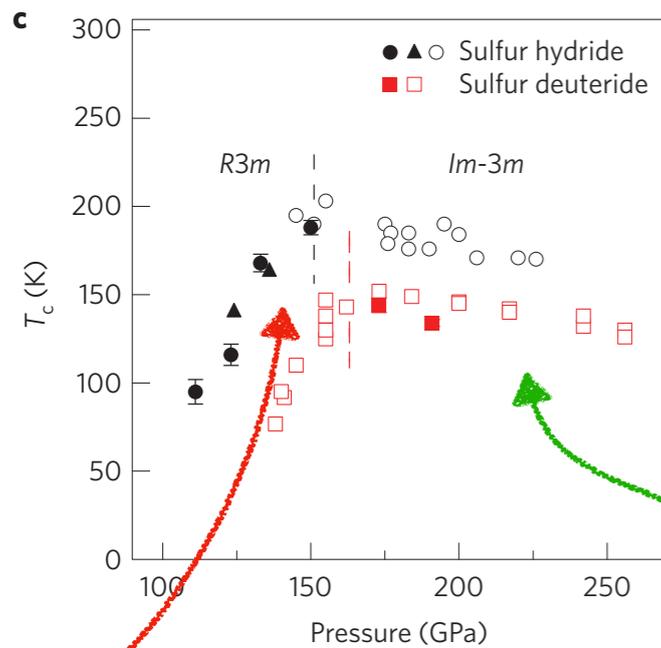
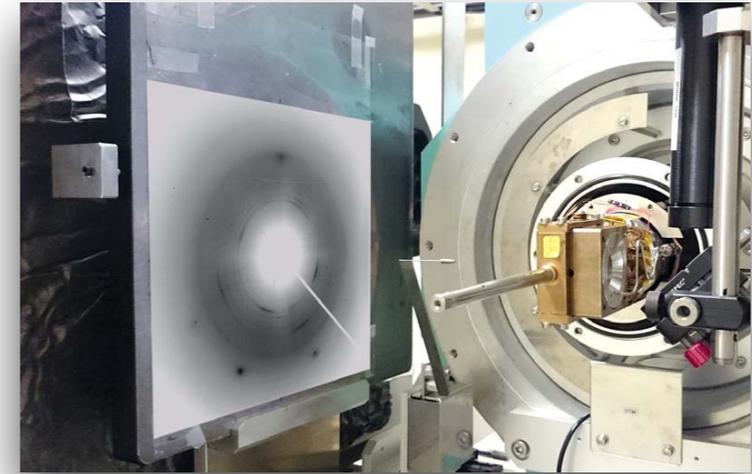
nature  
physics

LETTERS

PUBLISHED ONLINE: 9 MAY 2016 | DOI: 10.1038/NPHYS3760

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First-principles theories predict the **crystal structure** and **superconducting critical temperature** as a function of the pressure

Room temperature superconductivity? It depends on where the room is.



Vostok base in Antarctica

In 1983 a temperature of  $-89.2\text{ }^{\circ}\text{C}$  was registered

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# Flurry of predictions

Periodic table of binary hydride superconductors

H																	He
LiH <sub>6</sub> 82	BeH <sub>2</sub> 44											BH 21	C	N	O	F	Ne
Na	MgH <sub>4</sub> 30											AlH <sub>5</sub> 140	SiH <sub>x</sub> ~20	PH <sub>2</sub> 87	SH <sub>3</sub> 204	Cl	Ar
KH <sub>10</sub> 140	CaH <sub>6</sub> 235	ScH <sub>9</sub> 233	TiH <sub>14</sub> 54	VH <sub>8</sub> 72	CrH <sub>3</sub> 81	Mn	Fe	Co	Ni	Cu	Zn	GaH <sub>3</sub> 123	GeH <sub>4</sub> 220	AsH <sub>4</sub> 90	SeH <sub>3</sub> 120	BrH <sub>2</sub> 12	Kr
Rb	SrH <sub>10</sub> 259	YH <sub>10</sub> 326	ZrH <sub>14</sub> 88	NbH <sub>4</sub> 47	Mo	TcH <sub>2</sub> 11	RuH <sub>3</sub> 1.3	RhH 2.5	PdH 5	Ag	Cd	InH <sub>3</sub> 41	SnH <sub>14</sub> 90	SbH <sub>4</sub> 95	TeH <sub>4</sub> 100	IH <sub>2</sub> 30	XeH 29
Cs	BaH <sub>6</sub> 38		HfH <sub>2</sub> 76	TaH <sub>6</sub> 136	WH <sub>5</sub> 60	Re	OsH 2	IrH 7	PtH 25	AuH 21	Hg	Tl	PbH <sub>8</sub> 107	BiH <sub>5</sub> 110	PoH <sub>4</sub> 50	At	Rn
FrH <sub>7</sub> 63	RaH <sub>12</sub> 116		Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
Lanthanides	LaH <sub>10</sub> 288	CeH <sub>8</sub> 117	PrH <sub>8</sub> 31	NdH <sub>8</sub> 6	Pm	Sm	Eu	Gd	Tb	Dy	HoH <sub>4</sub> 37	ErH <sub>15</sub> 30	TmH <sub>8</sub> 21	Yb	LuH <sub>12</sub> 7		
Actinides	AcH <sub>10</sub> 250	ThH <sub>10</sub> 221	PaH <sub>9</sub> 62	UH <sub>8</sub> 35	NpH <sub>7</sub> 10	Pu	AmH <sub>8</sub> 0.3	CmH <sub>8</sub> 0.9	Bk	Cf	Es	Fm	Md	No	Lr		

T<sub>c</sub> (K) Theoretically predicted

# Potential high- $T_c$ superconducting lanthanum and yttrium hydrides at high pressure

Hanyu Liu<sup>a</sup>, Ivan I. Naumov<sup>a</sup>, Roald Hoffmann<sup>b</sup>, N. W. Ashcroft<sup>c</sup>, and Russell J. Hemley<sup>d,e,1</sup>

PRL 119, 107001 (2017)

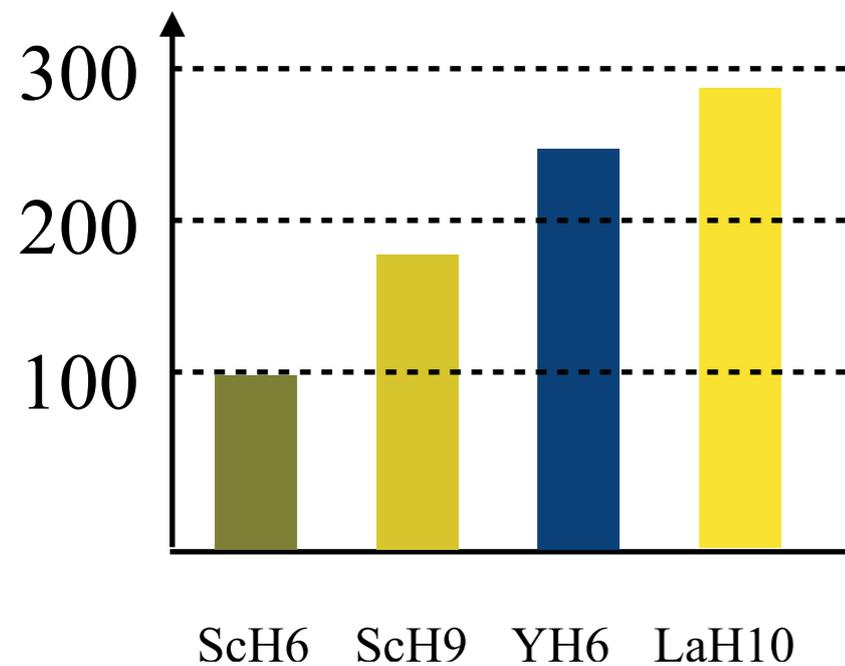
PHYSICAL REVIEW LETTERS

week ending  
8 SEPTEMBER 2017

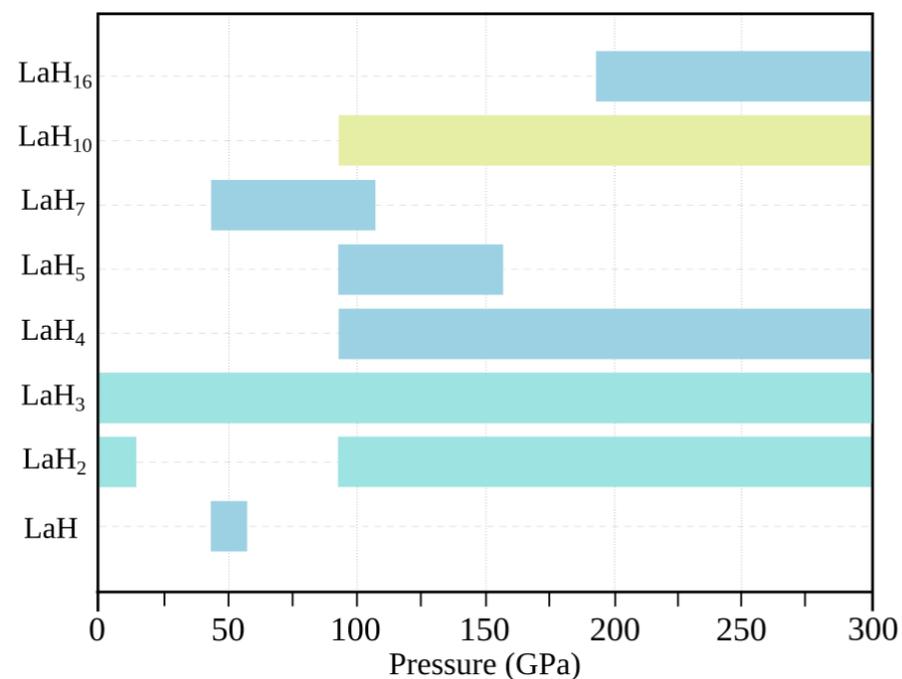
## Hydrogen Clathrate Structures in Rare Earth Hydrides at High Pressures: Possible Route to Room-Temperature Superconductivity

Feng Peng,<sup>1,2,3</sup> Ying Sun,<sup>3</sup> Chris J. Pickard,<sup>4</sup> Richard J. Needs,<sup>5</sup> Qiang Wu,<sup>6</sup> and Yanming Ma<sup>3,7,\*</sup>

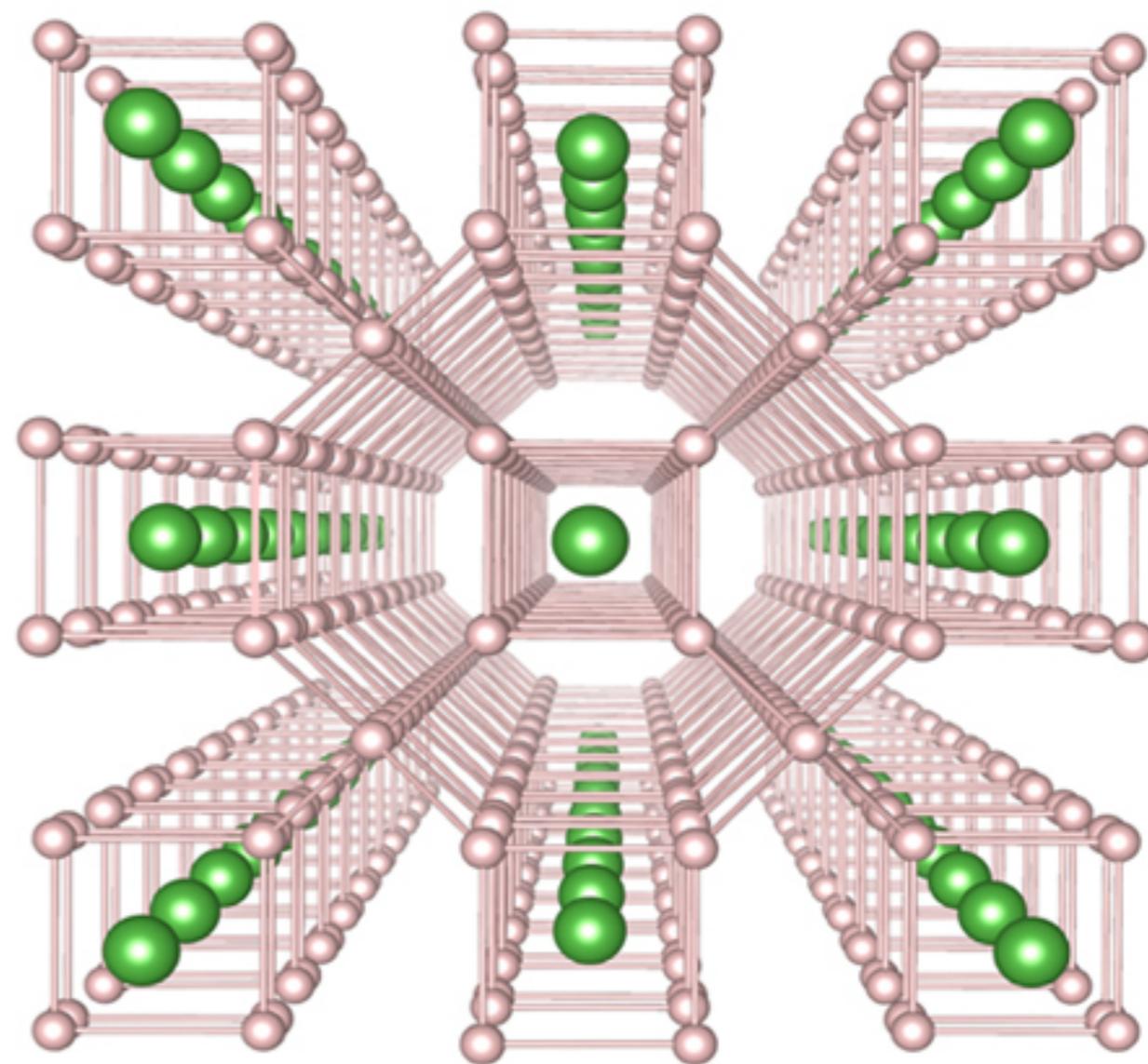
$T_c=260$  K



Stability range

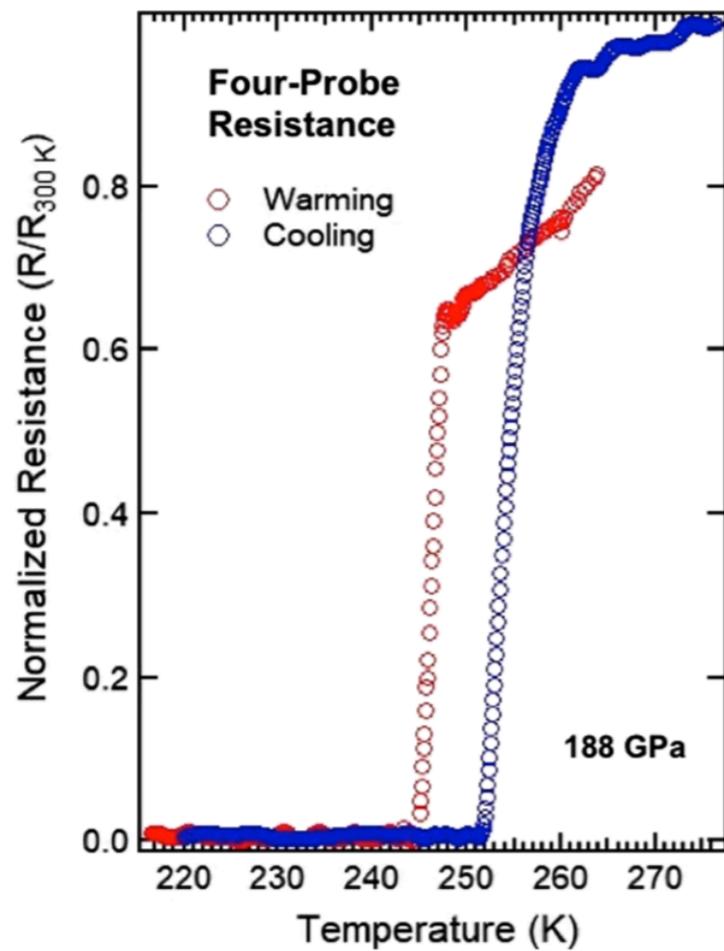
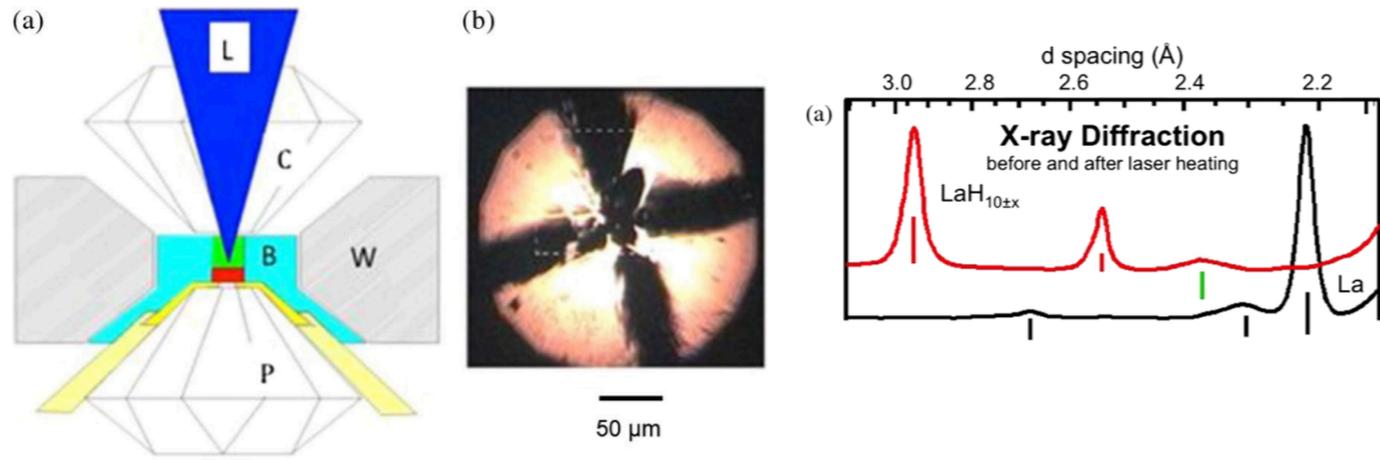


# LaH<sub>10</sub>



### Evidence for Superconductivity above 260 K in Lanthanum Superhydride at Megabar Pressures

Maddury Somayazulu,<sup>1,\*</sup> Muhtar Ahart,<sup>1</sup> Ajay K. Mishra,<sup>2,‡</sup> Zachary M. Geballe,<sup>2</sup> Maria Baldini,<sup>2,§</sup> Yue Meng,<sup>3</sup> Viktor V. Struzhkin,<sup>2</sup> and Russell J. Hemley<sup>1,†</sup>



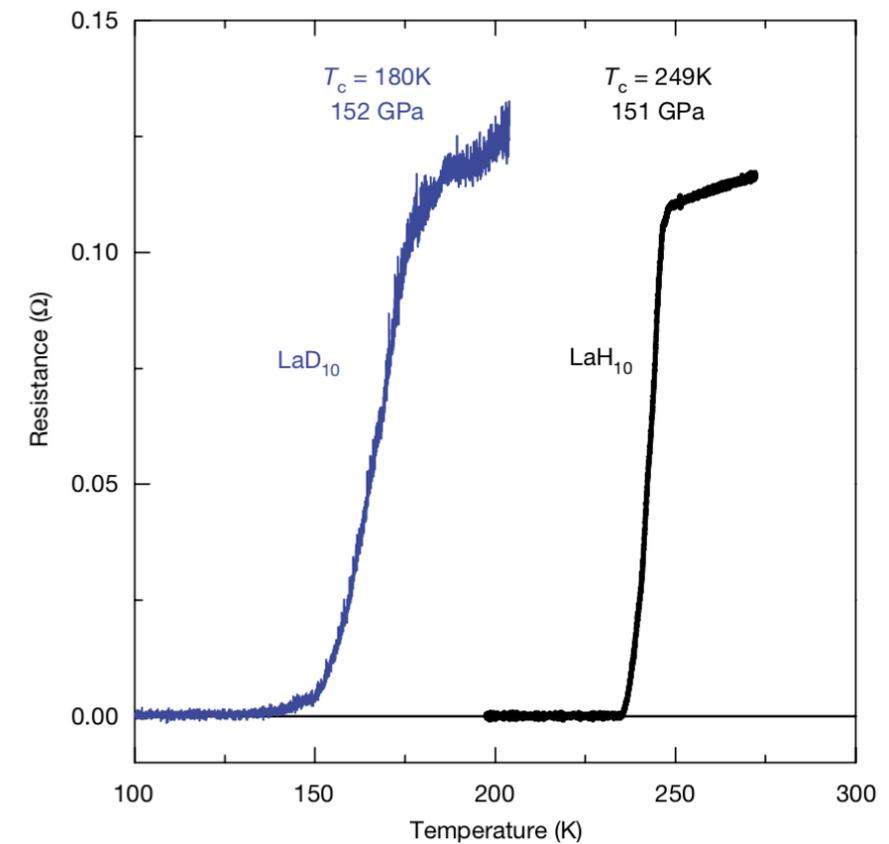
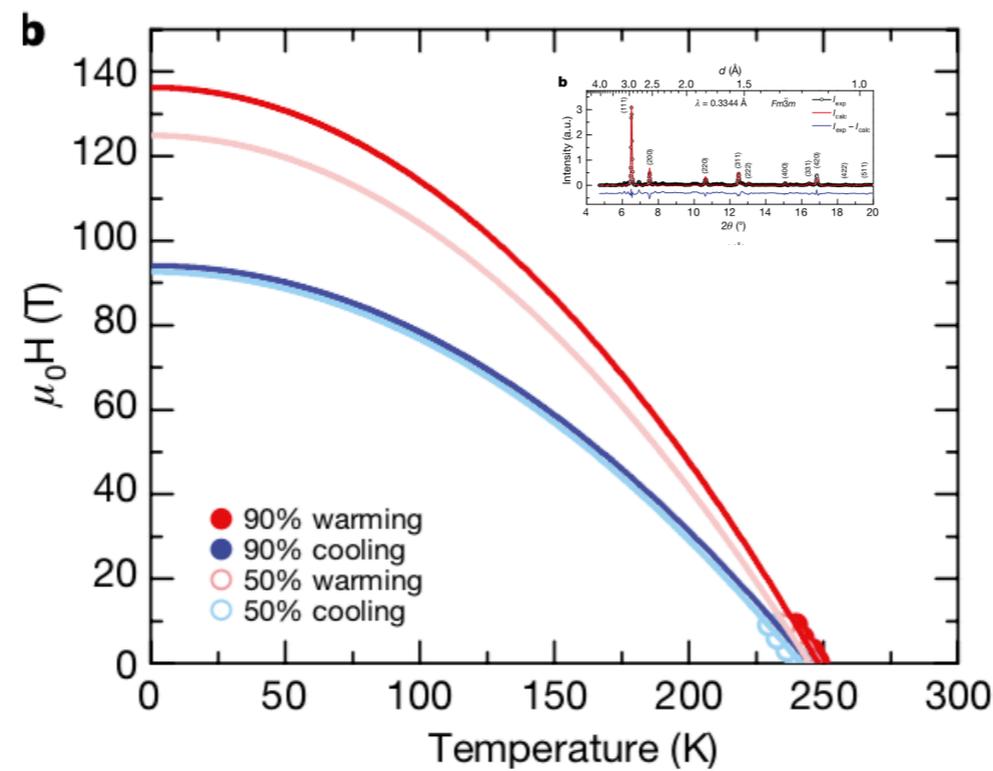
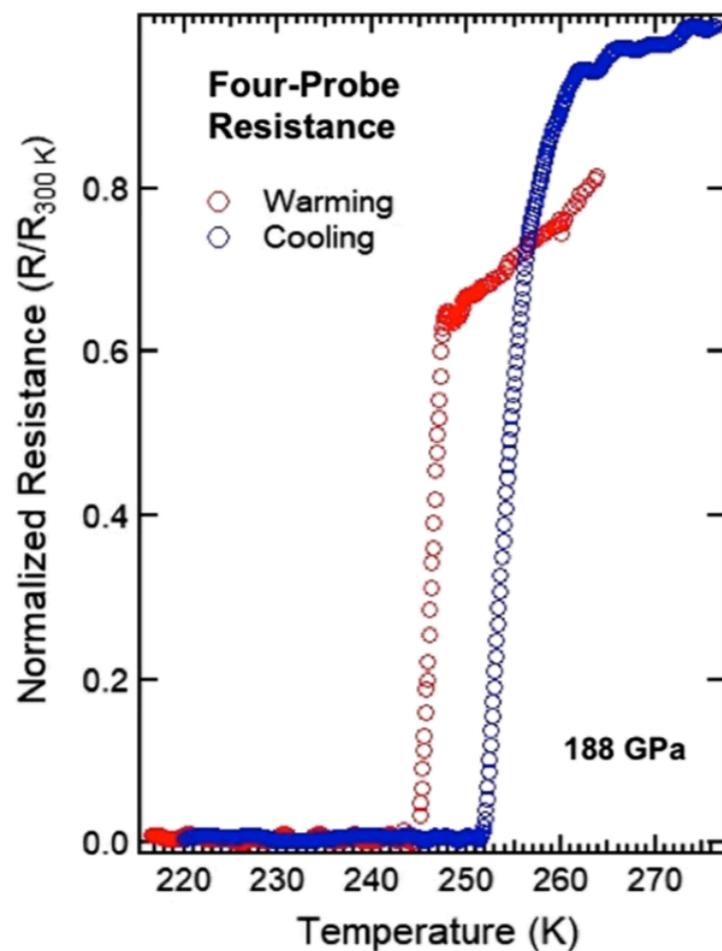
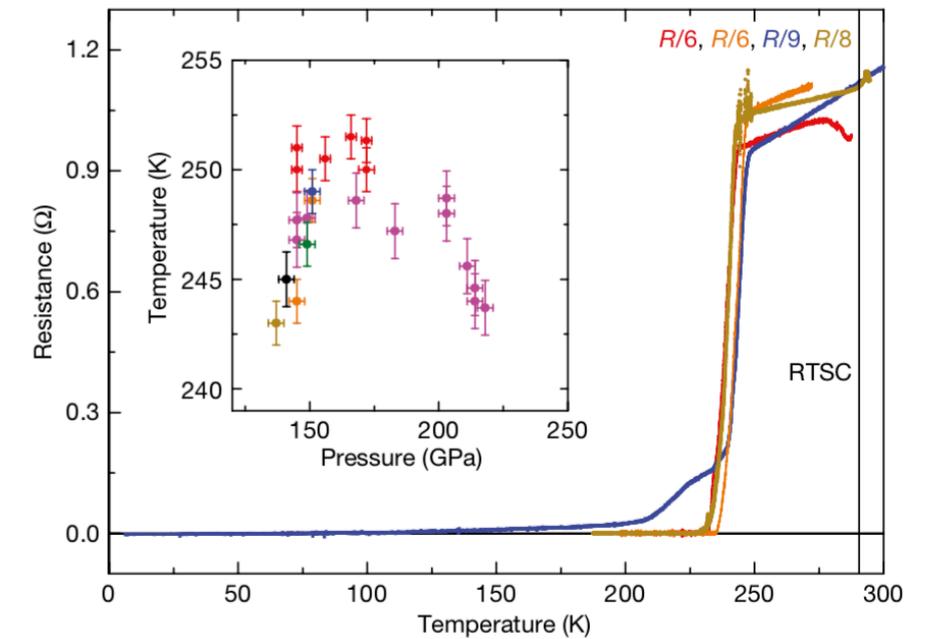
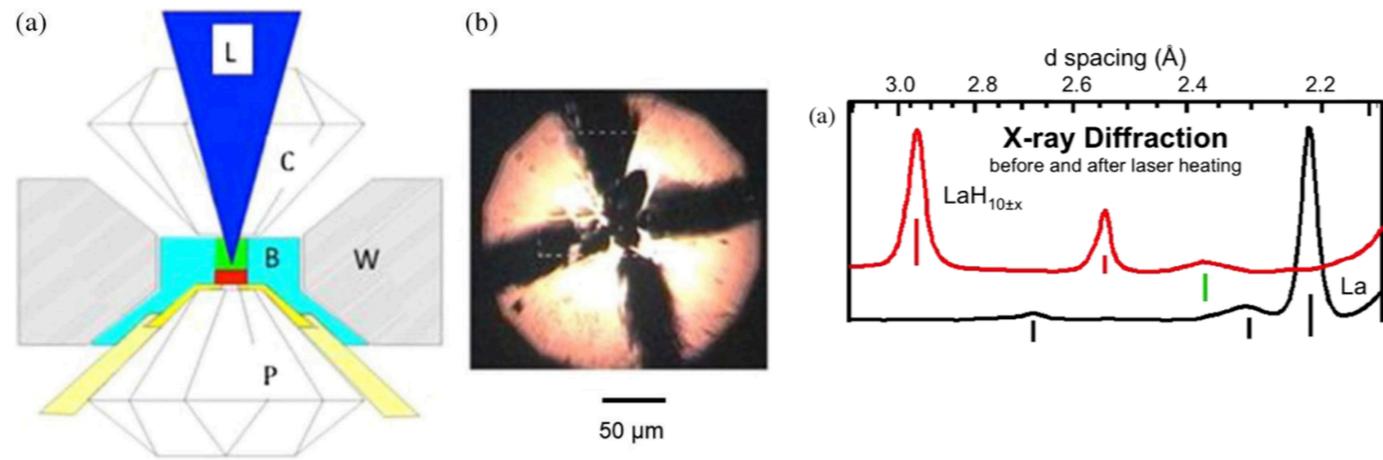
Editors' Suggestion    Featured in Physics

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### Superconductivity at 250 K in lanthanum hydride under high pressures

A. P. Drozdov<sup>1,7</sup>, P. P. Kong<sup>1,7</sup>, V. S. Minkov<sup>1,7</sup>, S. P. Besedin<sup>1,7</sup>, M. A. Kuzovnikov<sup>1,6,7</sup>, S. Mozaffari<sup>2</sup>, L. Balicas<sup>2</sup>, F. F. Balakirev<sup>3</sup>, D. E. Graf<sup>2</sup>, V. B. Prakapenka<sup>4</sup>, E. Greenberg<sup>4</sup>, D. A. Knyazev<sup>1</sup>, M. Tkacz<sup>5</sup> & M. I. Erements<sup>1,\*</sup>



# Quantum crystal structure in the 250-kelvin superconducting lanthanum hydride

<https://doi.org/10.1038/s41586-020-1955-z>

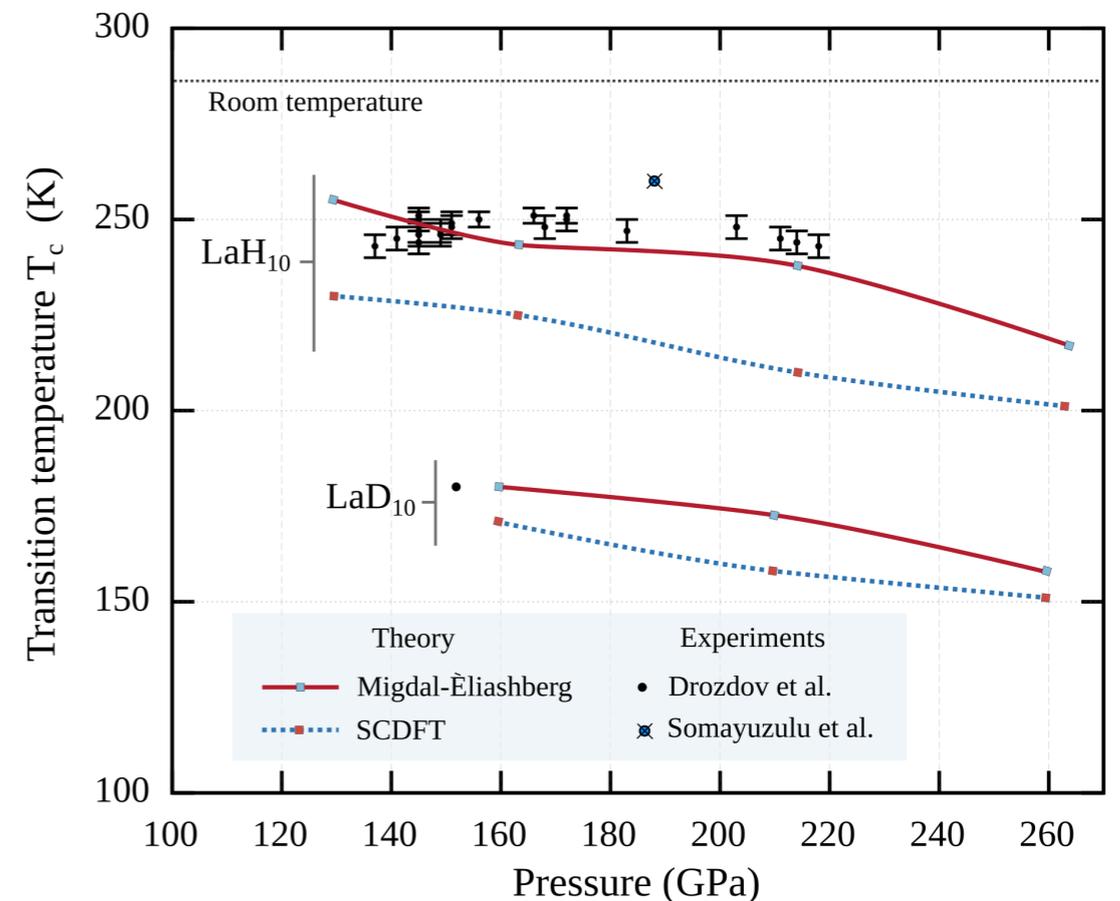
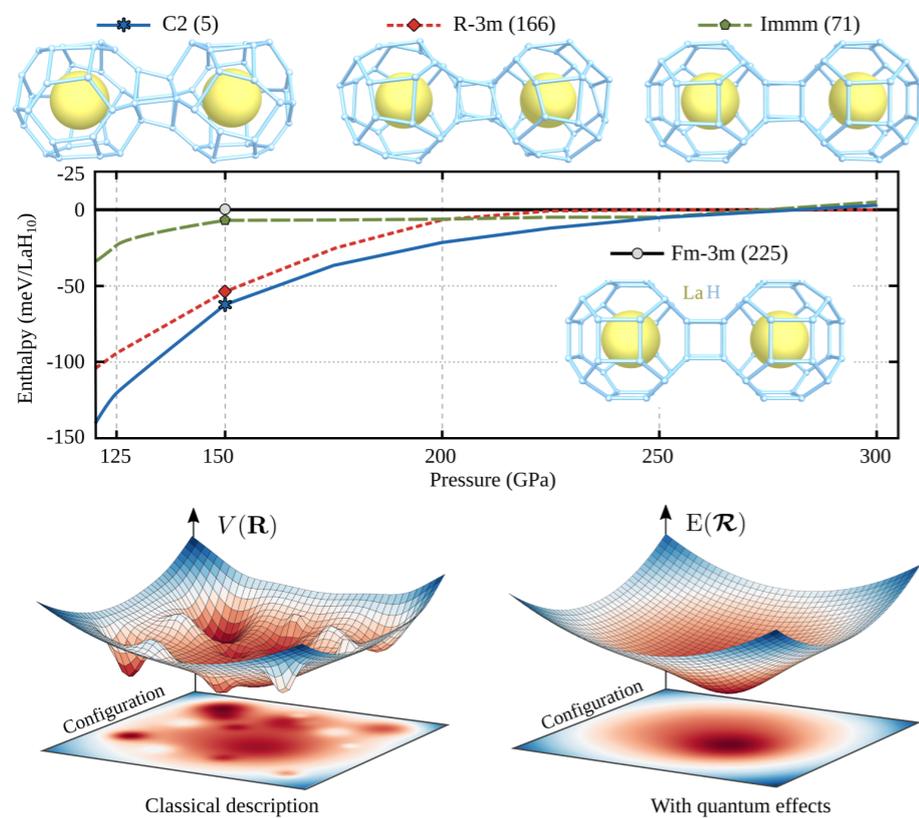
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Ion Errea<sup>1,2,3</sup>, Francesco Belli<sup>1,2</sup>, Lorenzo Monacelli<sup>4</sup>, Antonio Sanna<sup>5</sup>, Takashi Koretsune<sup>6</sup>, Terumasa Tadano<sup>7</sup>, Raffaello Bianco<sup>2</sup>, Matteo Calandra<sup>8</sup>, Ryotaro Arita<sup>9,10</sup>, Francesco Mauri<sup>4,11</sup> & José A. Flores-Livas<sup>4\*</sup>

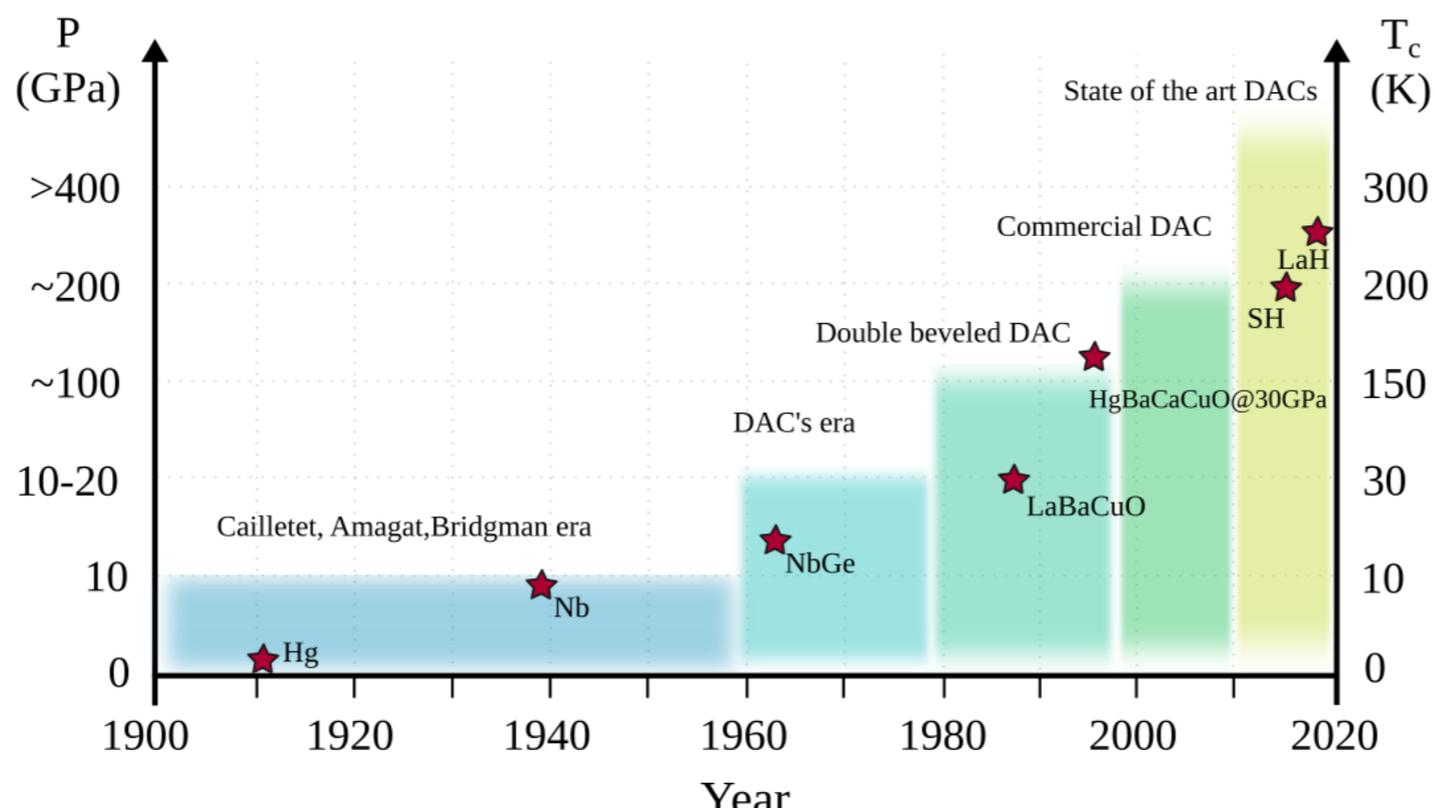
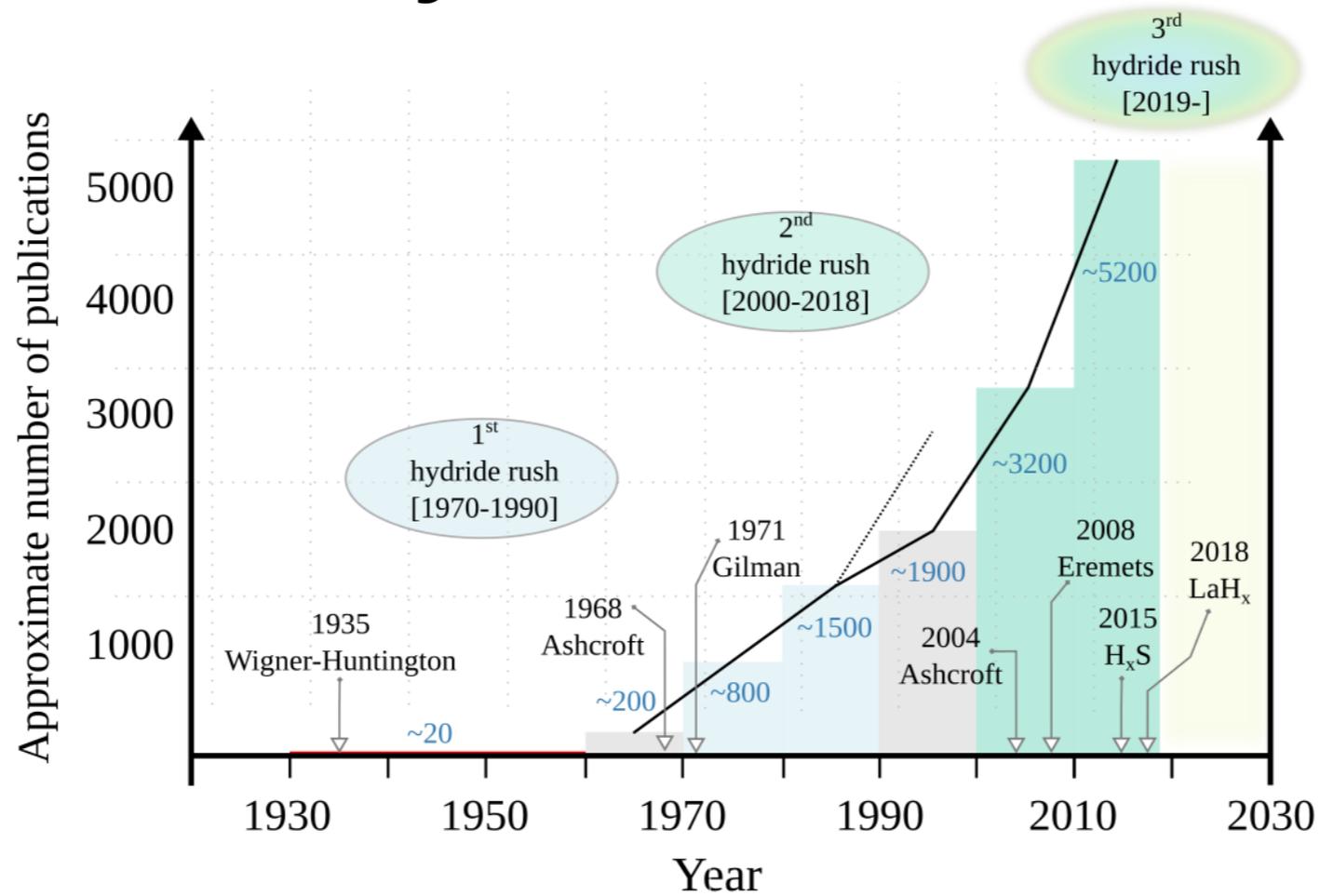
The discovery of superconductivity at 200 kelvin in the hydrogen sulfide system at high pressures<sup>1</sup> demonstrated the potential of hydrogen-rich materials as high-temperature superconductors. Recent theoretical predictions of rare-earth hydrides with hydrogen cages<sup>2,3</sup> and the subsequent synthesis of LaH<sub>10</sub> with a superconducting critical temperature ( $T_c$ ) of 250 kelvin<sup>4,5</sup> have placed these materials on the verge of achieving the long-standing goal of room-temperature superconductivity. Electrical and X-ray diffraction measurements have revealed a weakly pressure-dependent  $T_c$  for LaH<sub>10</sub> between 137 and 218 gigapascals in a structure that has a face-centred cubic arrangement of lanthanum atoms<sup>5</sup>. Here we show that quantum atomic fluctuations stabilize a highly symmetrical  $Fm\bar{3}m$  crystal structure over this pressure range. The



# Room temperature superconductivity

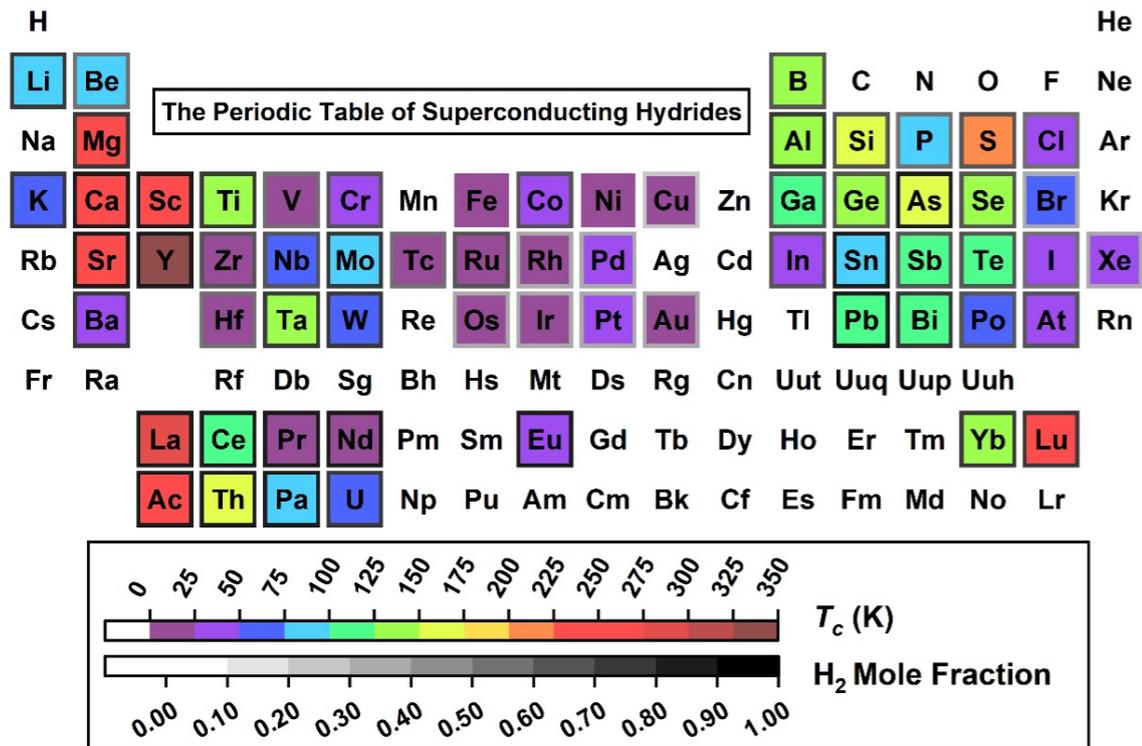


# Hydride revolution



Better call....

# Better call....



# Better call....

Periodic table of binary hydride superconductors

H																	He
LiH <sub>6</sub> 82	BeH <sub>2</sub> 44											BH 21	C	N	O	F	Ne
Na	MgH <sub>4</sub> 30											AlH <sub>5</sub> 140	SiH <sub>x</sub> ~20	PH <sub>2</sub> 90	SH <sub>3</sub> 200	Cl	Ar
KH <sub>10</sub> 140	CaH <sub>6</sub> 235	ScH <sub>9</sub> 233	TiH <sub>14</sub> 54	VH <sub>8</sub> 72	CrH <sub>3</sub> 81	Mn	Fe	Co	Ni	Cu	Zn	GaH <sub>3</sub> 123	GeH <sub>4</sub> 220	AsH <sub>4</sub> 90	SeH <sub>3</sub> 120	BrH <sub>2</sub> 12	Kr
Rb	SrH <sub>10</sub> 259	YH <sub>10</sub> 240	ZrH <sub>14</sub> 88	NbH <sub>4</sub> 47	Mo	TcH <sub>2</sub> 11	RuH <sub>3</sub> 1.3	RhH 2.5	PdH 5	Ag	Cd	InH <sub>3</sub> 41	SnH <sub>14</sub> 90	SbH <sub>4</sub> 95	TeH <sub>4</sub> 100	IH <sub>2</sub> 30	XeH 29
Cs	BaH <sub>6</sub> 38		HfH <sub>2</sub> 76	TaH <sub>6</sub> 136	WH <sub>5</sub> 60	Re	OsH 2	IrH 7	PtH 25	AuH 21	Hg	Tl	PbH <sub>8</sub> 107	BiH <sub>5</sub> 110	PoH <sub>4</sub> 50	At	Rn
FrH <sub>7</sub> 63	RaH <sub>12</sub> 116		Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
Lanthanides	LaH <sub>10</sub> 250	CeH <sub>8</sub> 117	PrH <sub>8</sub> 31	NdH <sub>8</sub> 6	Pm	Sm	Eu	Gd	Tb	Dy	HoH <sub>4</sub> 37	ErH <sub>15</sub> 30	TmH <sub>8</sub> 21	Yb	LuH <sub>12</sub> 7		
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T<sub>c</sub> (K) Experimentally confirmed

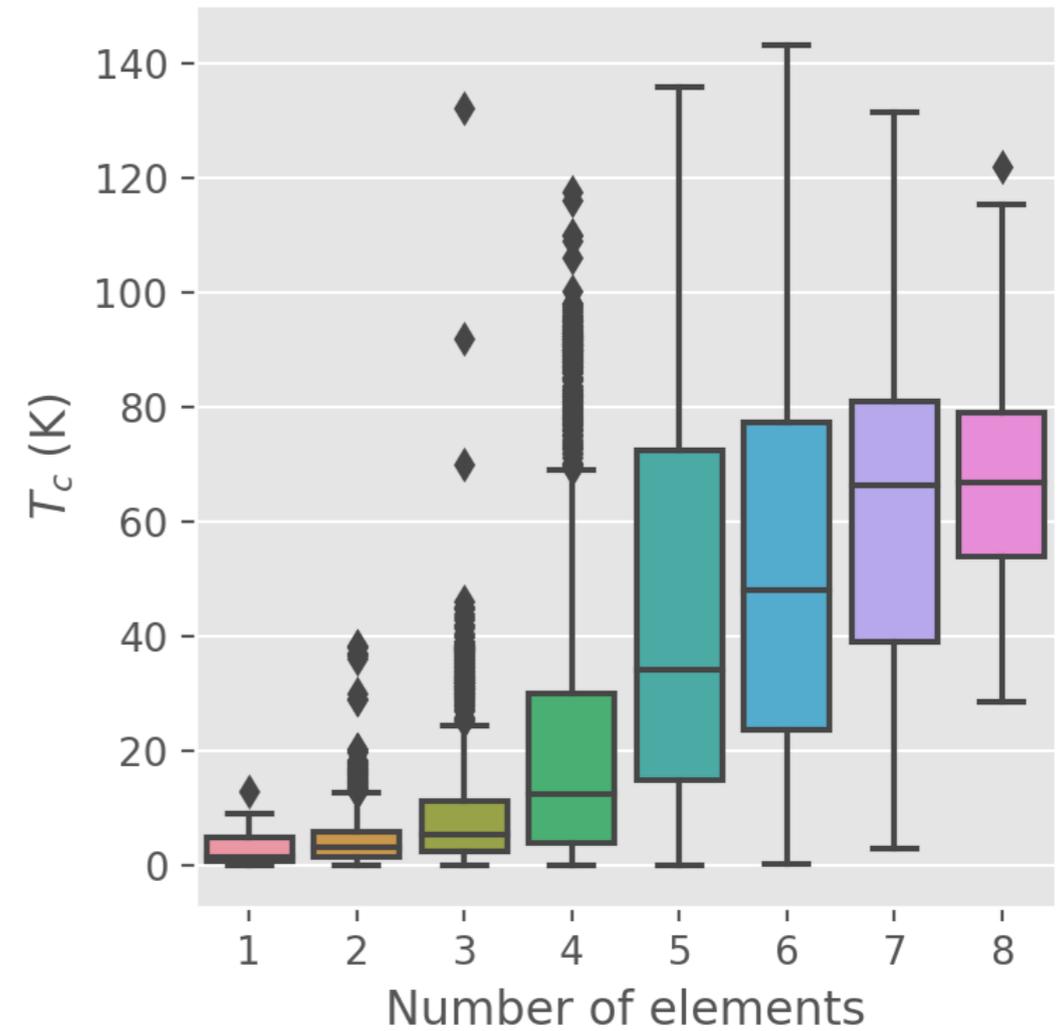
T<sub>c</sub> (K) Theoretically predicted

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# Another actor: Al

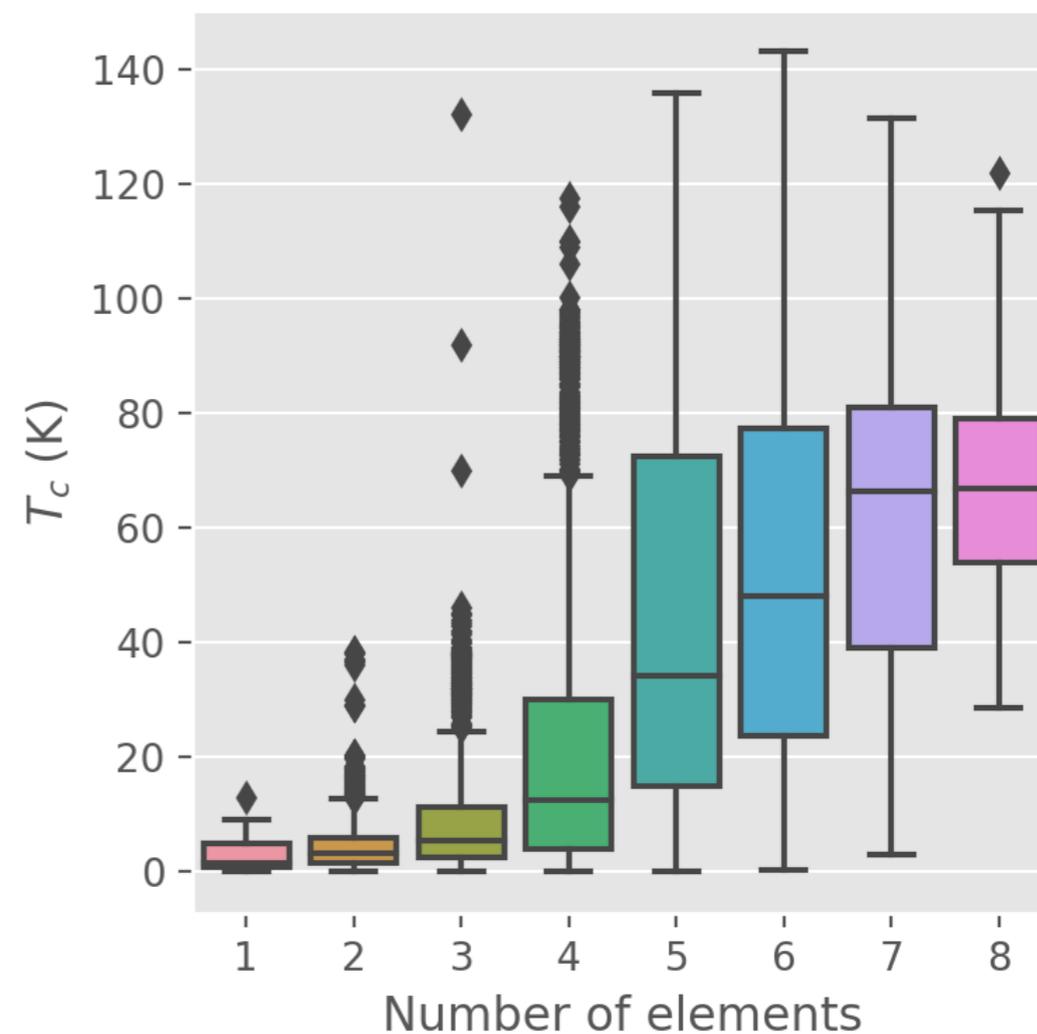
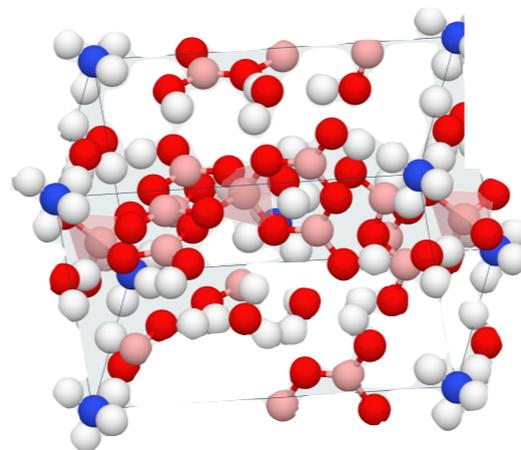
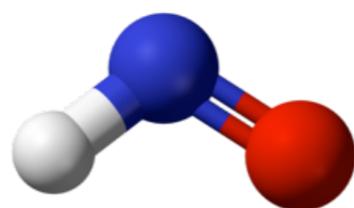
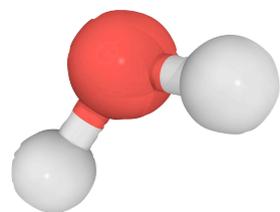
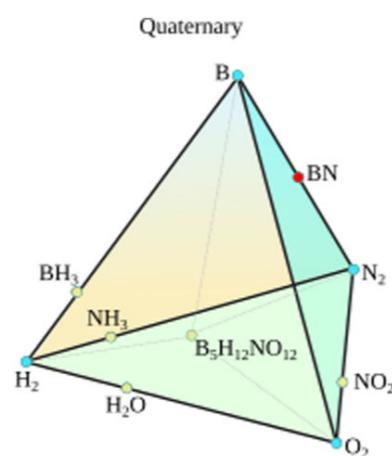
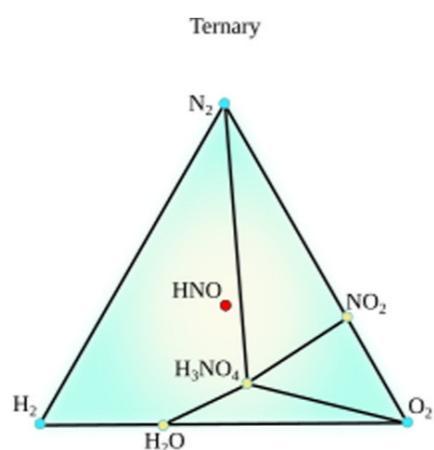
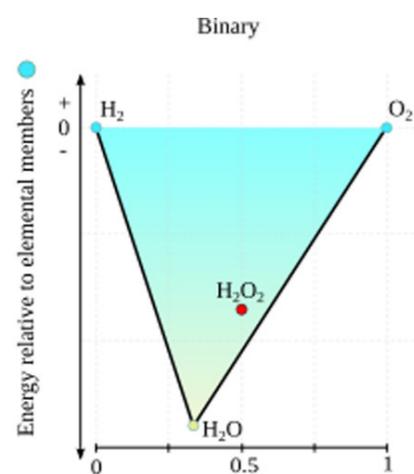


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# Another actor: Al



# Room-temperature superconductivity in a carbonaceous sulfur hydride

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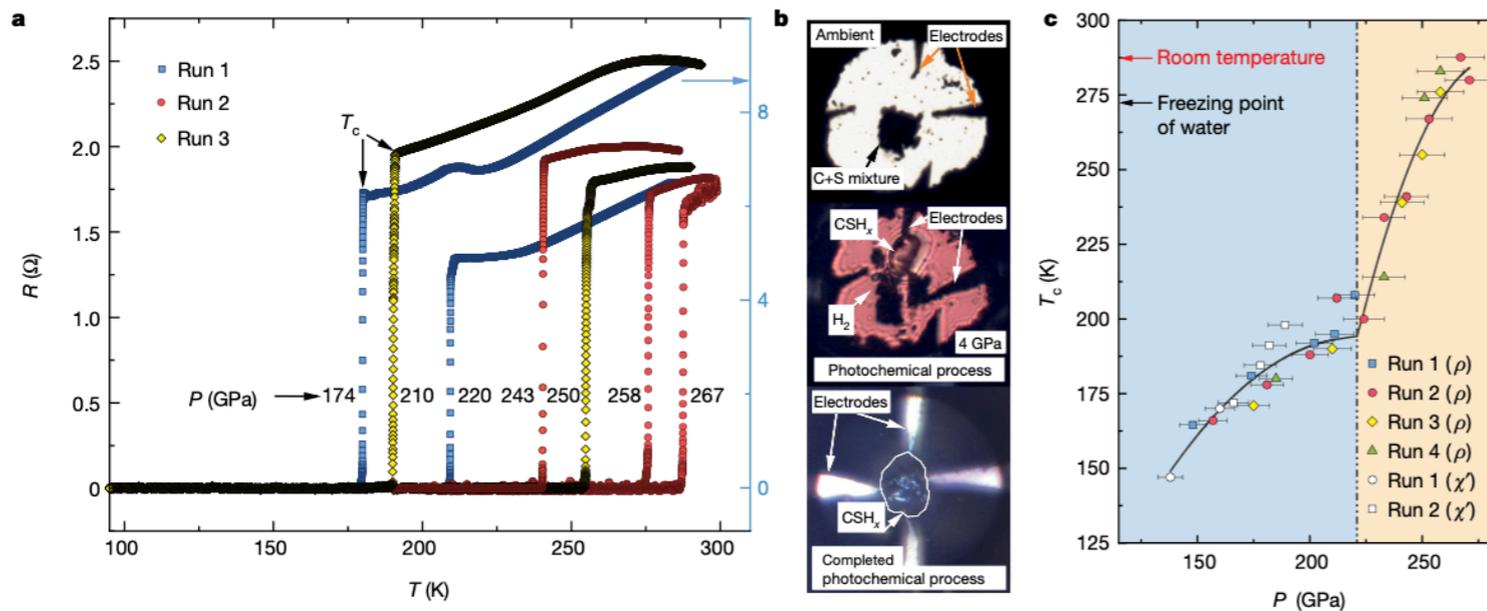
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Elliot Snider<sup>1,6</sup>, Nathan Dasenbrock-Gammon<sup>2,6</sup>, Raymond McBride<sup>1,6</sup>, Mathew Debessai<sup>3</sup>, Hiranya Vindana<sup>2</sup>, Kevin Vencatasamy<sup>2</sup>, Keith V. Lawler<sup>4</sup>, Ashkan Salamat<sup>5</sup> & Ranga P. Dias<sup>1,2,6</sup>

One of the long-standing challenges in experimental physics is the observation of room-temperature superconductivity<sup>1,2</sup>. Recently, high-temperature conventional superconductivity in hydrogen-rich materials has been reported in several systems under high pressure<sup>3–5</sup>. An important discovery leading to room-temperature superconductivity is the pressure-driven disproportionation of hydrogen sulfide ( $\text{H}_2\text{S}$ ) to  $\text{H}_3\text{S}$ , with a confirmed transition temperature of 203 kelvin at 155 gigapascals<sup>3,6</sup>. Both  $\text{H}_2\text{S}$  and  $\text{CH}_4$  readily mix with hydrogen to form guest–host structures at lower pressures<sup>7</sup>, and are of comparable size at 4 gigapascals. By introducing methane at low pressures into the  $\text{H}_2\text{S} + \text{H}_2$  precursor mixture for  $\text{H}_3\text{S}$ , molecular exchange is allowed within a large assemblage of van der Waals solids that are hydrogen-rich with  $\text{H}_2$  inclusions; these guest–host structures become the building blocks of superconducting compounds at extreme conditions. Here we report superconductivity in a photochemically transformed carbonaceous sulfur hydride system, starting from elemental precursors, with a maximum superconducting transition temperature of  $287.7 \pm 1.2$  kelvin (about 15 degrees Celsius) achieved at  $267 \pm 10$  gigapascals. The superconducting state is observed over a broad pressure range in the diamond anvil cell, from 140 to 275 gigapascals, with a sharp upturn in transition temperature above 220 gigapascals. Superconductivity is established by the observation of zero resistance, a magnetic susceptibility of up to 190 gigapascals, and reduction of the transition temperature under an external magnetic field of up to 9 tesla, with an upper critical magnetic field of about 62 tesla according to the Ginzburg–Landau model at zero temperature. The light, quantum nature of hydrogen limits the structural and stoichiometric determination of the system by X-ray scattering techniques, but Raman spectroscopy is used to probe the chemical and structural transformations before metallization. The introduction of chemical tuning within our ternary system could enable the preservation of the properties of room-temperature superconductivity at lower pressures.



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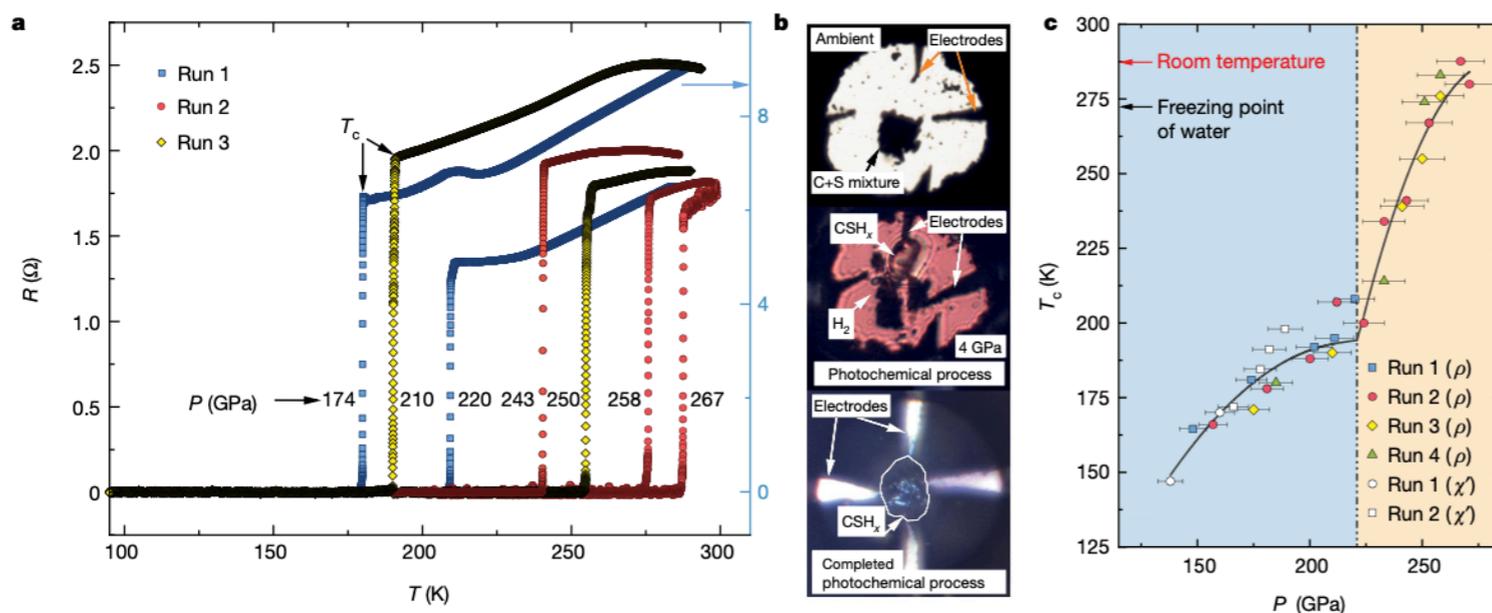
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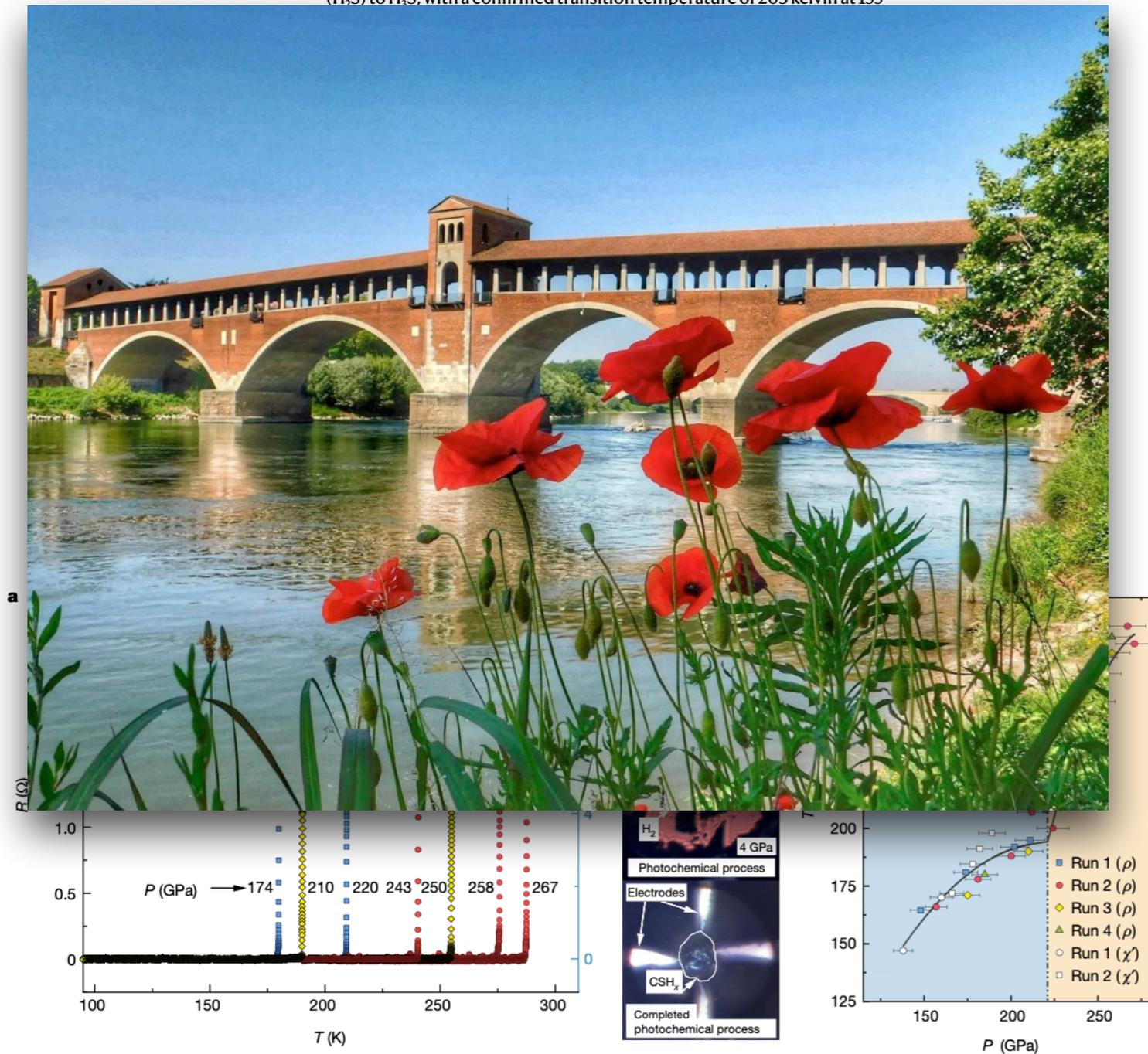
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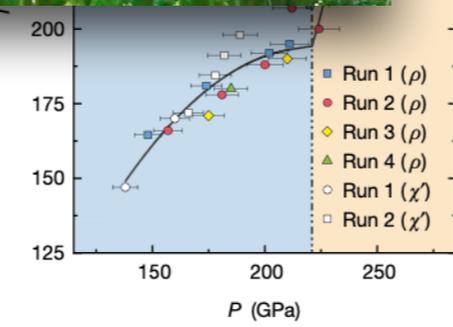
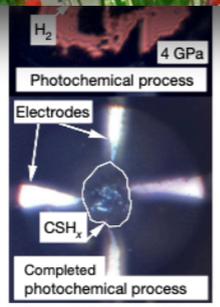
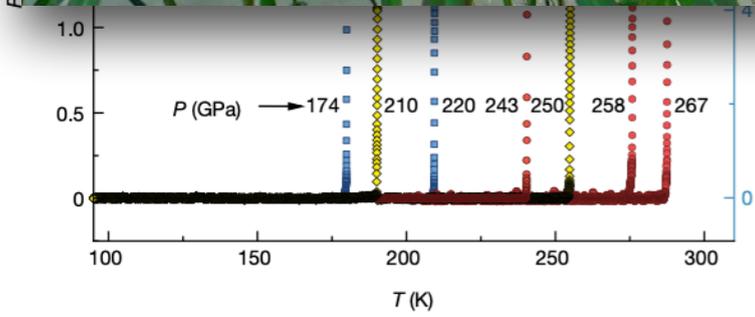
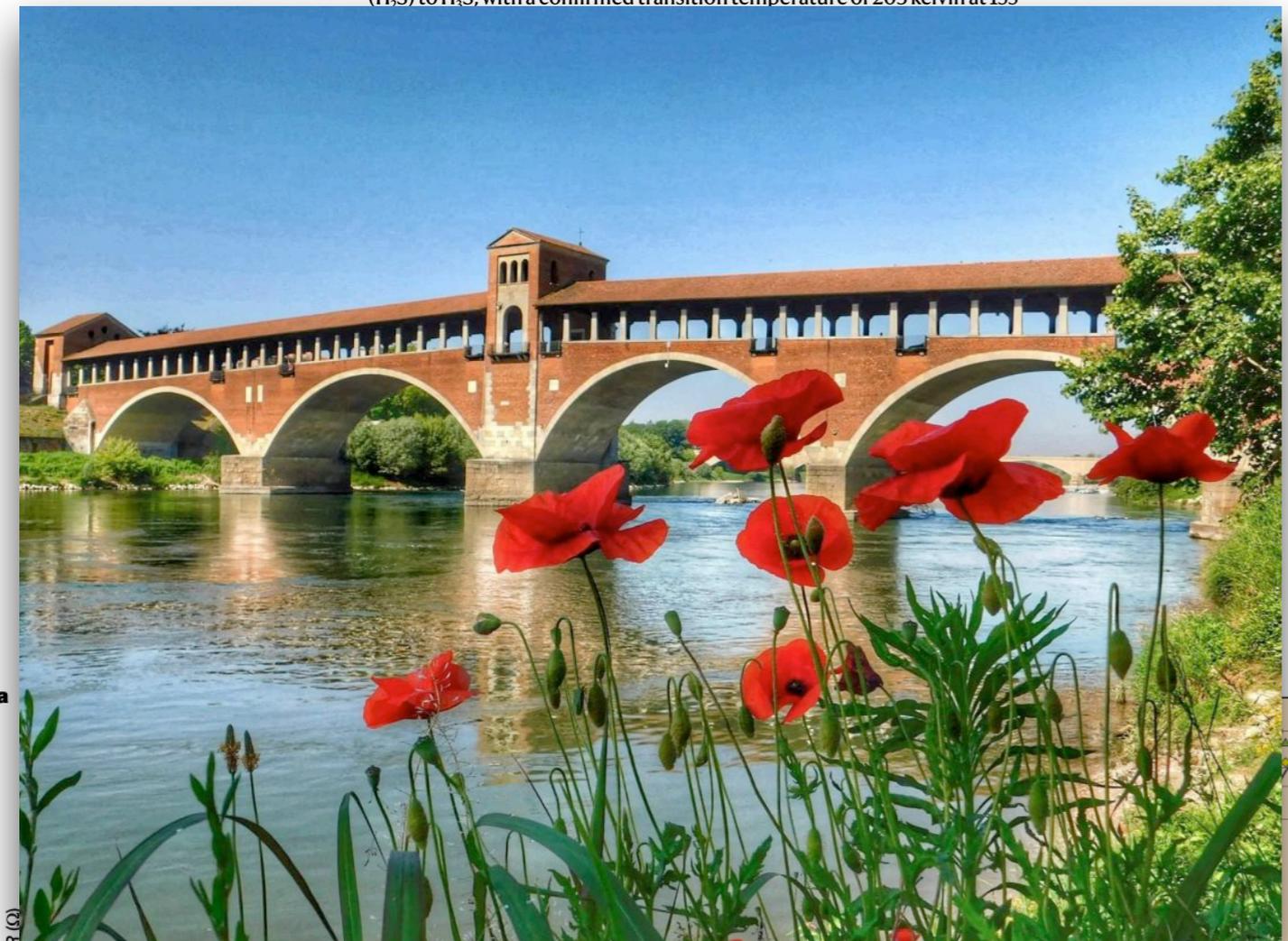
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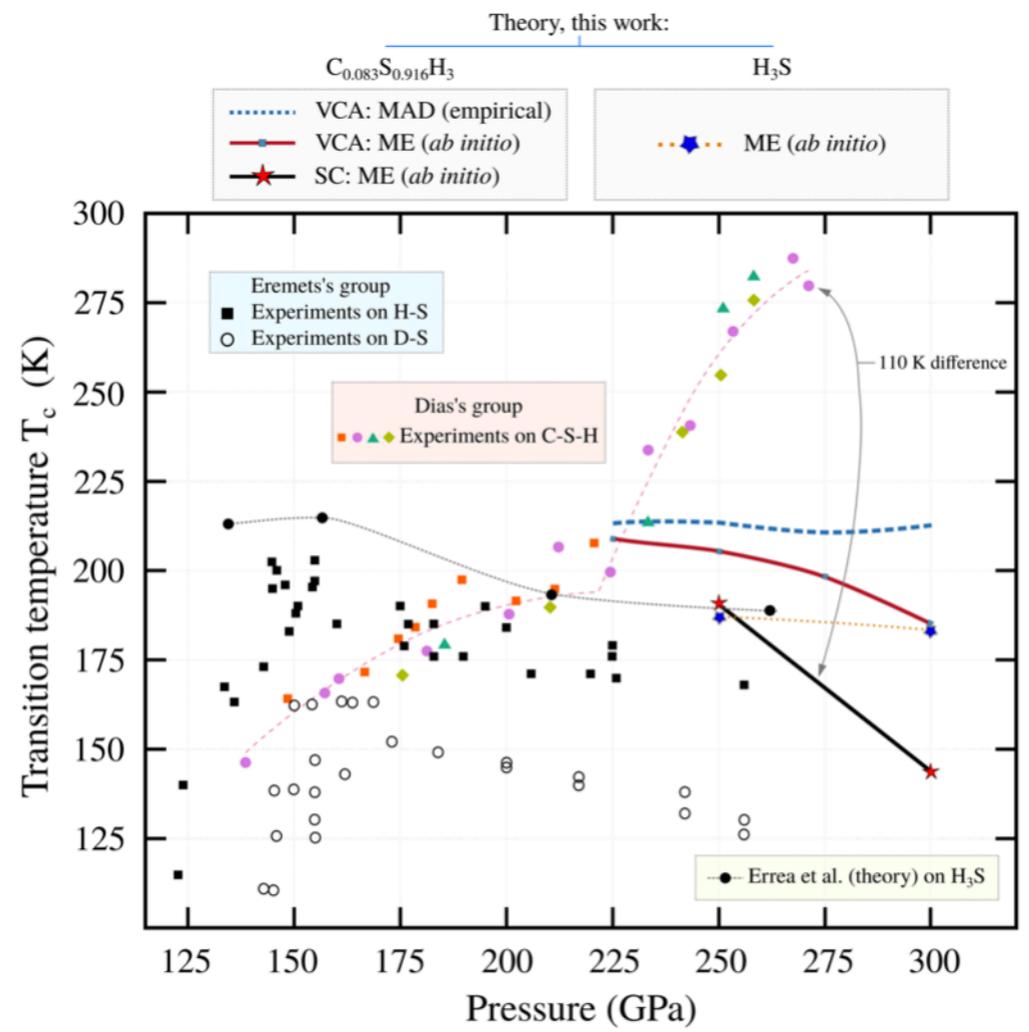
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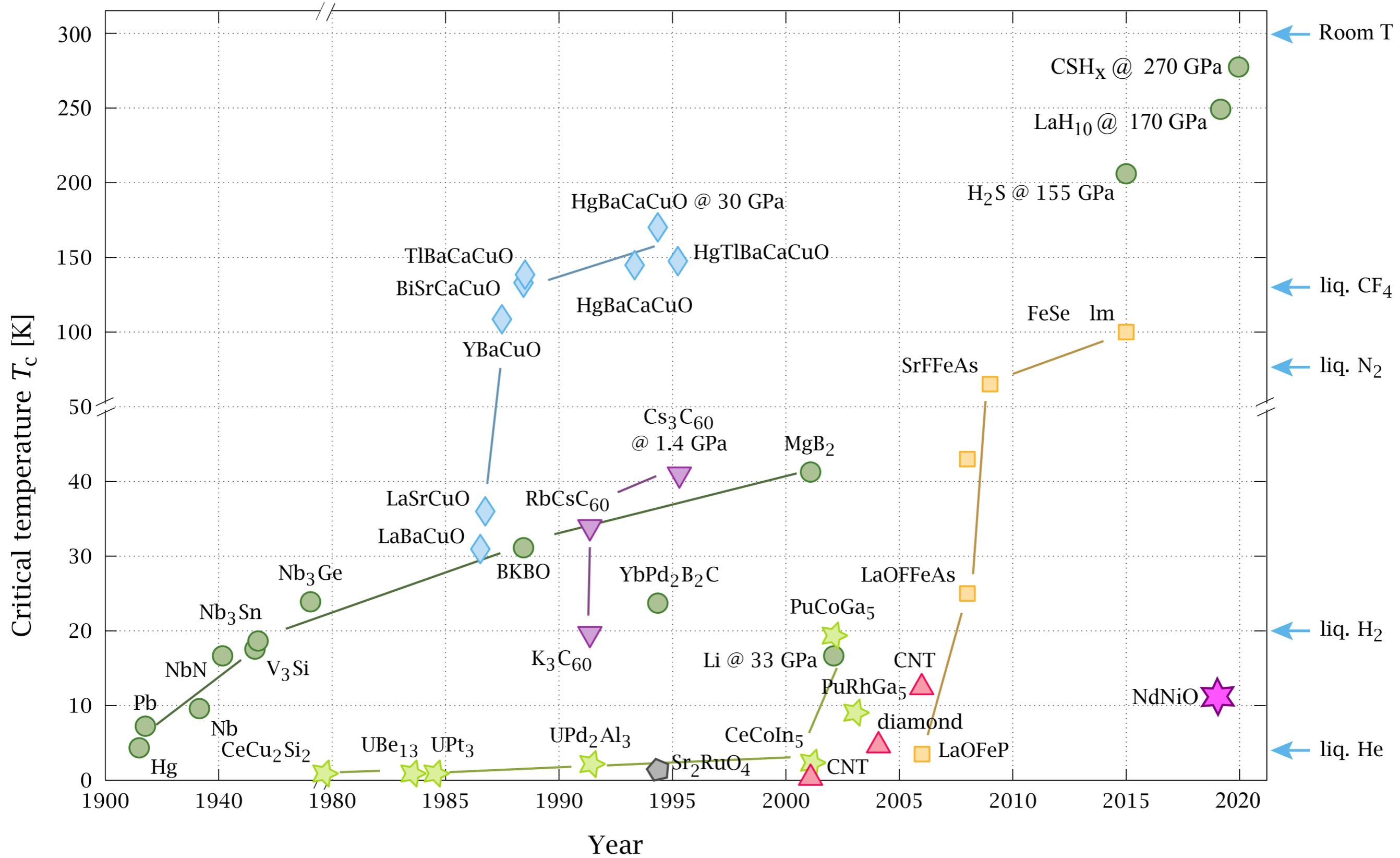


## Absence of conventional room-temperature superconductivity at high pressure in carbon-doped H<sub>3</sub>S

Tianchun Wang, Motoaki Hirayama, Takuya Nomoto, Takashi Koretsune, Ryotaro Arita, and José A. Flores-Livas  
Phys. Rev. B **104**, 064510 – Published 25 August 2021



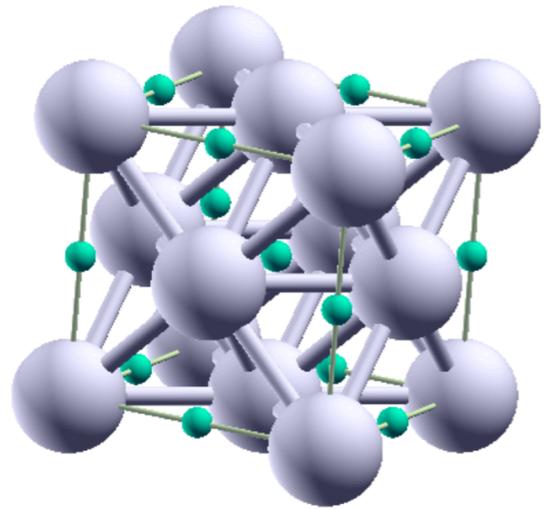
But theory fails (?)



From room temperature to ambient pressure: follow the hydrogen route

From room temperature to ambient pressure: follow the hydrogen route

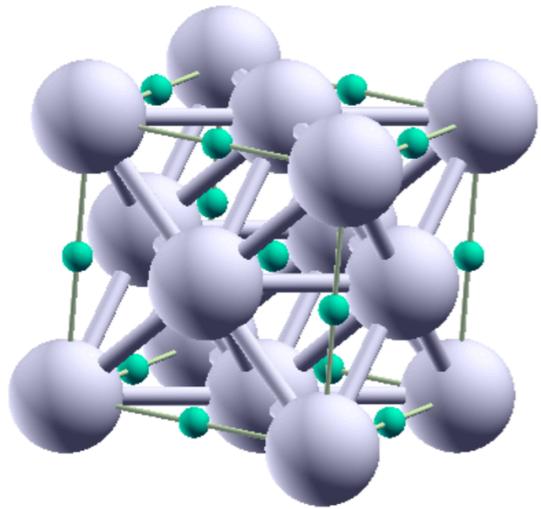
## Palladium hydride



**T<sub>c</sub>=9 K**

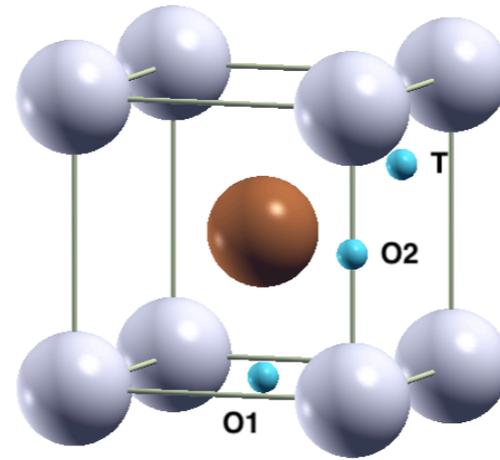
From room temperature to ambient pressure: follow the hydrogen route

### Palladium hydride

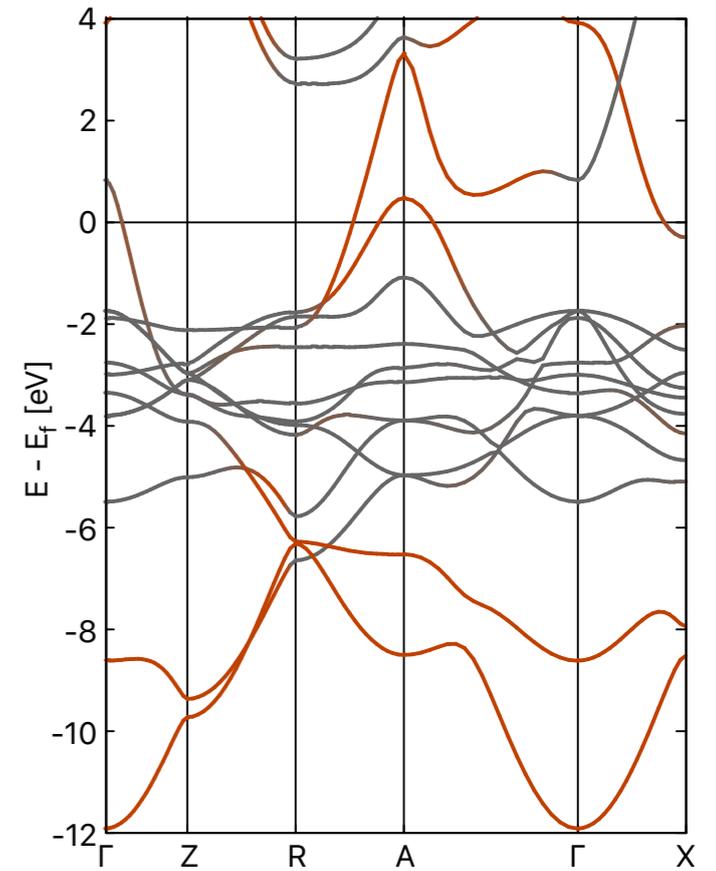


**T<sub>c</sub>=9 K**

### Palladium - Copper hydride

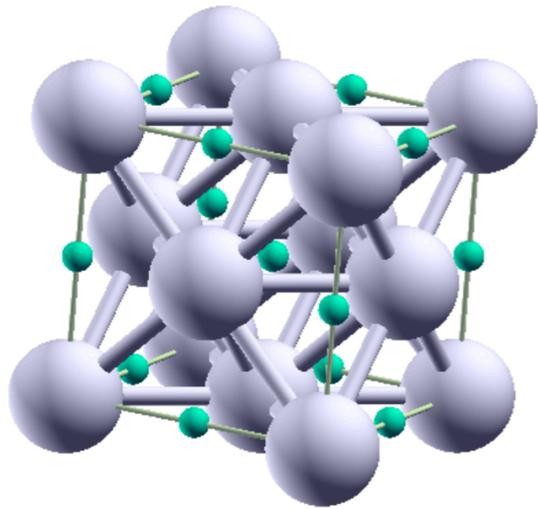


**T<sub>c</sub>=20 K**



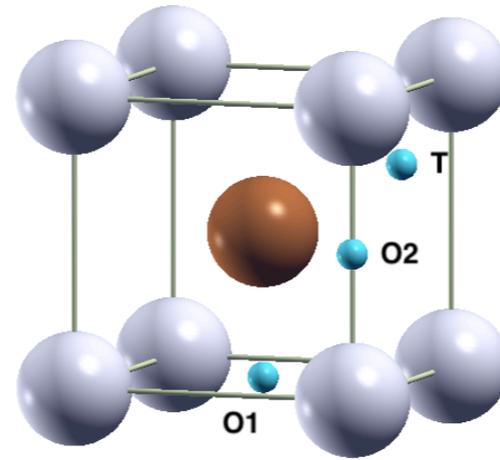
From room temperature to ambient pressure: follow the hydrogen route

### Palladium hydride

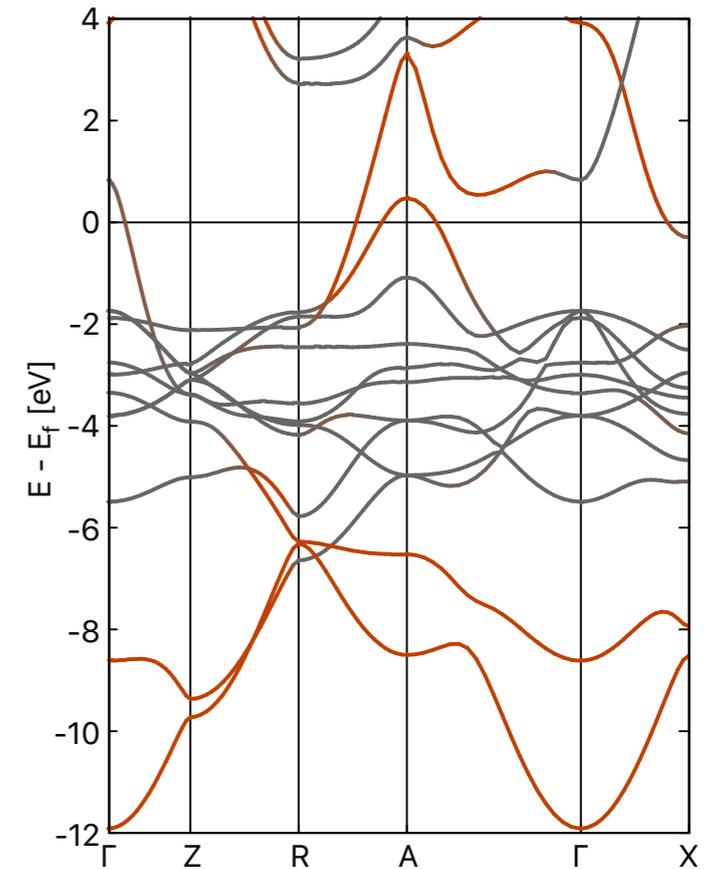


$T_c = 9 \text{ K}$

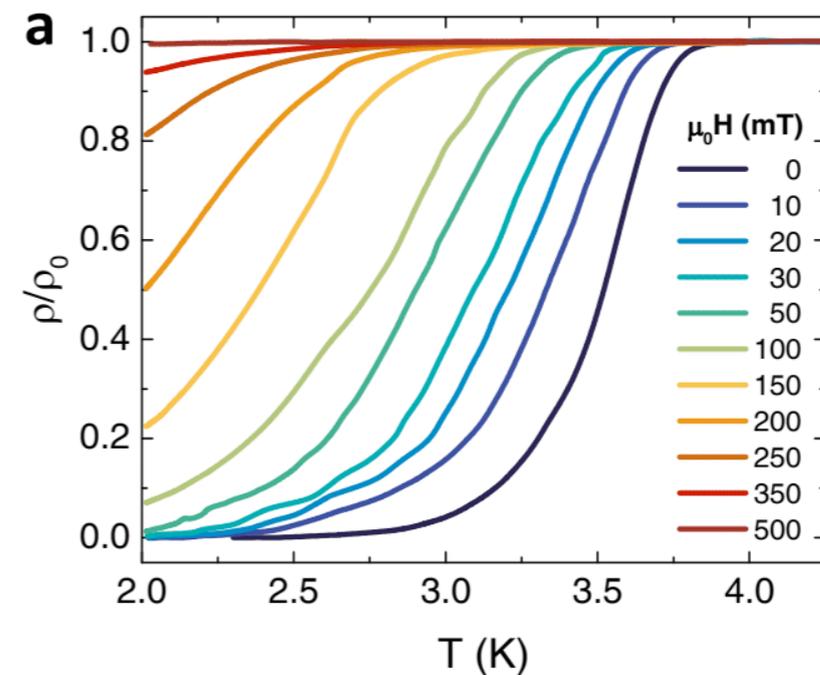
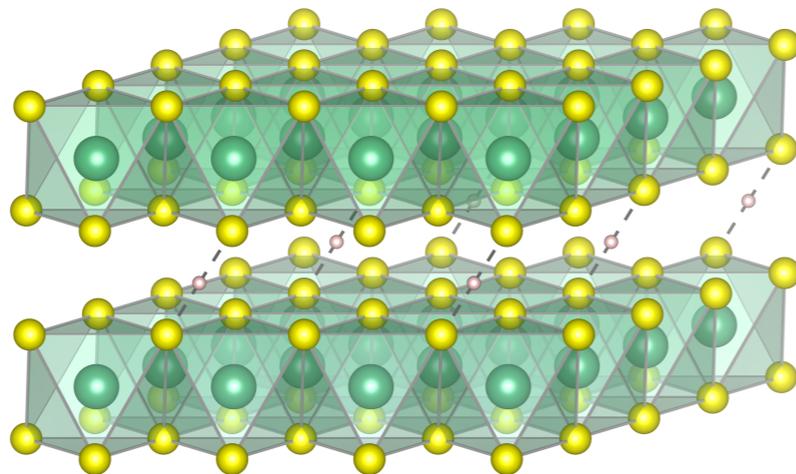
### Palladium - Copper hydride



$T_c = 20 \text{ K}$



### Hydrogenated TMD Titanium diselenide, $\text{TiSe}_2$



**Any suggestions?**

$$\frac{2}{\xi_k} \tanh\left(\frac{\xi_k}{2}\right) \sum_{\lambda q} |g_{kk'}^{\lambda q}|^2 [I(\xi_k, \xi_{k'}, \Omega_{\lambda q}) - I(\xi_k, -\xi_{k'}, \Omega_{\lambda q})] \quad (25)$$

does not depend on the nuclear masses (the sum is over the atoms of the crystal at cell position  $\mathbf{R}_l$ , with respect the reference cell  $\mathbf{R}_0$ ).

From equation (30), and using, again, the (mass-independent) normalization of the eigenvectors we obtain (see appendix B for the details of the calculations):

$$K_{kk'}^{cl} = v_{kk'}. \quad (26)$$

$$V_{\lambda q} = -\sum_{\mu} \frac{\zeta_{\alpha\mu}^{\lambda q} \zeta_{\alpha\mu}^{\lambda q}}{2M_{\alpha}} \Omega_{\lambda q}. \quad (32)$$

of the coupling constants  $\zeta_{\alpha\mu}^{\lambda q}$  and  $\chi_0(\mathbf{r}', \mathbf{r}'')$  are constants are given by the matrix elements of the self-consistent potential  $(V_{\lambda q}(\mathbf{r}))$  with respect to the phonon mode  $\lambda$  at wave-vector  $\mathbf{q}$  between electrons at wavevectors  $\mathbf{k}$  and  $\mathbf{k}' = \mathbf{k} + \mathbf{q}$ .

$$V_{\lambda q}(\mathbf{r}) = \int d^3 r' \varphi_k^*(\mathbf{r}) V_{\lambda q}(\mathbf{r}').$$

The self-consistent potential can be written as [45]:

$$V_{\lambda q}(\mathbf{r}) = V_{\lambda q}^0(\mathbf{r}) + \int d^3 r' \int d^3 r'' \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} + f_{\alpha}(\mathbf{r}, \mathbf{r}') \right) \chi_0(\mathbf{r}', \mathbf{r}'') V_{\lambda q}^0(\mathbf{r}'').$$

where  $\chi_0(\mathbf{r}, \mathbf{r}')$  is the full response function and

$$V_{\lambda q}^0(\mathbf{r}) = \sum_{\alpha, l, \mu} \frac{Z_{\alpha}}{\sqrt{2M_{\alpha} \Omega_{\lambda q}}} e^{i\mathbf{q} \cdot \mathbf{R}_l} \zeta_{\alpha\mu}^{\lambda q} \left( \frac{\partial}{\partial R_{\alpha l}^{\mu}} \frac{1}{|\mathbf{r} - \mathbf{R}_{\alpha l}|} \right) \quad (35)$$

the derivative of the bare potential. Since the full response function is not explicitly known, the response equation (34) is normally rewritten as

$$V_{\lambda q}(\mathbf{r}) = V_{\lambda q}^0(\mathbf{r}) + \int d^3 r' \int d^3 r'' \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} + f_{\alpha}(\mathbf{r}, \mathbf{r}') \right) \chi_0(\mathbf{r}', \mathbf{r}'') V_{\lambda q}(\mathbf{r}''). \quad (36)$$

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In order to calculate the partial derivative of the coupling constant with respect to the atomic masses, we write  $V_{\lambda q}^0(\mathbf{r})$  as the partial derivative of the potential,  $\frac{\partial V_{\lambda q}^0(\mathbf{r})}{\partial M_{\alpha}}$

$$\frac{\partial V_{\lambda q}^0(\mathbf{r})}{\partial M_{\alpha}} = \int d^3 r' \int d^3 r'' (\delta(\mathbf{r} - \mathbf{r}') \delta(\mathbf{r} - \mathbf{r}'') + \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} + f_{\alpha}(\mathbf{r}, \mathbf{r}') \right) \chi_0(\mathbf{r}', \mathbf{r}'')) \frac{\partial V_{\lambda q}^0(\mathbf{r}')}{\partial M_{\alpha}}. \quad (37)$$

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# THANK YOU FOR YOUR ATTENTION

see more @ <http://www.aquila.infn.it/profeta/>

(BSE) [45]:

$$T(1,2,3,4) = w(1,3) \delta_{13} \delta_{24} + w(1,2) G(1,5) G(2,6) T(5,6,3,4). \quad (8)$$

The coordinate  $l$  is a compact notation:  $l = \{\mathbf{r}_l, \tau_l, \sigma_l\}$ , where  $\mathbf{r}_l$  is the real space vector,  $\tau_l$  the Matsubara time, and  $\sigma_l$  the spin index. The diagrammatic form of this BSE and the self-energy contribution  $\tilde{\Sigma}^T = \tilde{G}T$  corresponding to the  $T$  matrix are shown in Eq. (9).

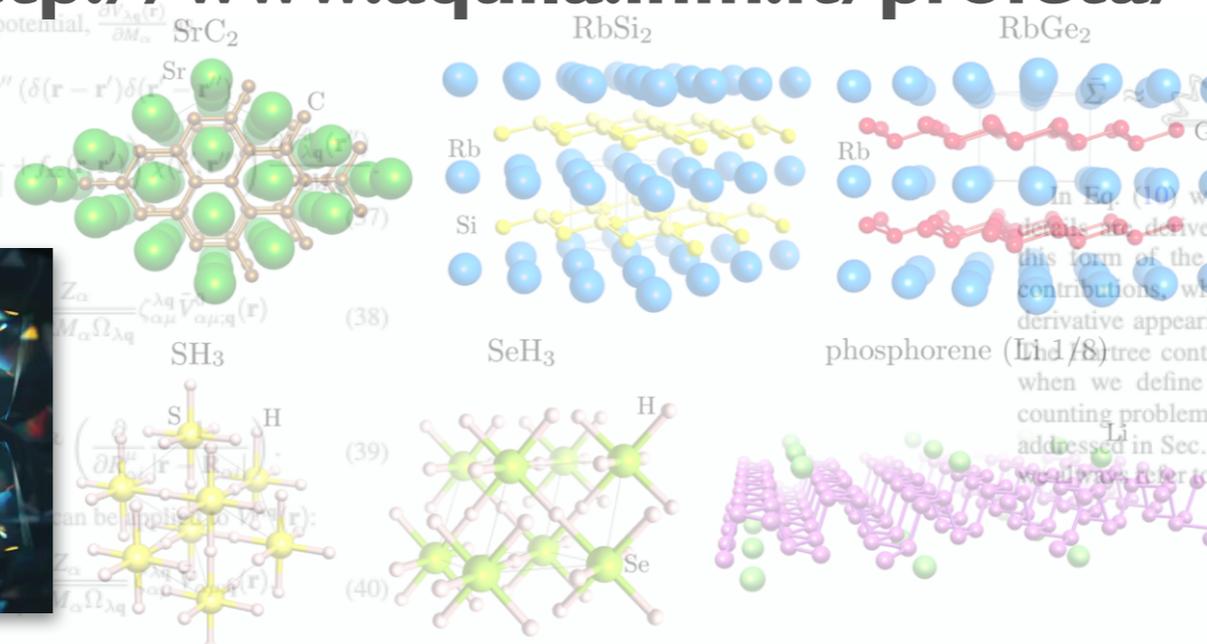


mathematically, it is well known that the matrix approximation of the magnetic response function is not sufficient for various studies to account for the systems [46–49].

However, for reasons of simplicity, we do not make direct use of the  $T$  matrix and the corresponding self-energy for constructing the effective interaction. Instead we consider a larger set of diagrams, by starting from the particle-hole propagator  $\Lambda^P$  [50–52]. This object contains all proper particle-hole contributions. These are all diagrams which are irreducible with respect to a bare Coulomb interaction and have two incoming and two outgoing open coordinates. The  $T$  matrix is fully contained in  $\Lambda^P$ . We use the analogy with  $\tilde{\Sigma}^T$  [see Eq. (9)], to formulate the self-energy containing magnetic contributions as

$$\Sigma^{\text{GW}} = \Sigma^{\text{SF}} + \Lambda^P + \Sigma^{\text{Ph}} \quad (10)$$

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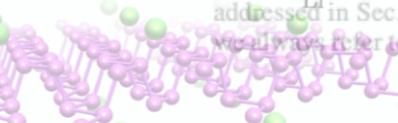
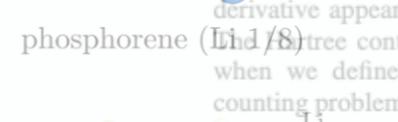
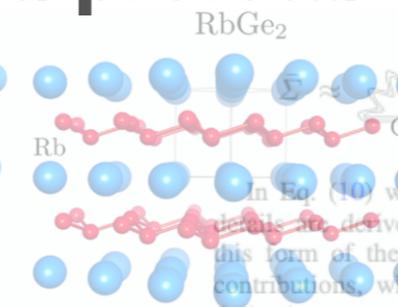
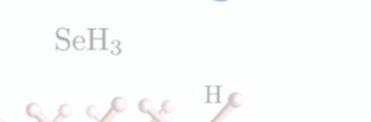
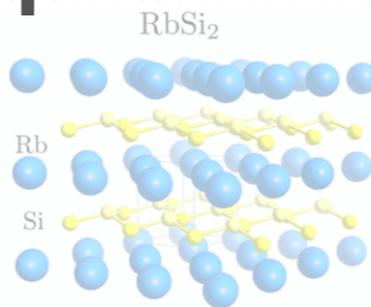
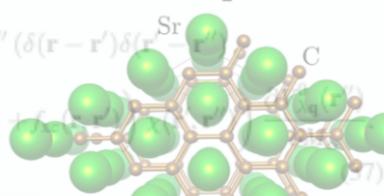
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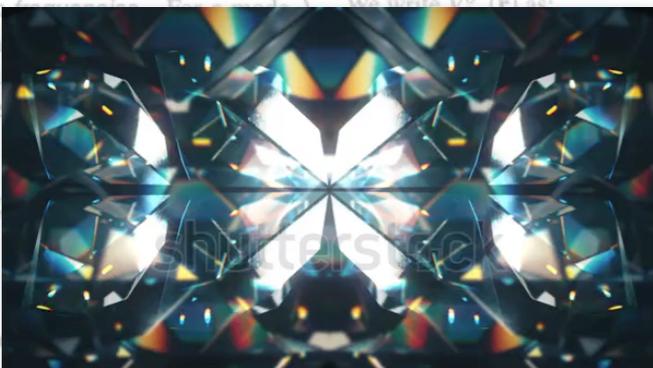
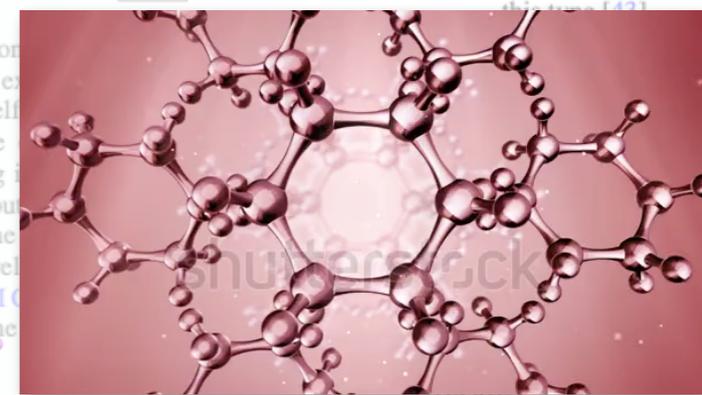


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# Why and how does it happen?

Frohlich (1952)

$$H_F = H_e + H_p + H_{ep}$$

$$H_e = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma}$$

$$H_p = \sum_{q,\lambda} \hbar\omega_{q,\lambda} [b_{q\lambda}^\dagger b_{q\lambda} + \frac{1}{2}]$$

$$H_{ep} = \sum_{k\sigma} \sum_{q,G\lambda} g_{k,\lambda}^{q+G} c_{k+q+G,\sigma}^\dagger c_{k\sigma} [b_{q\lambda} + b_{-q\lambda}^\dagger]$$

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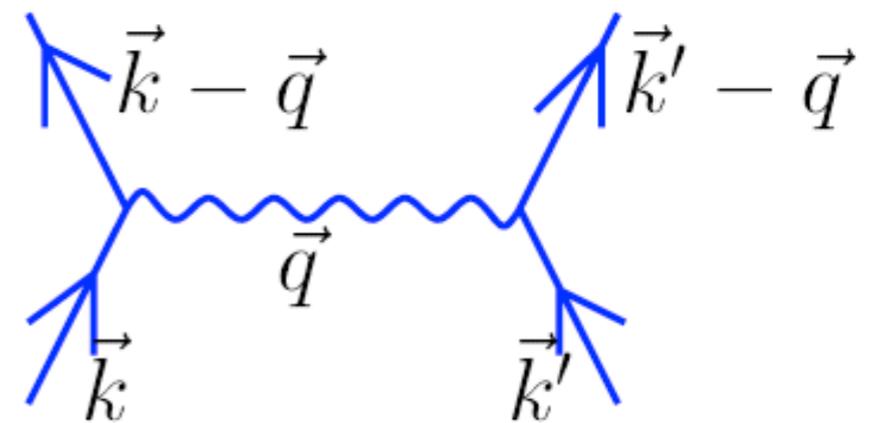
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Define an effective hamiltonian (not terms which couple e and ph)

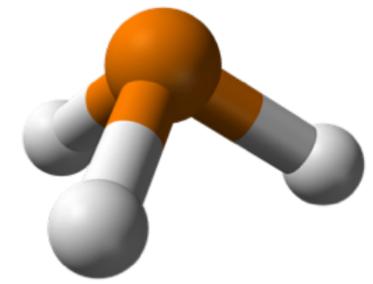
$$\tilde{H}_F = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{2} \sum_{\substack{kk'qG \\ \sigma\sigma'}} V_{ph} c_{k+q+G\sigma}^\dagger c_{k'-q-G\sigma'}^\dagger c_{k'\sigma'} c_{k\sigma}$$

$$V_{ph} = \sum_{\lambda} \frac{\hbar\omega_{q\lambda} |g(q+G;\lambda)|^2}{[\epsilon_k - \epsilon_{k+q+G}]^2 - [\hbar\omega_{q\lambda}]^2}$$

if < 0 attraction !!



# Isolated example? Superconducting phosphines ( $\text{PH}_3$ )



RAPID COMMUNICATIONS

PHYSICAL REVIEW B **93**, 020508(R) (2016)



## Superconductivity in metastable phases of phosphorus-hydride compounds under high pressure

José A. Flores-Livas,<sup>1</sup> Maximilian Amsler,<sup>2</sup> Christoph Heil,<sup>3</sup> Antonio Sanna,<sup>4</sup> Lilia Boeri,<sup>3</sup> Gianni Profeta,<sup>5</sup> Chris Wolverton,<sup>2</sup> Stefan Goedecker,<sup>1</sup> and E. K. U. Gross<sup>4</sup>

<sup>1</sup>Department of Physics, Universität Basel, Klingelbergstr. 82, 4056 Basel, Switzerland

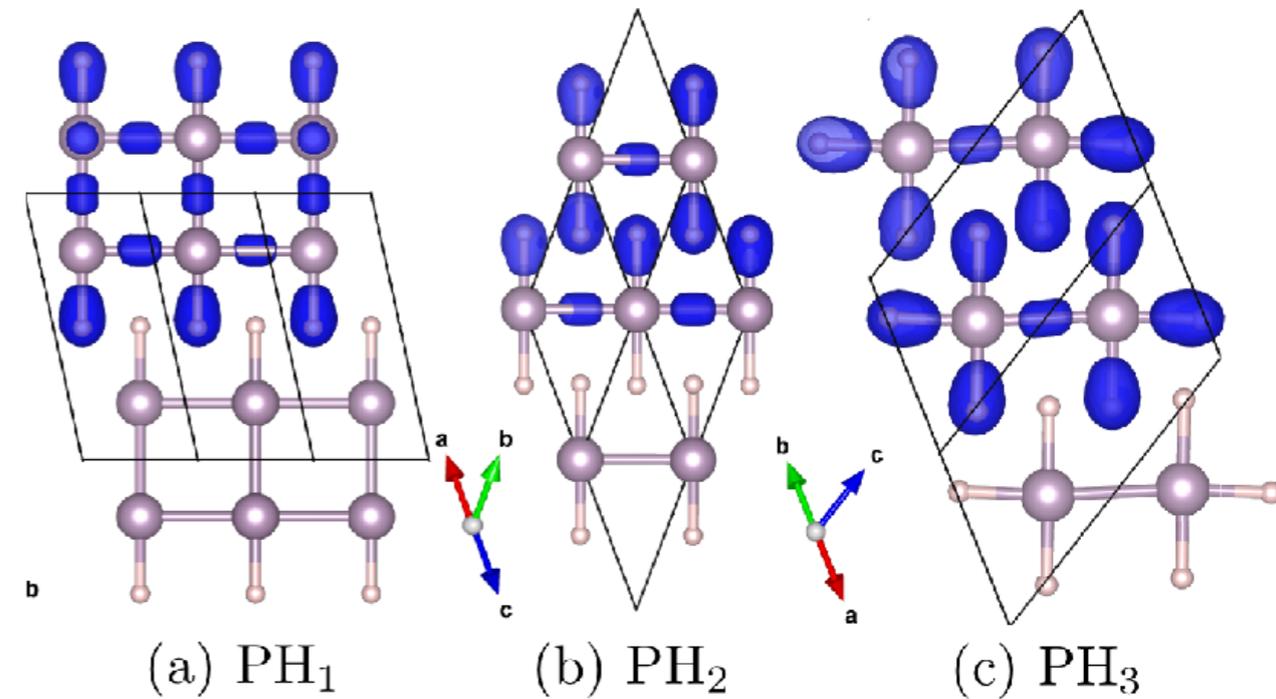
<sup>2</sup>Department of Materials Science and Engineering, Northwestern University, Evanston, Illinois 60208, United States

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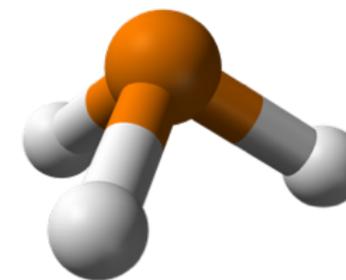
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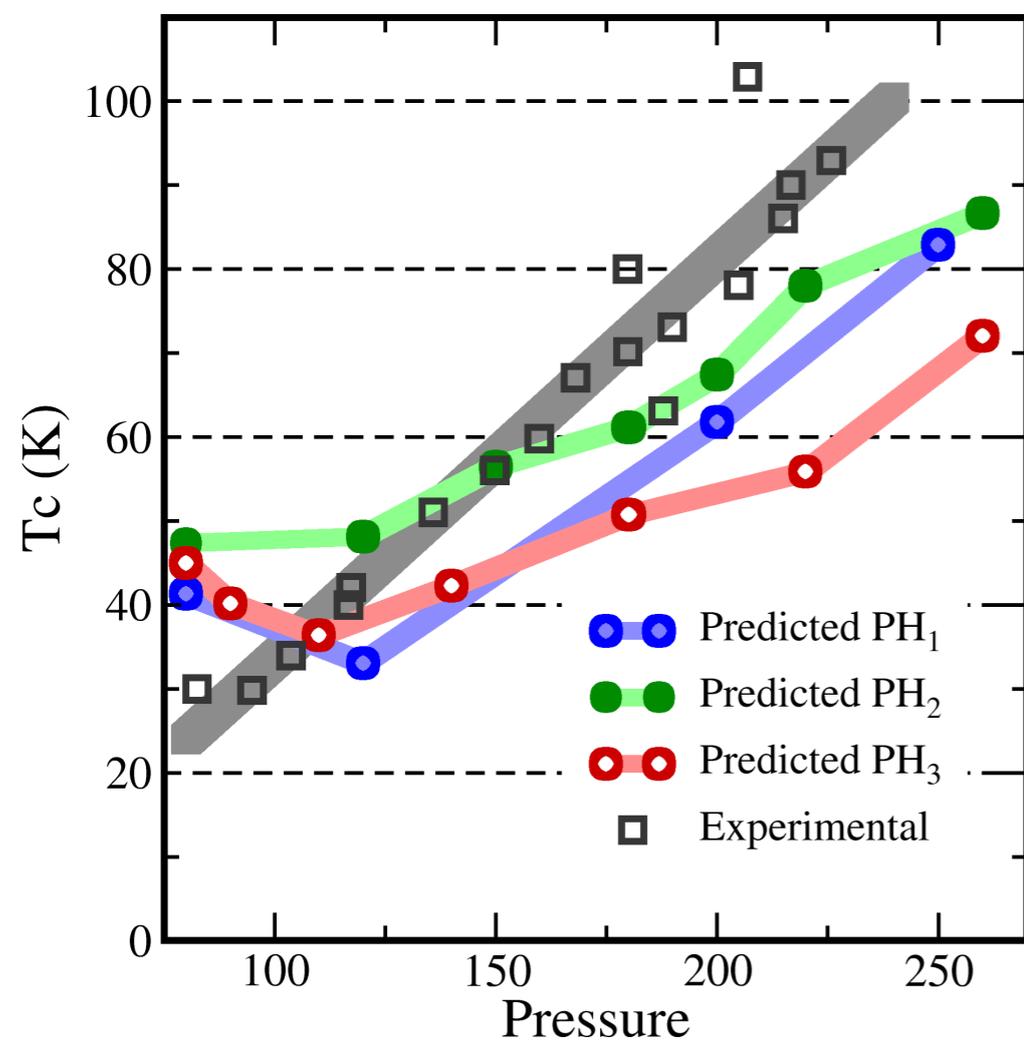
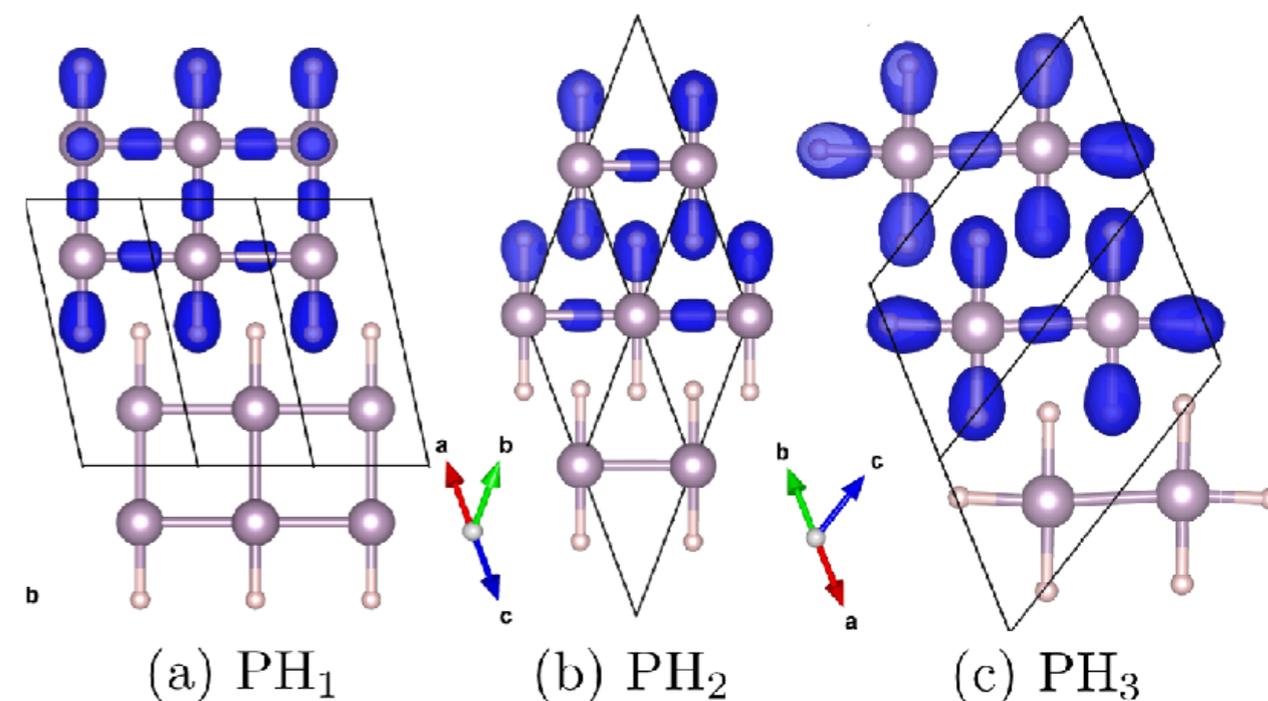
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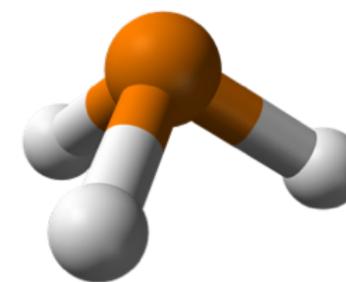
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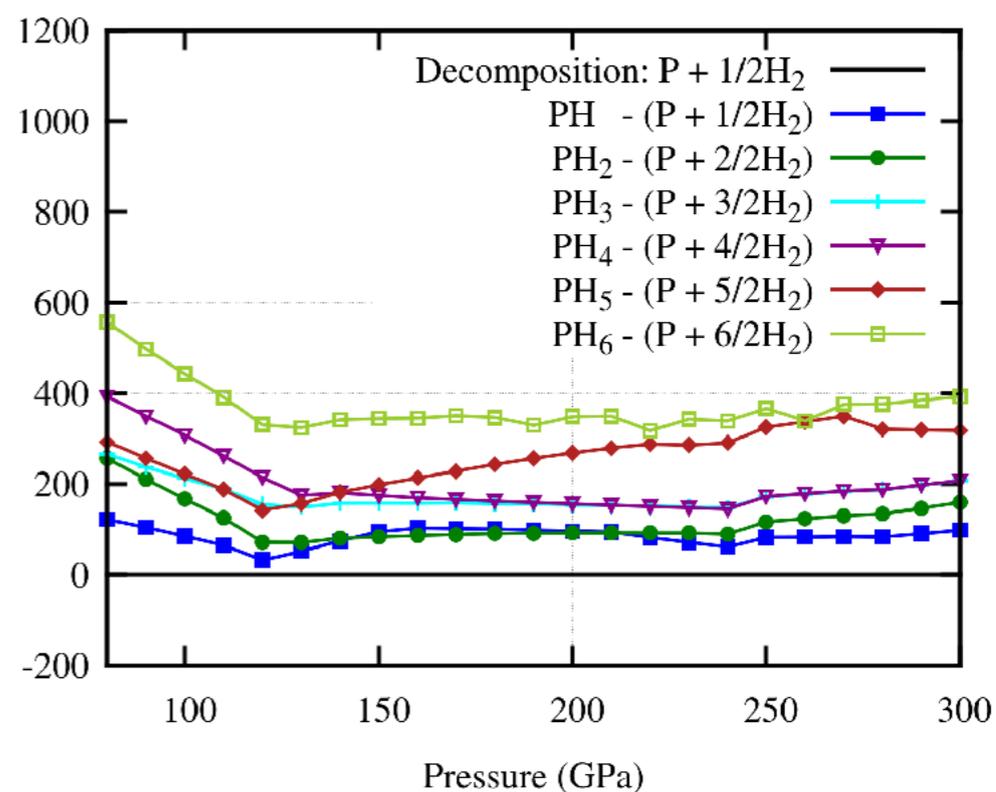
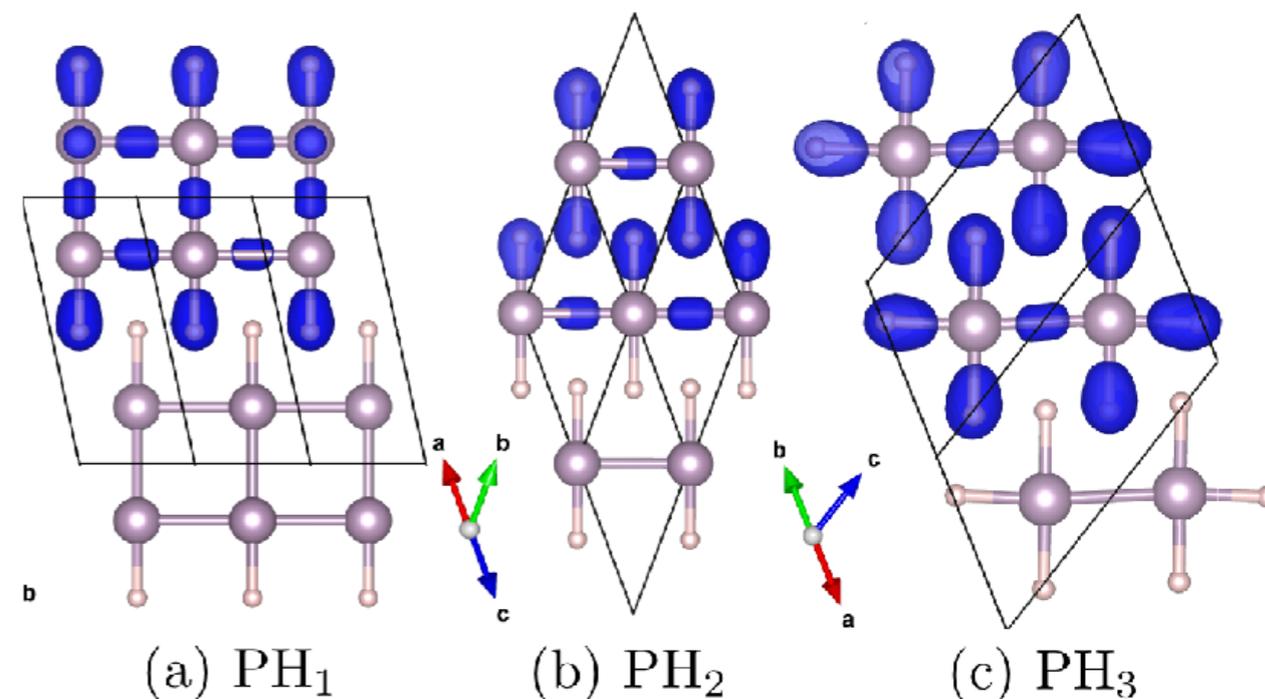
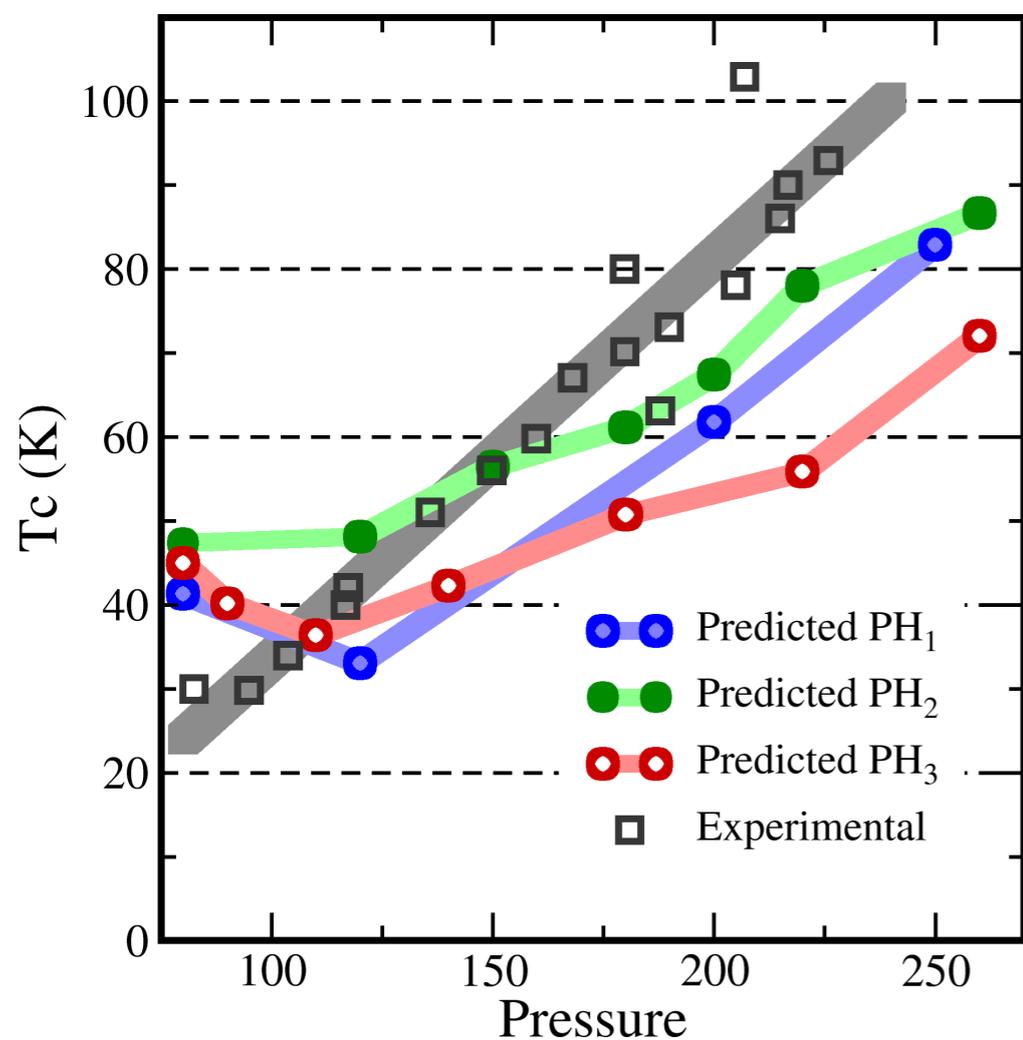
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# Superconductivity up to 243 K in the yttrium-hydrogen system under high pressure

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