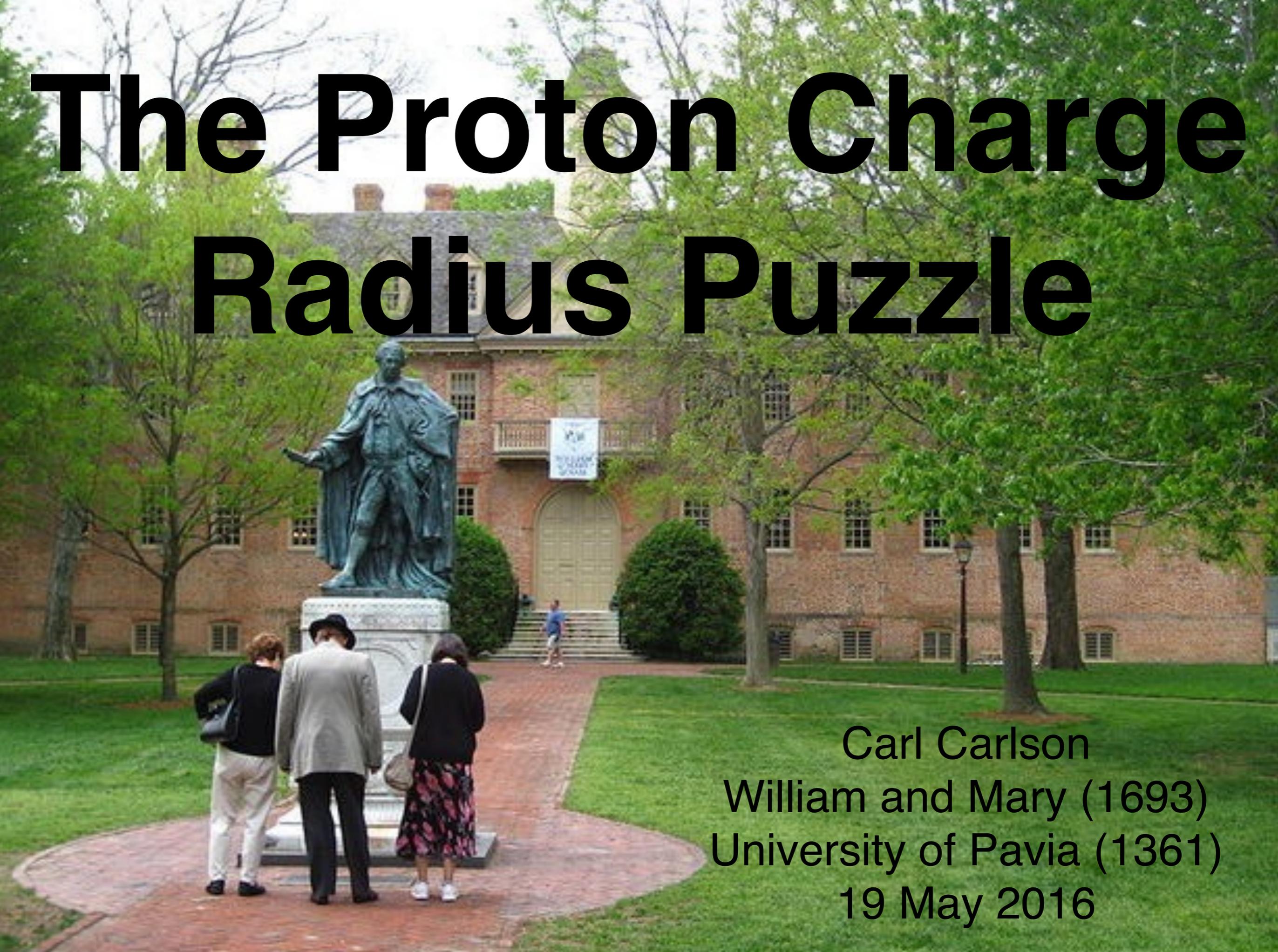


The Proton Charge Radius Puzzle



Carl Carlson
William and Mary (1693)
University of Pavia (1361)
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The puzzle

- Measure charge radius of the proton different ways, get different answers
- Difference is 7 s.d.
(was 5 s.d. when first announced, 2010)
- Why? Don't yet know.

This talk

1. The measurements:
where the differences came from
2. Suggested explanations
 - A. Ordinary explanations
 - Maybe some things are harder than they seem
 - B. Exotic explanations
 - Will discuss: Is it Physics Beyond the Standard Model?
 - Will mention: other possibilities (later)
3. Highlight: List of coming relevant data

This talk

1. The measurements:
where the differences came from

2. Suggested explanations

A. Ordinary explanations

- Maybe some things are harder than they seem

B. Exotic explanations

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- Will mention: other possibilities (later)

3. Highlight: List of coming relevant data

new stuff here

and here too

some arriving in 2016 (maybe!)

Measuring proton radius

- Two methods: scattering or atomic spectroscopy
- Two probes: electrons or muons
- *i.e.*,
 - $e-p$ elastic scattering
 - $\mu-p$ elastic scattering
 - spectroscopy of electronic Hydrogen
 - spectroscopy of muonic Hydrogen
- 4 categories of measurements, 3 done with sufficient accuracy (and more data coming), $\mu-p$ scattering coming

$e-p$ scattering

- Measure differential cross section, fit results to form factors,

$$\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2)$$

$$\left[\tau = Q^2 / 4m_p^2 ; \quad 1/\epsilon = 1 + 2(1 + \tau) \tan^2(\theta_e/2) \right]$$

- Low Q^2 , mainly sensitive to G_E .
- Extrapolate to $Q^2 = 0$, whence

$$R_E^2 = -6 \left(dG_E / dQ^2 \right)_{Q^2=0}$$

Extra: What is the proton radius?

- By this I mean, what is the definition?
- NR, easy. Given w. f., obtain RMS radius,

$$R^2 = \langle r^2 \rangle = \int d^3r r^2 |\psi(r)|^2$$

- In concept, obtaining proton radius by electron scattering same as obtaining radius of H-atom w. f. by scattering an external electron off the bound electron. Worked out by Bethe in 1930's
- Rutherford scattering cross section off pointlike target,

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{point}} = \left(\frac{k q Q}{4E \sin^2(\theta/2)} \right)^2$$

Straight out of Taylor's UG
Classical Mechanics text

More extra: Def'n of proton radius

- is modified for scattering off extended target, but just becomes

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} \Big|_{\text{point}} \times \left(G(Q^2)\right)^2$$

- Q = momentum transfer in scattering
- $G(Q^2)$ is "form factor", given NR by

$$G(Q^2) = \int d^3r e^{i\vec{Q}\cdot\vec{r}} |\psi(r)|^2$$

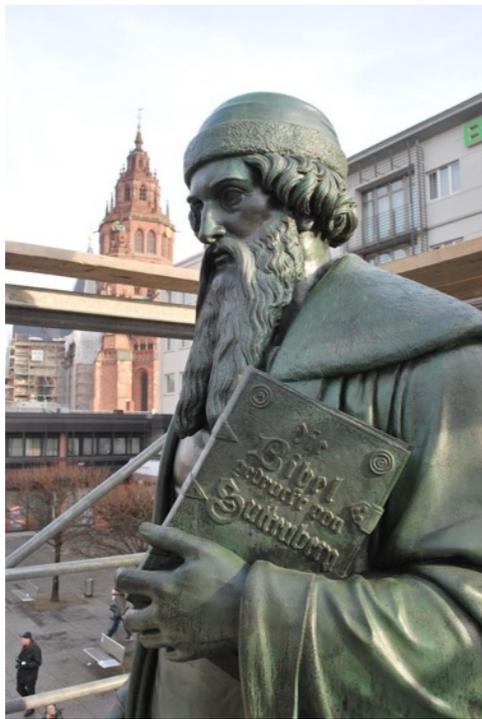
- easy:

$$G(Q^2) = 1 - \frac{1}{6} \langle r^2 \rangle Q^2 + \dots$$

De facto: measure radius by measuring form factor at small momentum transfer and looking at expansion

Low- Q^2 scattering data

- Most extensive current data comes from Mainz, which has an electron accelerator, and is also city of Gutenberg



- Data, Jan Bernauer *et al.*, PRL 2010 (and later articles).
- marked by low Q^2 data, range 0.004 to 1 GeV^2
- From their analysis,

$$R_E = 0.879(8) \text{ fm}$$

Atomic energy level splittings

- Basic: Schrödinger equation, H-atom, point protons

$$E = -\frac{\text{Ryd}}{n^2}, \quad \text{where} \quad \text{Ryd} = \frac{1}{2}m_e\alpha^2 \approx 13.6\text{eV}$$

- plus QED corrections
- plus finite size proton, pushing energy upward a bit.

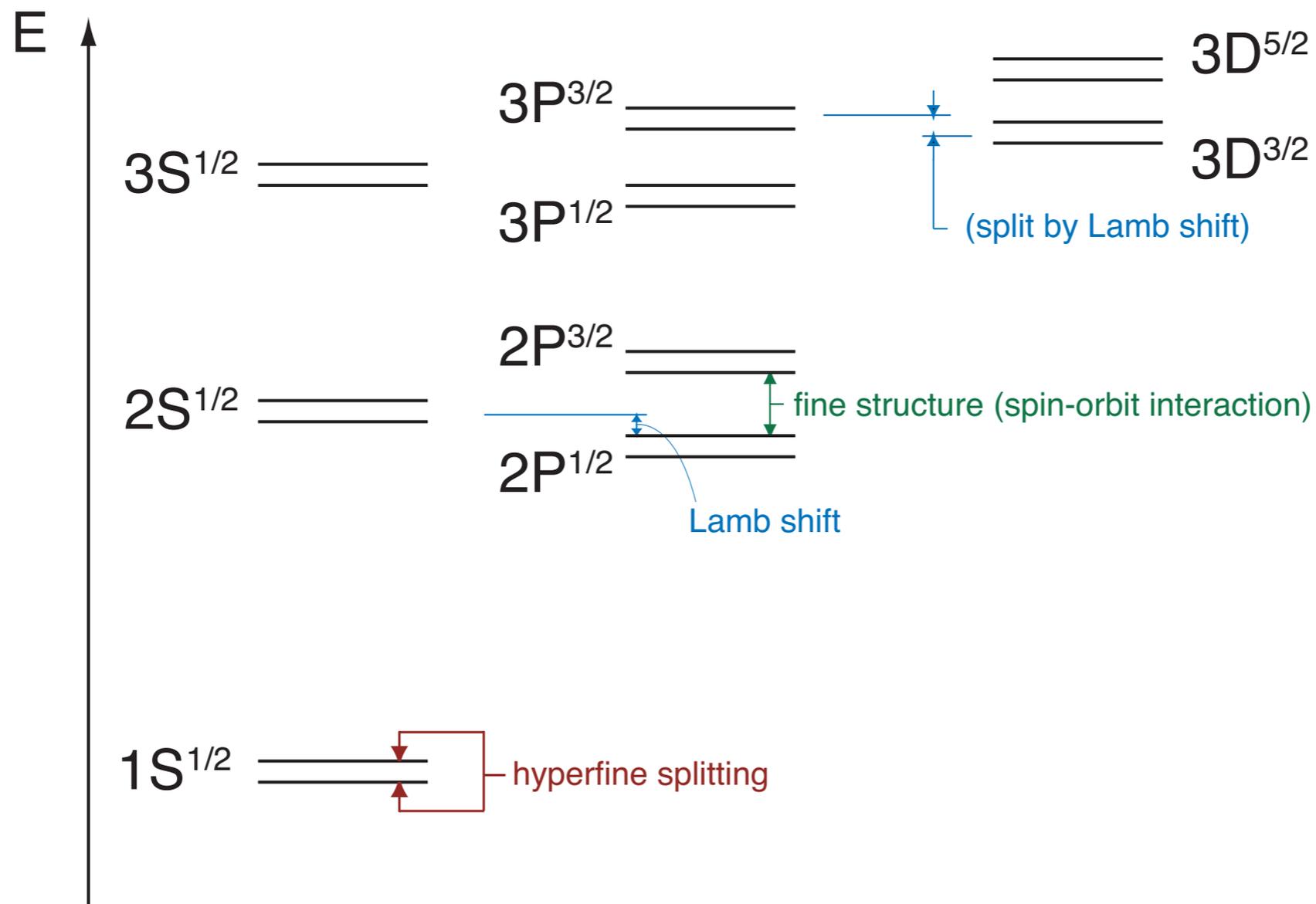
$$\Delta E_{\text{finite size}} = \frac{2\pi\alpha}{3}\phi_{nS}^2(0)R_E^2$$

fine print: $\phi_{nS}^2(0) = (m_r\alpha)^3 / (n^3\pi)$

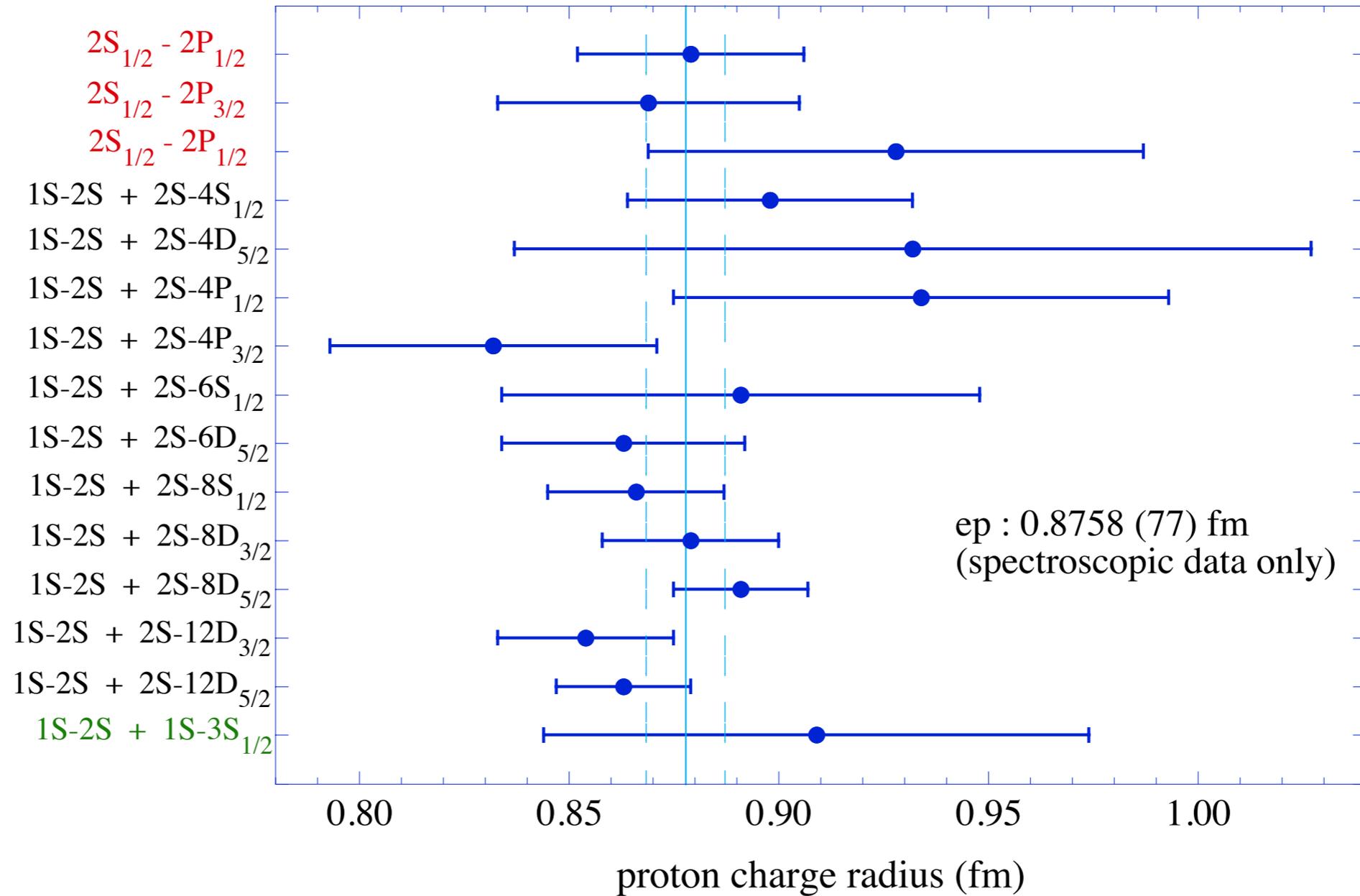
measure energy accurately

↔ measure radius

- Reminder, H-atom energy levels (diagram not to scale)



Atomic results



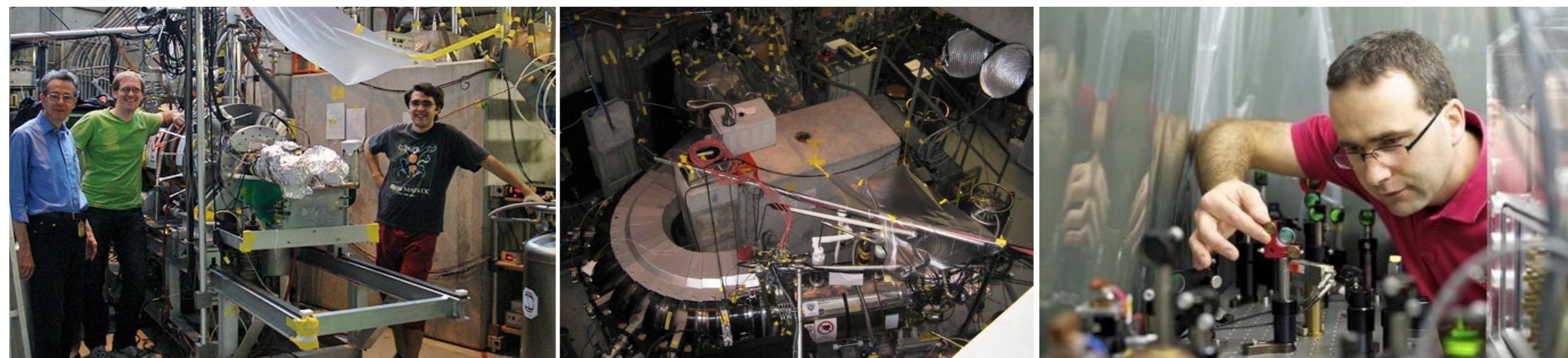
All electron results

- Consistent
- Combined by Committee on Data in Science and Technology (CODATA, 2014 value),

$$R_E = 0.8751(61) \text{ fm}$$

Then in 2010 ...

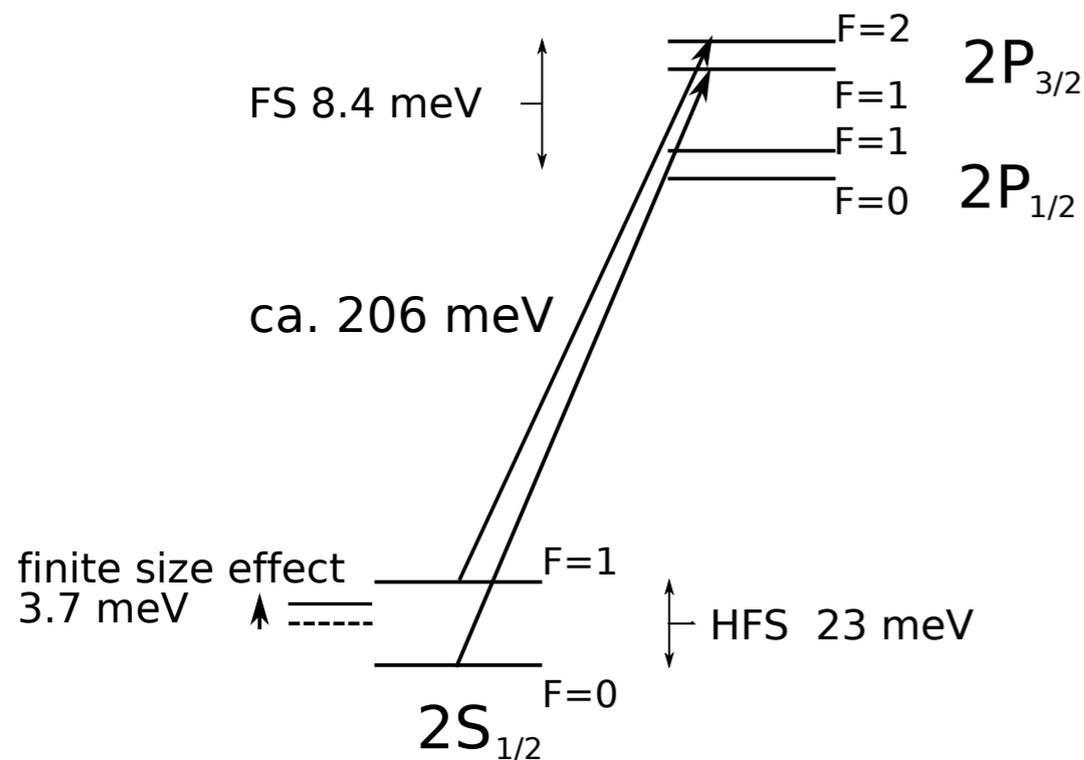
- CREMA = Charge Radius Experiment with Muonic Atoms



- Did atomic physics, specifically Lamb shift, with muons (muon = electron, but weighs 200 times more, orbits 200 times closer).
- Goal: measure proton radius with factor 10 smaller uncertainty

CREMA

- $2S$ - $2P$ Lamb shift in μ -H.
- Measured two lines,



- pubs:
upper line, Pohl *et al.*, Nature 2010
other line Antognini *et al.*, Science 2013

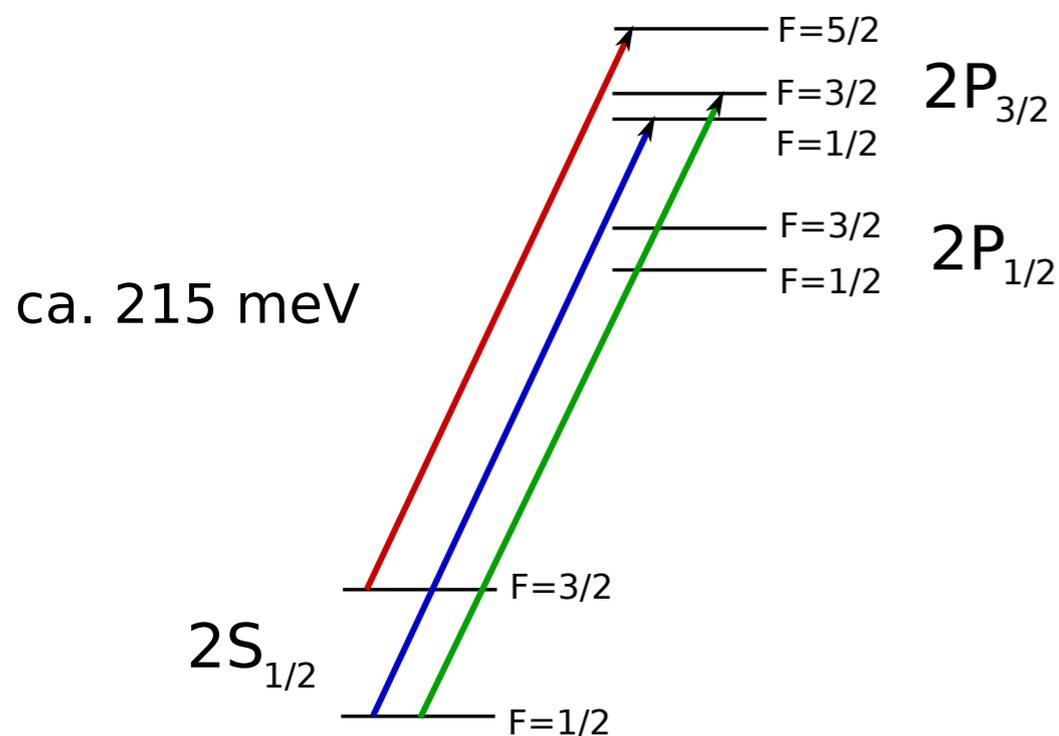
- Interpreting finite size effect in terms of proton radius,

$$R_E = 0.84087(39) \text{ fm}$$

- Whoops: result 4% or 7σ small

Other data-deuteron

- Reported at conferences 2013
- 2015 experimenters circulate draft of theory paper!
- Measured three lines



- Quick summary: if proton radius is shrunken, the deuteron radius is also.

Other data — Helium

- New 2013/2014 data
- μ -⁴He at Mainz Proton Radius Workshop, 2014
- μ -³He at Gordon Conference, N.H., 2014
- Quick summary: He radii from μ Lamb shift in accord with electron scattering radii.

Explanations?

- Hard to see problems with μ experiment
 - Hard to get working
 - But once working, easy to analyze
- Problems with analysis of electron experiments?
But there are a lot of them.
- BSM explanations?
 - If so, further tests?

Review $e-p$ scattering data

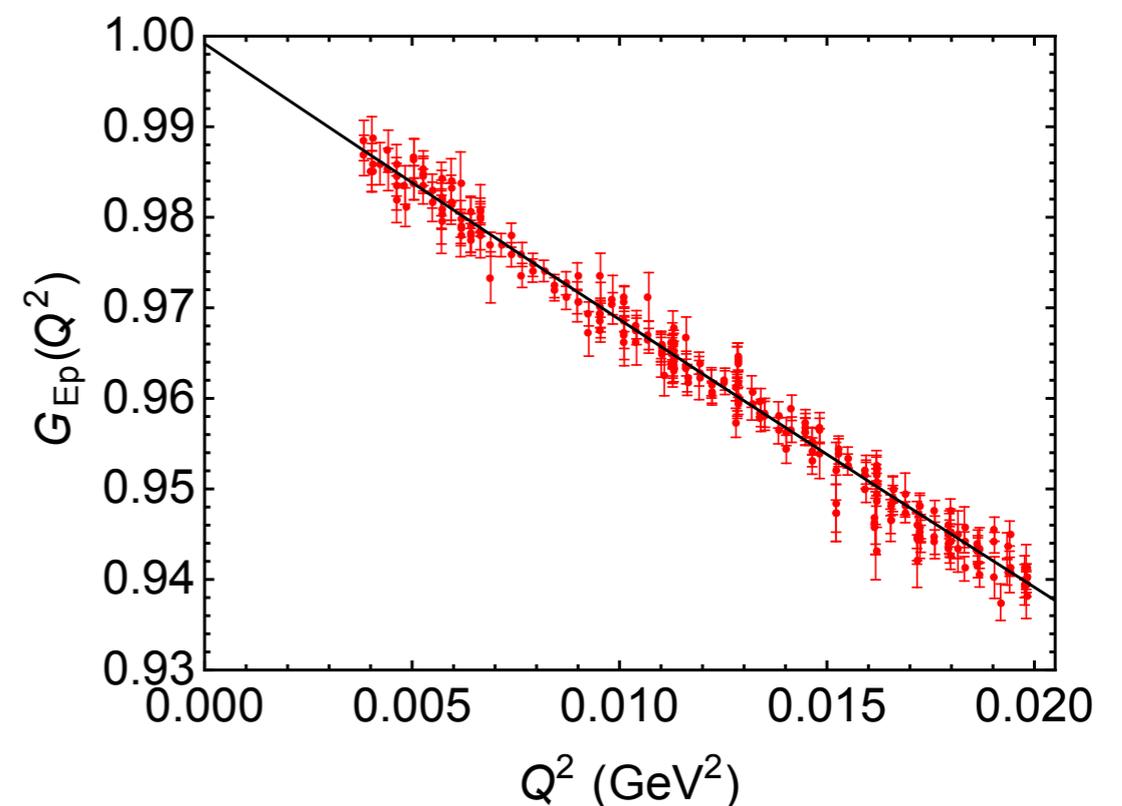
- Point: Measurements at finite Q^2 . Need to extrapolate to $Q^2 = 0$ to obtain charge radius. (Mainz group itself: $R_E = 0.879(8)$ fm.)
- Because of importance, others have tried, using different ways of fitting data. Three recent fits found “big” values:
- Graczyk & Juszczak (2014), using Bayesian ideas and pre-Mainz world data, obtained
$$R_E = 0.899(3) \text{ fm.}$$
- Lee, Arrington, & Hill (2015) using Mainz data and neat mapping ideas to ensure convergence of expansions, obtained
$$R_E = 0.895(20) \text{ fm.}$$
- Arrington & Sick (2015) found
$$R_E = 0.879(11) \text{ fm.}$$

But...

- Several recent fits found “small” values (*i.e.*, compatible with muonic Lamb shift experiment):
- Lorenz, Meißner, Hammer, & Dong (2015 and earlier), dispersive ideas, also using timelike data, obtained
$$R_E = 0.840(15) \text{ fm.}$$
- Horbatsch and Hessels (1509.05644)
- Carlson, Griffioen, Maddox (1509.06676)
- Higinbotham, Kabir, Lin, Meekins, Norum, Sawatzky (1510.01293)

Recent $e-p$ analyses, I

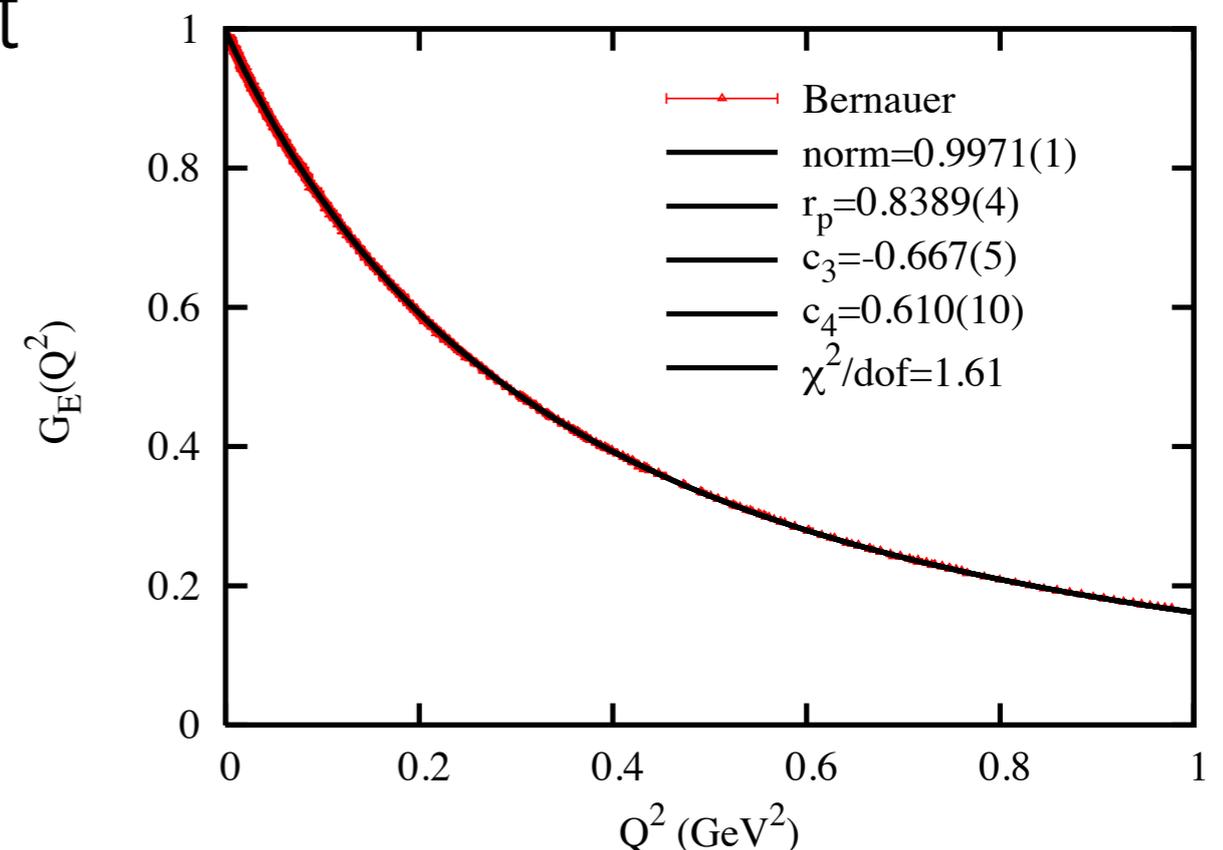
- Maddox et al. (1509.06676)
- First viewpoint: Charge radius is a $Q^2 = 0$ concept, should be able to obtain just from low Q^2 data.
- Technical: Form factor is analytic function of Q^2 , except for cut starting at $4m_\pi^2$. Hence, polynomial expansion in Q^2 converges for $Q^2 < 4m_\pi^2$.
- For low Q^2 data, use $Q^2 < 0.02 \text{ GeV}^2$ (243 data points) linear plus quadratic in Q^2 , get $R_E = 0.850(19) \text{ fm}$



Recent $e-p$ analyses, I

- Second viewpoint: fitting whole Q^2 data range with complicated (i.e., many parameters) function leads to dangers in extrapolation.
- Fit whole Mainz 2010 data set with simpler functions (i.e., 4 or so parameters), that extrapolate more reliably. From collection of such fits quote

$$R_E = 0.840(16) \text{ fm}$$



Recent $e-p$ analyses, II

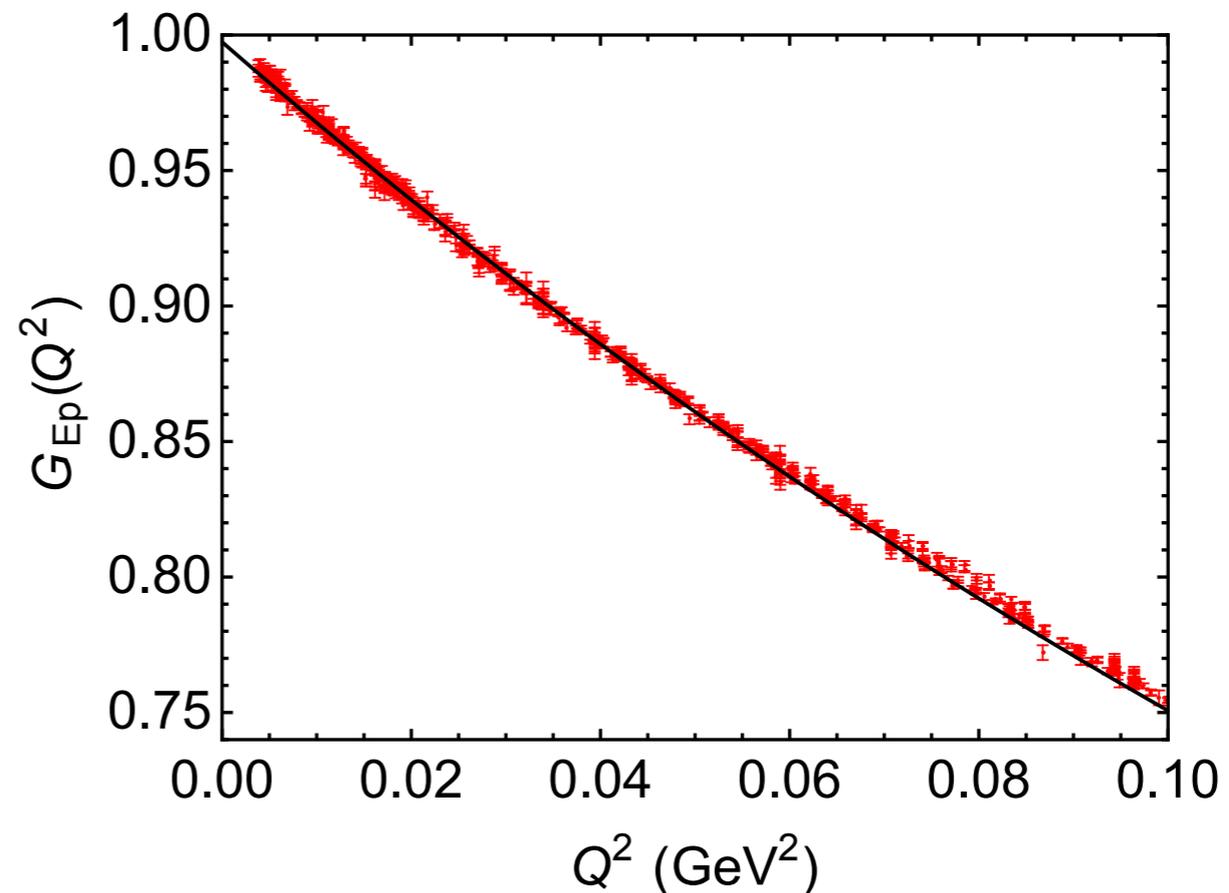
- Higinbotham, Kabir, Lin, Meekins, Norum, Sawatzky (1510.01293)
- Also emphasized use of low Q^2 range data.
- Additional contribution: resurrecting Saskatoon 1974 and Mainz 1980 data. Excellent data. $Q^2 < 0.031$ and < 0.055 GeV^2 , resp.
- Excellent discussion of statistics relevant to deciding how many parameters to use. Argued for reliability of even linear fits in this data range.
- Obtained R_E compatible with muonic atomic data, 0.84 fm

Recent e - p analyses, III

- Horbatsch and Hessels (1509.05644)
- Also believe “the rms charge radius of the proton is a small- Q^2 concept. Thus, if possible, it should be determined from low- Q^2 data.”
- Look at Mainz 2010 data restricting $Q^2 < 0.1 \text{ GeV}^2$. Analyze two ways, get bifurcated result.
- their take-away conclusion: scattering data can't help
- proton radius problem remains, but between electron atomic physics and muon atomic physics

H. H.

- dipole fit: $G_E = (1 + R_E^2 Q^2 / 12)^{-2}$, similarly for G_M
- Got $R_E = 0.842(2)$ fm and $R_M = 0.800(2)$ fm
- Fits look o.k.

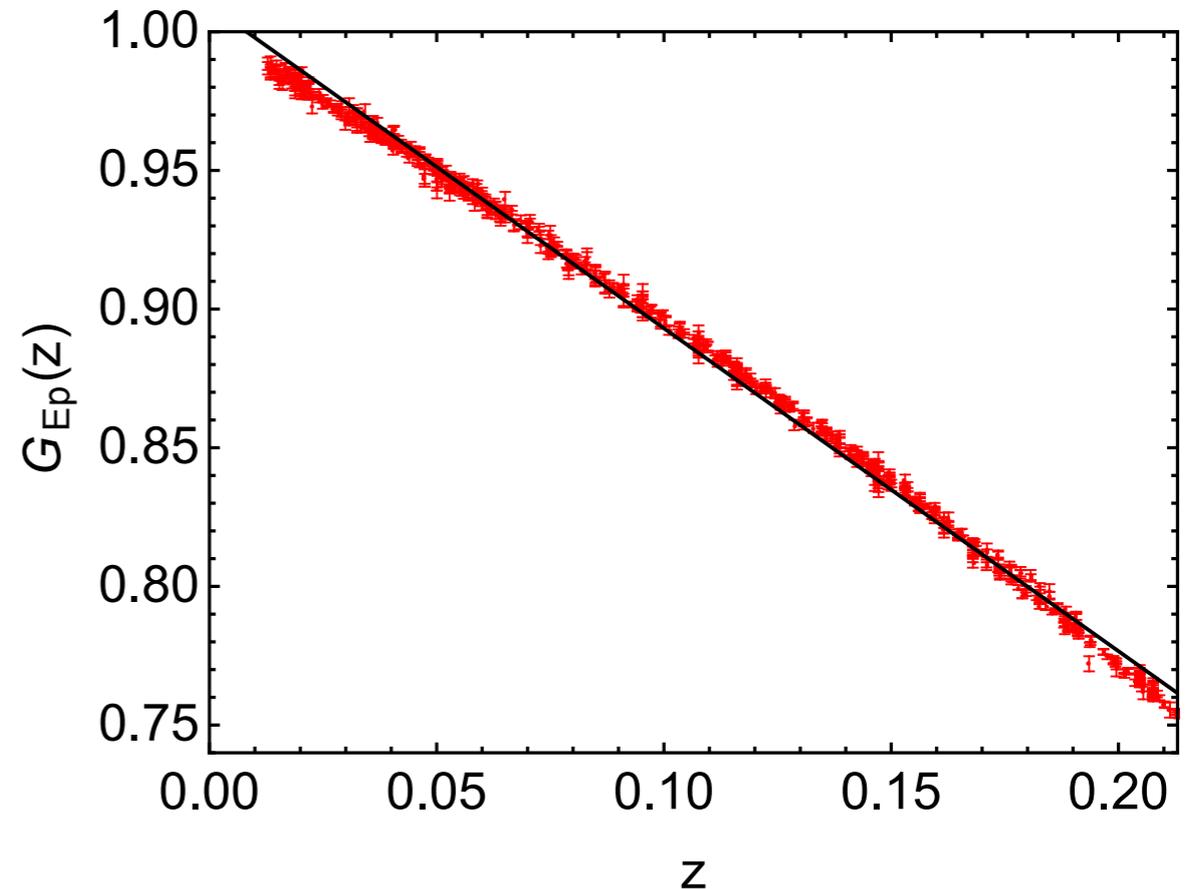


H. H.

- z variable expansion,
$$z = \frac{\sqrt{4m_\pi^2 + Q^2} - 2m_\pi}{\sqrt{4m_\pi^2 + Q^2} + 2m_\pi}$$
- reason: for functions like G_E , polynomial expansion in z converges for all $0 < z < 1$, *i.e.*, all spacelike Q^2
- Expansion linear in z ,
$$G_E = 1 - (8/3)m_\pi^2 R_E^2 z$$
- Now got $R_E = 0.888(1)$ fm and $R_M = 0.874(2)$ fm

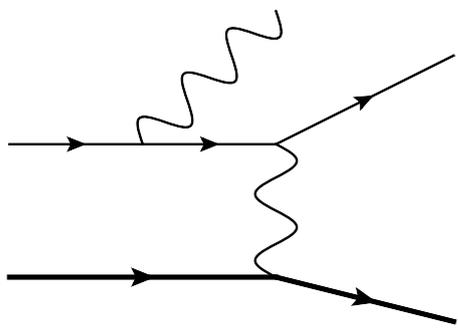
H.H.

- Fit looks not good
- This is $Q^2 < 0.1 \text{ GeV}^2$ data
- Concavity when plotted in this variable not well fit by linear polynomial
- Overly large R_E not surprise
- (Plot is mine; theirs would look better, but principal problem remains. Can explain.)
- My take-away 1: should include z^2 term if doing this way. My result when doing so is $R_E = 0.838 \text{ fm}$.
- My take-away 2: low R_E o.k., high R_E not o.k.



Scattering future

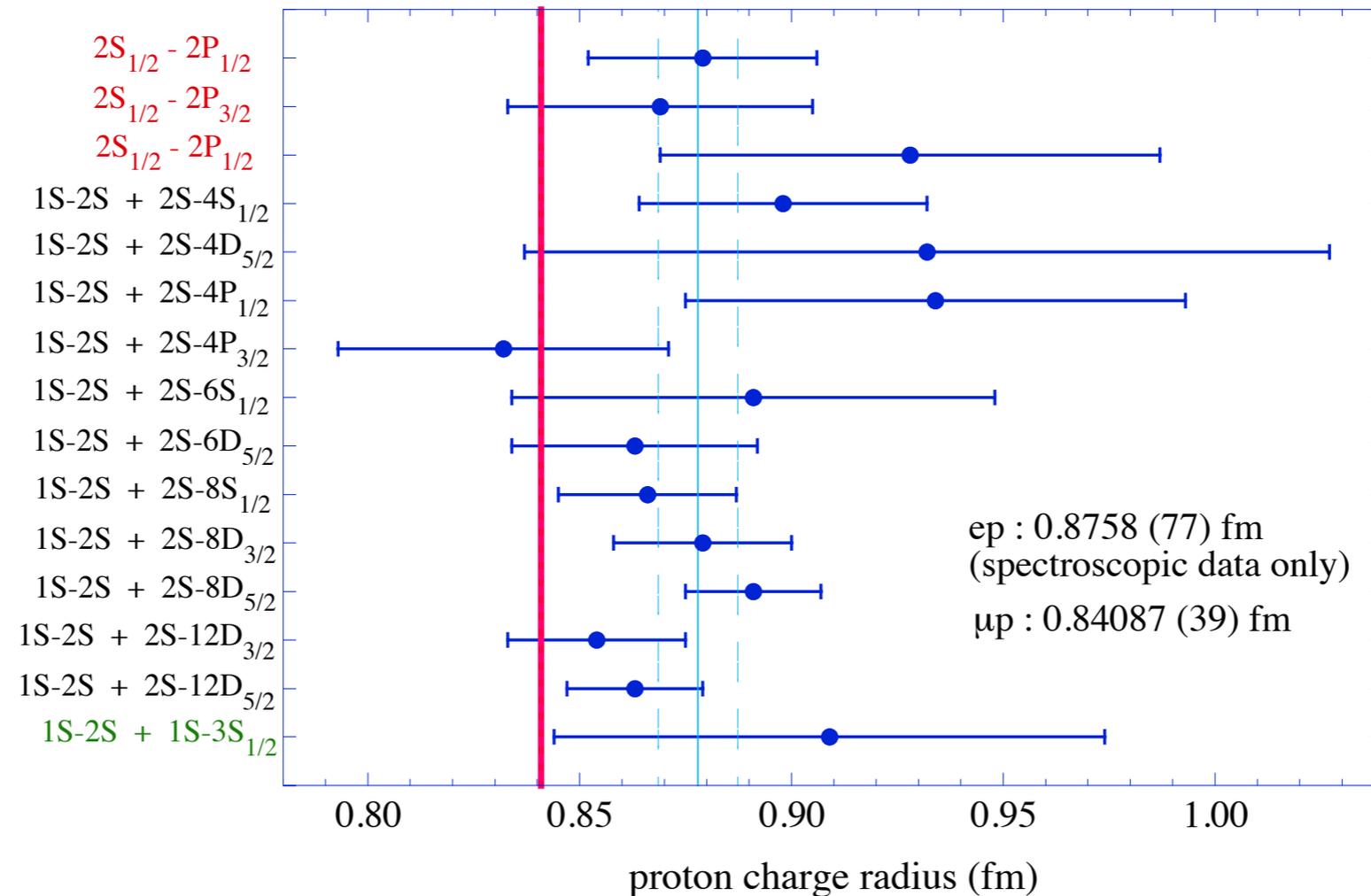
- *A*: Continue discussing statistics and extrapolations
- *B*: Do further experiments to lower lowest Q^2 , and also do μ scattering
- *1*: PRad at JLab: Just target and detector screen, allowing very small scattering angles. Anticipate $Q^2|_{\text{low}} \approx 0.0002 \text{ GeV}^2$. Running now!



- *2*: ISR (Initial State Radiation) at Mainz. Photon radiation takes energy out of electron, allowing lower Q at given scattering angle. Anticipate $Q^2|_{\text{low}} \approx 0.0001 \text{ GeV}^2$. Data taken, more data to be taken; under analysis.

- *3*: MUSE = Muon scattering experiment at the PSI. Anticipate $Q^2|_{\text{low}} \approx 0.002 \text{ GeV}^2$. Production runs 2017/2018.

Back to atomic spectroscopy



- Same plot, but μ -H value added
- Possible: correlated systematic errors. There are more measurements than independent expt'l groups.

Short term future

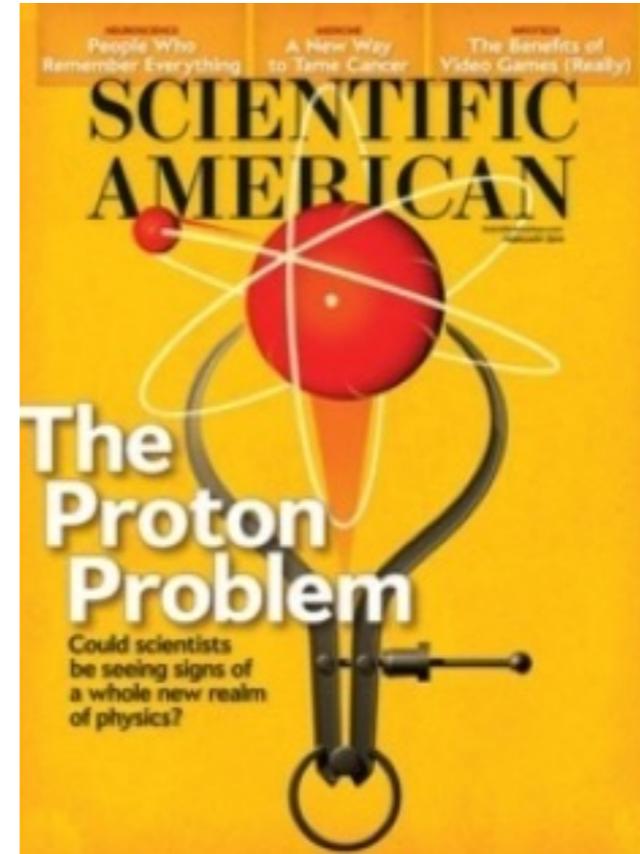
- Several independent groups are doing more precise experiments. The first 3 (at least) can individually get the proton radius to under 1%.
 - York University (Canada): Ordinary hydrogen $2S-2P$ Lamb shift
(“We have run into some systematic effects that we want to understand better”)
 - MPI Quantum Optics (Garching): $2S-4P$ transition
(“...about $2S-4P$: things are progressing great, but you haven't missed anything concerning publications. I will be happy to let you know as soon as there is some news from our side.”)
 - Laboratoire Kastler Brossel (Paris): $1S-3S$ transition
(“...In parallel, we have another failure with a RF amplifier, we put another which has failed after one week... We are fighting with a little bit of luck I hope to get a result for $1S-3S$ before the end of this year.”)
 - NIST (USA): Measure Rydberg using “Rydberg” states, very high n states, uncontaminated by proton size. (Very relevant: recall previous discussion.)
 - + National Physical Lab (U.K.), several $2S-nS, D$ transitions
- Under way, may see results soon. Will be important, one way or another.

Exotic possibilities

- Breakdown of Lorentz invariance? (Gomes, Kostelecky, & Vargas, 2014)
- Unanticipated QCD corrections? (G. Miller, 2013)
- Higher-dimensional gravity(?) (1509.08735, Dahia and Lemos)
- Renormalization group effects for effective particles (Glazek, 2014)
- Will consider breakdown of muon-electron universality. New particle coupling to muons and protons. Small or no coupling to other particles.
- References (optimistic or neutral): Tucker-Smith & Yavin (2011), Batell, McKeen, & Pospelov (2011), Brax & Burrage (2011), Rislw & Carlson (2012, 2014), Marfatia & Keung (2015), Pauk & Vanderhaeghen (2015)
- References (pessimistic): Barger, Chiang, Keung, Marfatia (2011, 2012), Karshenboim, McKeen, & Pospelov (2014)

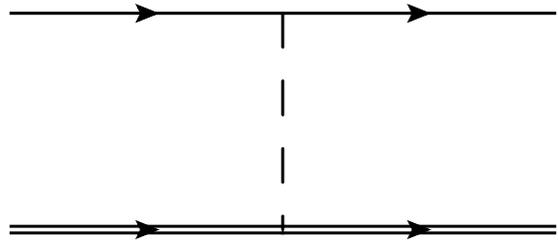
μ -H Lamb shift

- Point: Experimenters do not directly measure proton radius. Measure energy deficit, 310 μeV . Interpret as proton radius deficit.
- Idea: Proton radius unchanged. Energy deficit due to new force, carried by exchange of new particle.
- New particle is scalar or vector. Pseudoscalar or axial vector have little effect on Lamb shift for similar couplings.



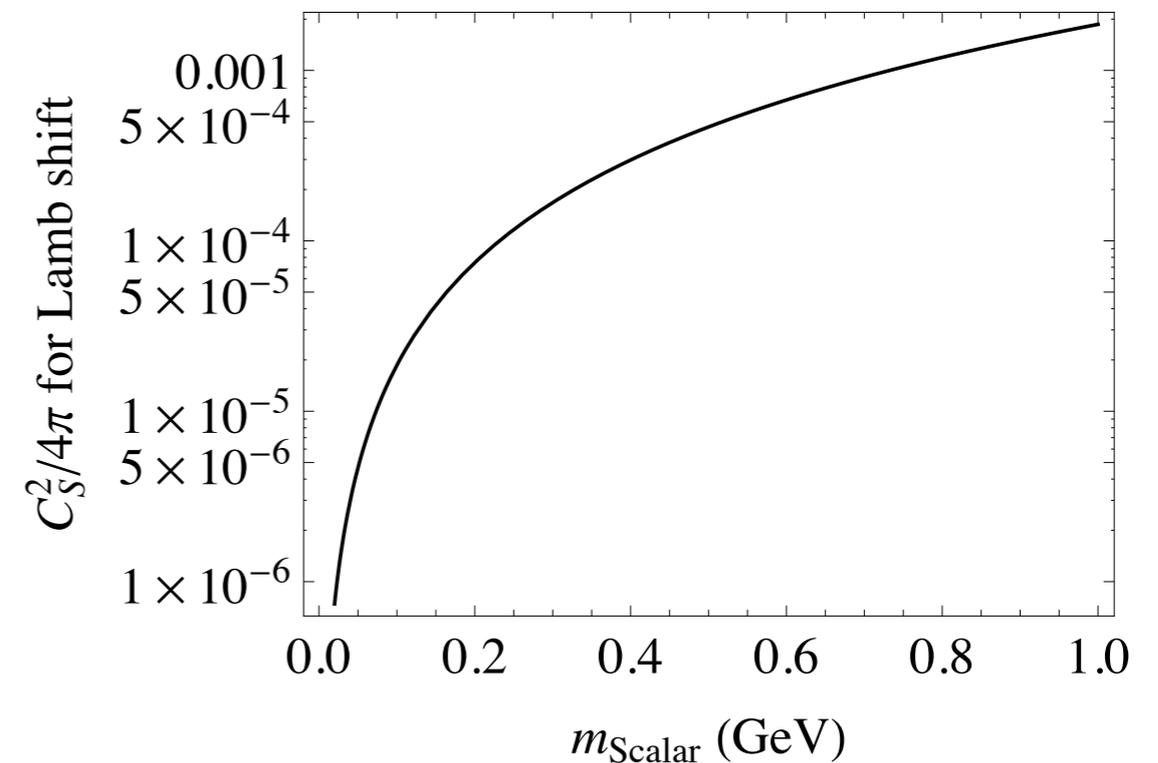
Energy shift

- e.g., scalar case



$$V(r) = -\frac{C_S^\mu C_S^p}{4\pi r} e^{-Mr}$$

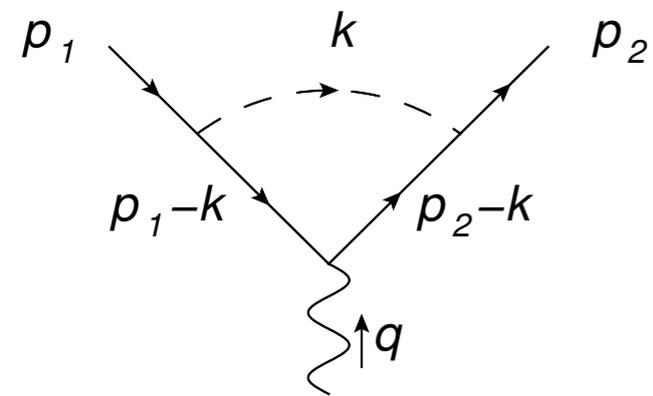
- Pick $C_S^\mu C_S^p$ to give 320 μeV for given m_ϕ .
(Plot for $C_S^\mu = C_S^p$.)



Other muon processes

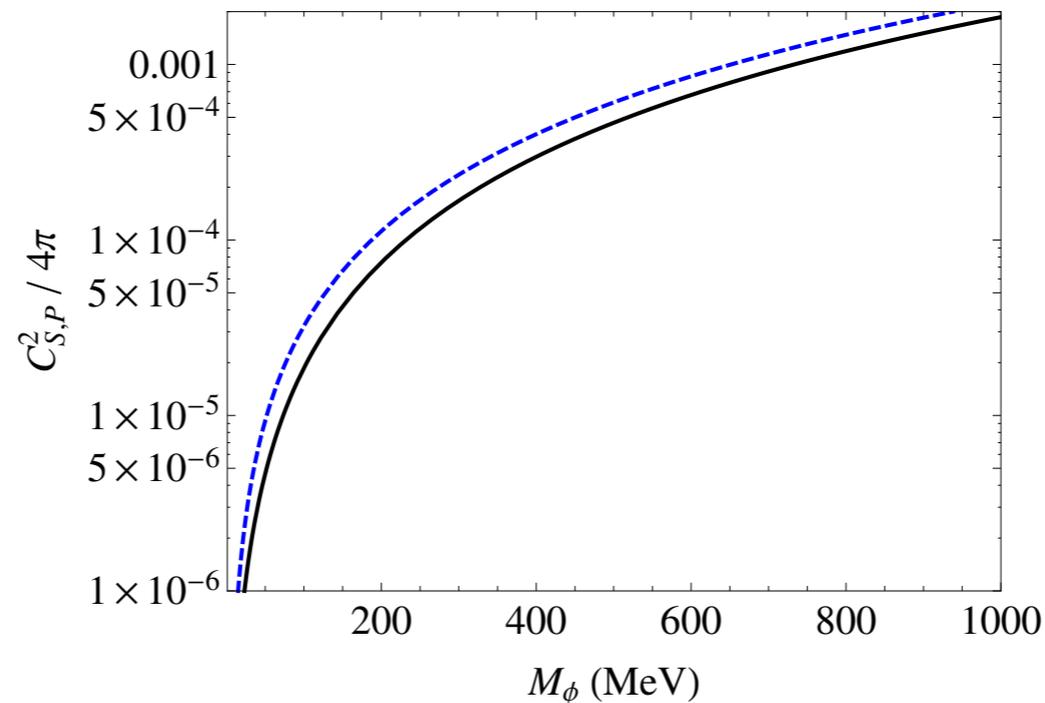
- Worry about other processes where new particle couples to muons. *First:*
 - Loop corrections to μ magnetic moment
 - (Reminder: 3 σ discrepancy between measured and standard model calculated $(g-2)_\mu$.)
 - If new exchange particle light, effect on $(g-2)_\mu$ small enough (Tucker-Smith & Yavin). Otherwise, need to fix by fine tuning.

Fixing $(g-2)_\mu$



- Will need extra particle and fine tuning
- Lucky break: corrections to $(g-2)$ from regular vector and axial vector have opposite sign. Same is true of scalar and pseudoscalar.
- With extra particle, have new coupling, say C_{P^j} . Choose coupling to cancel in $(g-2)_\mu$. Does not much affect Lamb shift.
- Couplings now fixed, albeit mass sensitive. Hence predictions for other processes fixed.

(Fine tuning plot)



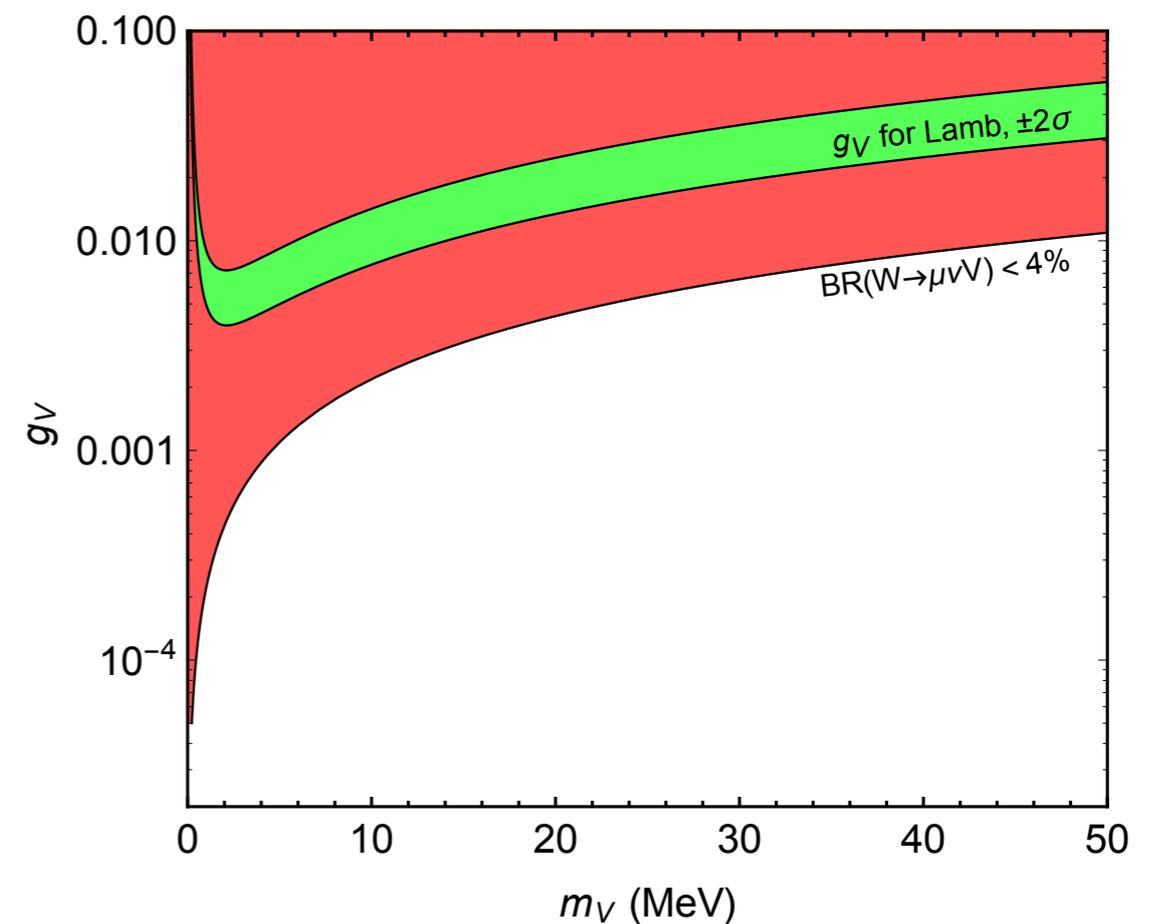
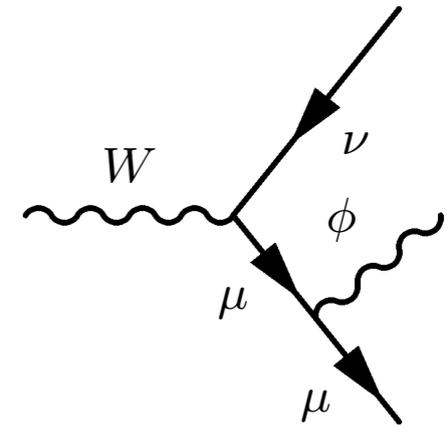
- Above for scalar-pseudoscalar
- Low enough mass, cancellation not needed (TSY)
- Couplings now fixed, albeit mass sensitive.
- \therefore Predictions for other processes now fixed.

BSM problems

1. Radiative corrections to W-decay
2. Non-effect in He

W decay

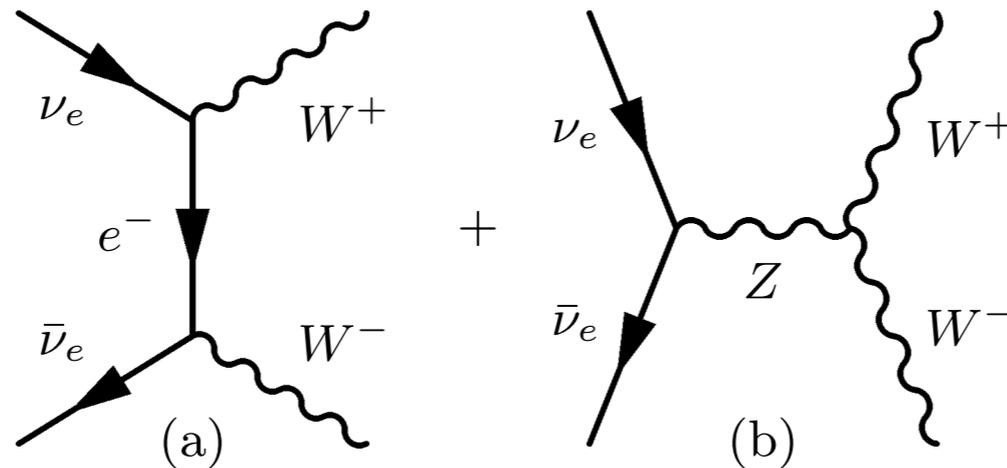
- Remark of Karshenboim, McKeen, and Pospelov: fast growth with energy of amplitudes involving massive vector particles
- If light new particle ϕ or V coupling to muon, it gives large radiative correction to W decay via $W \rightarrow \mu\nu V$, larger than measured error in W decay rate.



Red: forbidden
Fig. based on
Karshenboim et al. (2014)

W decay

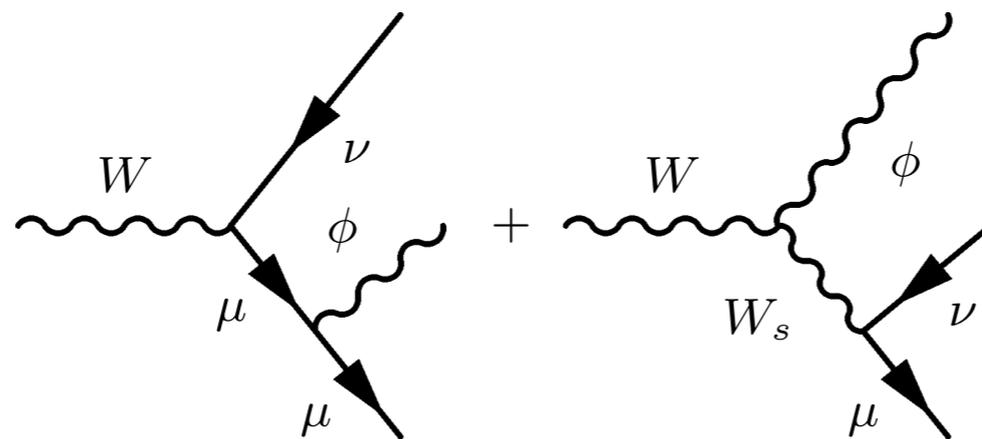
- Reminiscent of (from early days of W.S. model),



- Left diagram grew unpleasantly at high energy, right diagram cancelled it at high energy, was small at lower energy

Here

- Should have interaction also with W to make theory renormalizable.



- Problem ameliorated (see Freid and me (2015))

Helium Lamb shift

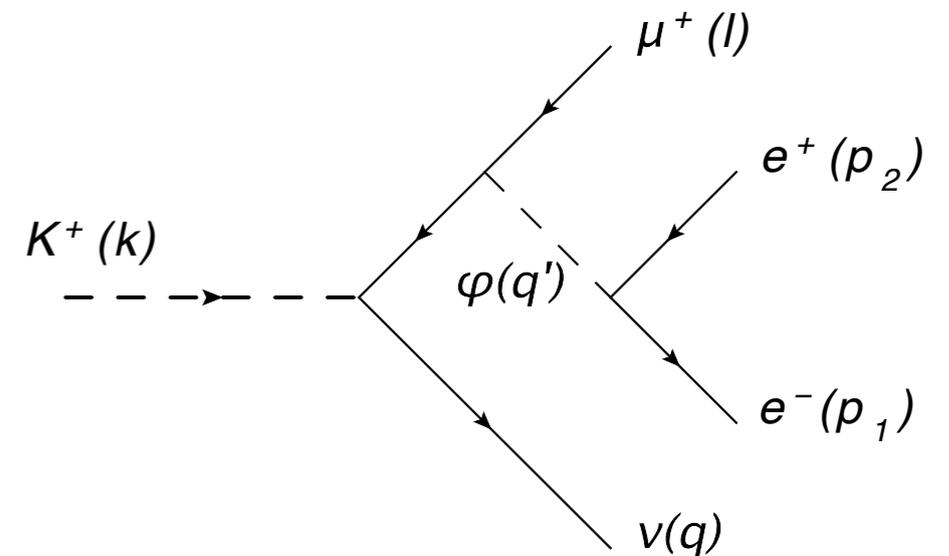
- A pair (^3He & ^4He) of non-contradictory results.
- He radii measured in electron scattering, to about 1/4%. These radii go into prediction for Lamb shift.
- Preliminary data on $\mu\text{-He}$ Lamb shift agrees with prediction, to about 1σ . If due to heavy BSM particle exchange, should disagree by about 5σ .
- How does mass creep in?

Heavy atom Lamb shift

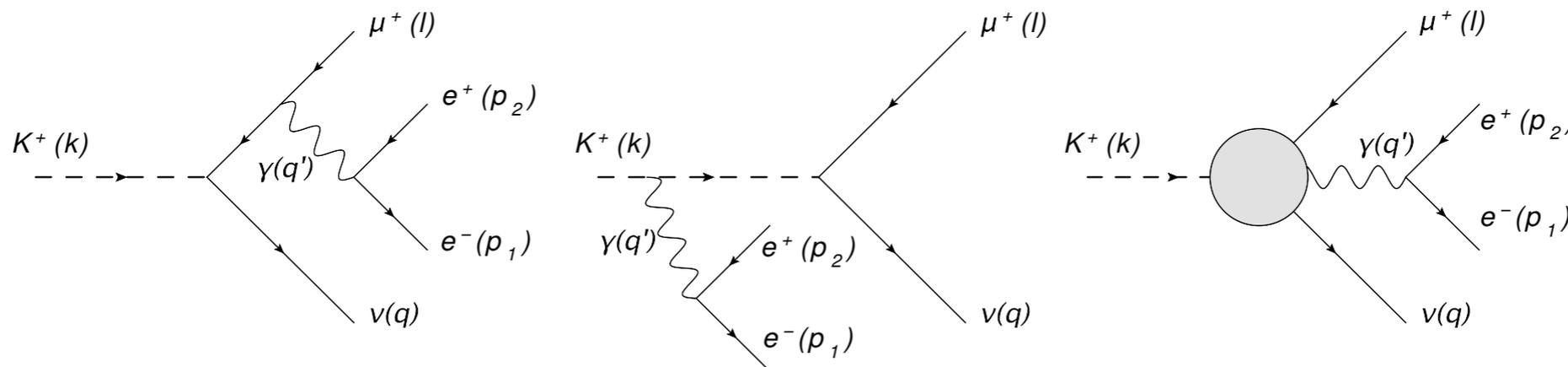
- Physics: Range of potential is controlled by mass. Light mass, long range, like Coulomb potential, does not split S and P states.
- Application: $Z=2$ helium has orbital muons closer to nucleus than $Z=1$ hydrogen. What looks like long range to helium is short range to hydrogen, if mass chosen correctly.
- Quick bottom line: Get result for proton big enough and for He small enough if $m_\phi \approx 1$ MeV.

New force seen elsewhere?

- Older suggestion: correction to K -decay, *viz.*, $K \rightarrow \mu \nu e^+ e^-$ as correction to $K \rightarrow \mu \nu$.

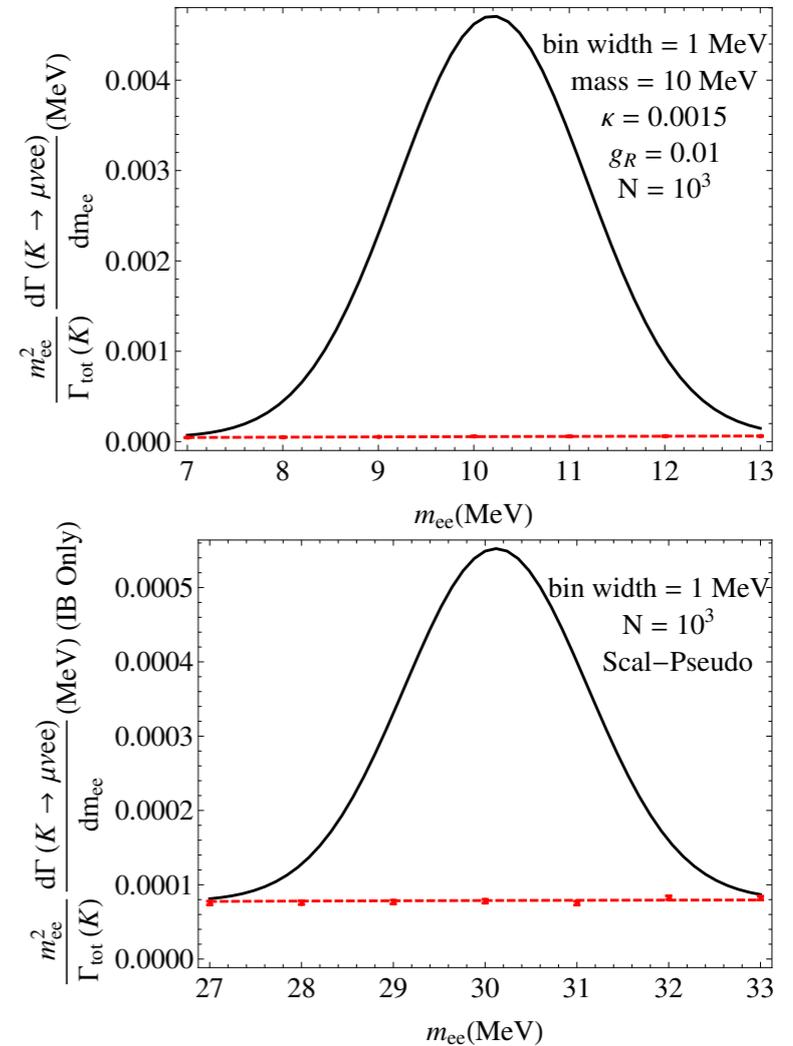


- Of course, QED gives same final state, with smooth (calculable) spectrum of e^+e^- .



φ visible?

- φ (new BSM particle) will give bump. Size calculable.
- Is it observable?
Wow, Yes. (If it exists.)
[Red = QED background, solid = bump from φ]



- Note: TREK experiment (E36) at JPARC (Japan) will observe 10^{10} kaon decays, or about 200,000 $K \rightarrow \mu \nu e^+ e^-$ events, about 1000 per MeV bin in the mass range we are considering. (Thanks to M. Kohl)

Reminder: new data coming

- New CREMA measurements (out at conferences, 2013/14)
- 3 scattering expts. underway or coming
- Electron deuteron scattering (Griffioen *et al.*, Mainz) (data taken)
- 5 atomic energy level measurements
- TREK at JPARC
- Maybe also: Trumuonium ($\mu^+\mu^-$) at JLab

Ending

- Remarkable: 6 years after the first announcement, the problem persists.
- Interestingly little discussion of the correctness of the μ -H Lamb shift data.
- Serious and good new data coming.
- Opinion: Either
 - All radii correct, and BSM—muonic specific force—is explanation despite problems, or
 - The electron based radius measurements will reduce to the muonic value.
- Comment: the theory for $(g-2)_\mu$ cannot be considered settled until the proton radius problem is settled. Further, there may be striking corrections to other processes that involve muons.

The end for now!