The Proton Charge

Radius Puzzle

Carl Carlson
William and Mary (1693)
University of Pavia (1361)
19 May 2016
The puzzle

- Measure charge radius of the proton different ways, get different answers
- Difference is 7 s.d. (was 5 s.d. when first announced, 2010)
- Why? Don’t yet know.
This talk

1. The measurements: where the differences came from

2. Suggested explanations
   
   A. Ordinary explanations
      
      • Maybe some things are harder than they seem

   B. Exotic explanations
      
      • Will discuss: Is it Physics Beyond the Standard Model?
      
      • Will mention: other possibilities (later)

3. Highlight: List of coming relevant data
This talk

1. The measurements: where the differences came from

2. Suggested explanations
   A. Ordinary explanations
      • Maybe some things are harder than they seem
   B. Exotic explanations
      • Will discuss: Is it Physics Beyond the Standard Model?
      • Will mention: other possibilities (later)

3. Highlight: List of coming relevant data
   • some arriving in 2016 (maybe!)
Measuring proton radius

- Two methods: scattering or atomic spectroscopy
- Two probes: electrons or muons
- *I.e.*,
  - $e$-$p$ elastic scattering
  - $\mu$-$p$ elastic scattering
  - spectroscopy of electronic Hydrogen
  - spectroscopy of muonic Hydrogen

- 4 categories of measurements, 3 done with sufficient accuracy (and more data coming), $\mu$-$p$ scattering coming
e-\( p \) scattering

• Measure differential cross section, fit results to form factors,

\[
\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2)
\]

\[
\left[ \tau = \frac{Q^2}{4m_p^2} ; \quad \frac{1}{\epsilon} = 1 + 2(1 + \tau) \tan^2(\theta_e/2) \right]
\]

• Low \( Q^2 \), mainly sensitive to \( G_E \).

• Extrapolate to \( Q^2 = 0 \), whence

\[
R_E^2 = -6 \left( \frac{dG_E}{dQ^2} \right)_{Q^2=0}
\]
Extra: What is the proton radius?

- By this I mean, what is the definition?
- NR, easy. Given w. f., obtain RMS radius,

\[ R^2 = \langle r^2 \rangle = \int d^3r r^2 |\psi(r)|^2 \]

- In concept, obtaining proton radius by electron scattering same as obtaining radius of H-atom w. f. by scattering an external electron off the bound electron. Worked out by Bethe in 1930's

- Rutherford scattering cross section off pointlike target,

\[ \left. \frac{d\sigma}{d\Omega} \right|_{\text{point}} = \left( \frac{k q Q}{4E \sin^2(\theta/2)} \right)^2 \]

Straight out of Taylor's UG Classical Mechanics text
More extra: Def’n of proton radius

- is modified for scattering off extended target, but just becomes

\[
\frac{d\sigma}{d\Omega} = \left. \frac{d\sigma}{d\Omega} \right|_{\text{point}} \times \left( G(Q^2) \right)^2
\]

- \( Q \) = momentum transfer in scattering
- \( G(Q^2) \) is “form factor”, given NR by

\[
G(Q^2) = \int d^3r \, e^{i\vec{Q} \cdot \vec{r}} \, |\psi(r)|^2
\]

- easy:

\[
G(Q^2) = 1 - \frac{1}{6} \langle r^2 \rangle Q^2 + \ldots
\]

De facto: measure radius by measuring form factor at small momentum transfer and looking at expansion
Low-$Q^2$ scattering data

- Most extensive current data comes from Mainz, which has an electron accelerator, and is also city of Gutenberg

- Data, Jan Bernauer et al., PRL 2010 (and later articles).

- marked by low $Q^2$ data, range 0.004 to 1 GeV$^2$

- From their analysis,

$$R_E = 0.879(8) \text{ fm}$$
Atomic energy level splittings

• Basic: Schrödinger equation, H-atom, point protons

\[ E = -\frac{\text{Ryd}}{n^2}, \quad \text{where} \quad \text{Ryd} = \frac{1}{2} m_e \alpha^2 \approx 13.6 \text{eV} \]

• plus QED corrections

• plus finite size proton, pushing energy upward a bit.

\[ \Delta E_{\text{finite size}} = \frac{2\pi \alpha}{3} \phi_{nS}^2(0) R_E^2 \]

fine print: \[ \phi_{nS}^2(0) = (m_r \alpha)^3 / (n^3 \pi) \]
measure energy accurately

\[\Leftrightarrow\] measure radius

- Reminder, H-atom energy levels (diagram not to scale)
Atomic results

\[ 2S_{1/2} - 2P_{1/2} \]
\[ 2S_{1/2} - 2P_{3/2} \]
\[ 2S_{1/2} - 2P_{1/2} \]
\[ 1S-2S + 2S-4S_{1/2} \]
\[ 1S-2S + 2S-4D_{5/2} \]
\[ 1S-2S + 2S-4P_{1/2} \]
\[ 1S-2S + 2S-4P_{3/2} \]
\[ 1S-2S + 2S-6S_{1/2} \]
\[ 1S-2S + 2S-6D_{5/2} \]
\[ 1S-2S + 2S-8S_{1/2} \]
\[ 1S-2S + 2S-8D_{3/2} \]
\[ 1S-2S + 2S-8D_{5/2} \]
\[ 1S-2S + 2S-12D_{3/2} \]
\[ 1S-2S + 2S-12D_{5/2} \]
\[ 1S-2S + 1S-3S_{1/2} \]

ep : 0.8758 (77) fm
(spectroscopic data only)
All electron results

- Consistent
- Combined by Committee on Data in Science and Technology (CODATA, 2014 value),

\[ R_E = 0.8751(61) \text{ fm} \]
Then in 2010 …

• CREMA = Charge Radius Experiment with Muonic Atoms

• Did atomic physics, specifically Lamb shift, with muons (muon = electron, but weighs 200 times more, orbits 200 times closer).

• Goal: measure proton radius with factor 10 smaller uncertainty
CREMA

- $2S-2P$ Lamb shift in $\mu$-H.
- Measured two lines,

  - $F=2 \rightarrow F=1$ 3.7 meV
  - $F=1 \rightarrow F=0$ finite size effect 23 meV
  - $F=1 \rightarrow F=0$ HFS 23 meV
  - $F=1 \rightarrow F=0$ HFS 3.7 meV
  - $F=1 \rightarrow F=0$ ca. 206 meV

- pubs: upper line, Pohl et al., Nature 2010
  other line Antognini et al., Science 2013

- Interpreting finite size effect in terms of proton radius,
  \[ R_E = 0.84087(39) \text{ fm} \]
- Whoops: result 4% or 7σ small
Other data-deuteron

- Reported at conferences 2013
- 2015 experimenters circulate draft of theory paper!
- Measured three lines

\[
\begin{align*}
2S_{1/2} & \quad F=1/2, F=3/2 \\
2P_{1/2} & \quad F=3/2, F=1/2, F=5/2 \\
2P_{3/2} & \quad F=3/2, F=1/2
\end{align*}
\]

ca. 215 meV

- Quick summary: if proton radius is shrunken, the deuteron radius is also.
Other data — Helium

• New 2013/2014 data

• $\mu$-$^4$He at Mainz Proton Radius Workshop, 2014

• $\mu$-$^3$He at Gordon Conference, N.H., 2014

• Quick summary: He radii from $\mu$ Lamb shift in accord with electron scattering radii.
Explanations?

• Hard to see problems with $\mu$ experiment
  • Hard to get working
  • But once working, easy to analyze

• Problems with analysis of electron experiments? But there are a lot of them.

• BSM explanations?
  • If so, further tests?
Review e-\(p\) scattering data

• Point: Measurements at finite \(Q^2\). Need to extrapolate to \(Q^2 = 0\) to obtain charge radius. (Mainz group itself: \(R_E = 0.879(8)\) fm.)

• Because of importance, others have tried, using different ways of fitting data. Three recent fits found “big” values:

• Graczyk & Juszczak (2014), using Bayesian ideas and pre-Mainz world data, obtained

\[ R_E = 0.899(3) \text{ fm.} \]

• Lee, Arrington, & Hill (2015) using Mainz data and neat mapping ideas to ensure convergence of expansions, obtained

\[ R_E = 0.895(20) \text{ fm.} \]

• Arrington & Sick (2015) found

\[ R_E = 0.879(11) \text{ fm.} \]
But...

• Several recent fits found “small” values \((i.e., \text{compatible with muonic Lamb shift experiment})\):

  • Lorenz, Meißner, Hammer, & Dong (2015 and earlier), dispersive ideas, also using timelike data, obtained \(R_E = 0.840(15) \text{ fm}\).

  • Horbatsch and Hessels (1509.05644)

  • Carlson, Griffioen, Maddox (1509.06676)

  • Higinbotham, Kabir, Lin, Meekins, Norum, Sawatzky (1510.01293)
Recent e-\(p\) analyses, I

- Maddox et al. (1509.06676)

- First viewpoint: Charge radius is a \(Q^2 = 0\) concept, should be able to obtain just from low \(Q^2\) data.

- Technical: Form factor is analytic function of \(Q^2\), except for cut starting at \(4m_{\pi}^2\). Hence, polynomial expansion in \(Q^2\) converges for \(Q^2 < 4m_{\pi}^2\).

- For low \(Q^2\) data, use \(Q^2 < 0.02 \text{ GeV}^2\) (243 data points) linear plus quadratic in \(Q^2\), get \(R_E = 0.850(19) \text{ fm}\)
Recent $e-p$ analyses, I

- Second viewpoint: fitting whole Q2 data range with complicated (i.e., many parameters) function leads to dangers in extrapolation.

- Fit whole Mainz 2010 data set with simpler functions (i.e., 4 or so parameters), that extrapolate more reliably. From collection of such fits quote

$$R_E = 0.840(16) \text{ fm}$$
Recent $e-p$ analyses, II

- Higinbotham, Kabir, Lin, Meekins, Norum, Sawatzky (1510.01293)

- Also emphasized use of low $Q^2$ range data.

- Additional contribution: resurrecting Saskatoon 1974 and Mainz 1980 data. Excellent data. $Q^2 < 0.031$ and $< 0.055$ GeV$^2$, resp.

- Excellent discussion of statistics relevant to deciding how many parameters to use. Argued for reliability of even linear fits in this data range.

- Obtained $R_E$ compatible with muonic atomic data, 0.84 fm
Recent e-\(p\) analyses, III

- Horbatsch and Hessels (1509.05644)

- Also believe “the rms charge radius of the proton is a small-\(Q^2\) concept. Thus, if possible, it should be determined from low-\(Q^2\) data.”

- Look at Mainz 2010 data restricting \(Q^2 < 0.1\) GeV\(^2\). Analyze two ways, get bifurcated result.

- their take-away conclusion: scattering data can’t help

- proton radius problem remains, but between electron atomic physics and muon atomic physics
H. H.

- dipole fit: $G_E = (1 + R_E^2 Q^2 /12)^{-2}$, similarly for $G_M$
- Got $R_E = 0.842(2)$ fm and $R_M = 0.800(2)$ fm
- Fits look o.k.
• $z$ variable expansion, $z = \frac{\sqrt{4m^2_\pi + Q^2 - 2m_\pi}}{\sqrt{4m^2_\pi + Q^2 + 2m_\pi}}$

• reason: for functions like $G_E$, polynomial expansion in $z$ converges for all $0 < z < 1$, i.e., all spacelike $Q^2$

• Expansion linear in $z$, $G_E = 1 - \frac{8}{3}m^2_\pi R^2_E z$

• Now got $R_E = 0.888(1)$ fm and $R_M = 0.874(2)$ fm
H. H.

- Fit looks not good
- This is $Q^2 < 0.1$ GeV$^2$ data
- Concavity when plotted in this variable not well fit by linear polynomial
- Overly large $R_E$ not surprise
- (Plot is mine; theirs would look better, but principal problem remains. Can explain.)
- My take-away 1: should include $z^2$ term if doing this way. My result when doing so is $R_E = 0.838$ fm.
- My take-away 2: low $R_E$ o.k., high $R_E$ not o.k.
Scattering future

• A: Continue discussing statistics and extrapolations

• B: Do further experiments to lower lowest $Q^2$, and also do $\mu$ scattering

  1: PRad at JLab: Just target and detector screen, allowing very small scattering angles. Anticipate $Q^2_{\text{low}} \approx 0.0002 \text{ GeV}^2$. Running now!

  2: ISR (Initial State Radiation) at Mainz. Photon radiation takes energy out of electron, allowing lower $Q$ at given scattering angle. Anticipate $Q^2_{\text{low}} \approx 0.0001 \text{ GeV}^2$. Data taken, more data to be taken; under analysis.

  3: MUSE = Muon scattering experiment at the PSI. Anticipate $Q^2_{\text{low}} \approx 0.002 \text{ GeV}^2$. Production runs 2017/2018.
Back to atomic spectroscopy

- Same plot, but $\mu$-H value added

- Possible: correlated systematic errors. There are more measurements than independent expt’l groups.
Short term future

• Several independent groups are doing more precise experiments. The first 3 (at least) can individually get the proton radius to under 1%.

• York University (Canada): Ordinary hydrogen 2S-2P Lamb shift
  ("We have run into some systematic effects that we want to understand better")

• MPI Quantum Optics (Garching): 2S-4P transition
  ("…about 2S-4P: things are progressing great, but you haven’t missed anything concerning publications. I will be happy to let you know as soon as there is some news from our side.")

• Laboratoire Kastler Brossel (Paris): 1S-3S transition
  ("…In parallel, we have another failure with a RF amplifier, we put another which has failed after one week… We are fighting with a little bit of luck I hope to get a result for 1S-3S before the end of this year.")

• NIST (USA): Measure Rydberg using “Rydberg” states, very high n states, uncontaminated by proton size. (Very relevant: recall previous discussion.)

• + National Physical Lab (U.K.), several 2S–nS,D transitions

• Under way, may see results soon. Will be important, one way or another.
Exotic possibilities

• Breakdown of Lorentz invariance? (Gomes, Kostelecky, & Vargas, 2014)
• Unanticipated QCD corrections? (G. Miller, 2013)
• Higher-dimensional gravity(?) (1509.08735, Dahia and Lemos)
• Renormalization group effects for effective particles (Glazek, 2014)

• Will consider breakdown of muon-electron universality. New particle coupling to muons and protons. Small or no coupling to other particles.


µ-H Lamb shift

• Point: Experimenters do not directly measure proton radius. Measure energy deficit, 310 µeV. Interpret as proton radius deficit.

• Idea: Proton radius unchanged. Energy deficit due to new force, carried by exchange of new particle.

• New particle is scalar or vector. Pseudoscalar or axial vector have little effect on Lamb shift for similar couplings.
Energy shift

• e.g., scalar case

\[ V(r) = -\frac{C_S^\mu C_S^p}{4\pi r} e^{-Mr} \]

• Pick \( C_S^\mu C_S^p \) to give 320 \( \mu eV \) for given \( m_\phi \).
  (Plot for \( C_S^\mu = C_S^p \).)
Other muon processes

• Worry about other processes where new particle couples to muons. First:

  • Loop corrections to $\mu$ magnetic moment

  • (Reminder: $3 \sigma$ discrepancy between measured and standard model calculated $(g-2)_\mu$.)

  • If new exchange particle light, effect on $(g-2)_\mu$ small enough (Tucker-Smith & Yavin). Otherwise, need to fix by fine tuning.
Fixing \((g-2)_\mu\)

- Will need extra particle and fine tuning

- Lucky break: corrections to \((g-2)\) from regular vector and axial vector have opposite sign. Same is true of scalar and pseudoscalar.

- With extra particle, have new coupling, say \(C_P\). Choose coupling to cancel in \((g-2)\). Does not much affect Lamb shift.

- Couplings now fixed, albeit mass sensitive. Hence predictions for other processes fixed.
(Fine tuning plot)

- Above for scalar-pseudoscalar
- Low enough mass, cancellation not needed (TSY)
- Couplings now fixed, albeit mass sensitive.
- ∴ Predictions for other processes now fixed.
BSM problems

1. Radiative corrections to W-decay
2. Non-effect in He
W decay

- Remark of Karshenboim, McKeen, and Pospelov: fast growth with energy of amplitudes involving massive vector particles

- If light new particle $\phi$ or $V$ coupling to muon, it gives large radiative correction to $W$ decay via $W \rightarrow \mu\nu V$, larger than measured error in $W$ decay rate.
W decay

- Reminiscent of (from early days of W.S. model),

\[ \text{FIG. 1: The illustrative process} \]

- Left diagram grew unpleasantly at high energy, right diagram cancelled it at high energy, was small at lower energy
Here

- Should have interaction also with $W$ to make theory renormalizable.

- Problem ameliorated (see Freid and me (2015))
Helium Lamb shift

• A pair ($^3$He & $^4$He) of non-contradictory results.

• He radii measured in electron scattering, to about 1/4%. These radii go into prediction for Lamb shift.

• Preliminary data on $\mu$-He Lamb shift agrees with prediction, to about 1$\sigma$. If due to heavy BSM particle exchange, should disagree by about 5$\sigma$.

• How does mass creep in?
Heavy atom Lamb shift

• Physics: Range of potential is controlled by mass. Light mass, long range, like Coulomb potential, does not split S and P states.

• Application: \(Z=2\) helium has orbital muons closer to nucleus than \(Z=1\) hydrogen. What looks like long range to helium is short range to hydrogen, if mass chosen correctly.

• Quick bottom line: Get result for proton big enough and for He small enough if \(m\phi \approx 1\) MeV.
New force seen elsewhere?

- Older suggestion: correction to $K$-decay, viz., $K \rightarrow \mu \nu e^+e^-$ as correction to $K \rightarrow \mu \nu$.

- Of course, QED gives same final state, with smooth (calculable) spectrum of $e^+e^-$. 
**φ** visible?

- **φ** (new BSM particle) will give bump. Size calculable.

- Is it observable? Wow, Yes. (If it exists.) [Red = QED background, solid = bump from φ]

- Note: TREK experiment (E36) at JPARC (Japan) will observe $10^{10}$ kaon decays, or about 200,000 $K \rightarrow \mu^+ e^- e^-$ events, about 1000 per MeV bin in the mass range we are considering. (Thanks to M. Kohl)

Plots from Rislow and me (2014)
Reminder: new data coming

- New CREMA measurements (out at conferences, 2013/14)
- 3 scattering expts. underway or coming
- Electron deuteron scattering (Griffioen et al., Mainz) (data taken)
- 5 atomic energy level measurements
- TREK at JPARC
- Maybe also: Trumuonium ($\mu^+\mu^-$) at JLab

(14 experiments)
Ending

• Remarkable: 6 years after the first announcement, the problem persists.

• Interestingly little discussion of the correctness of the $\mu$-H Lamb shift data.

• Serious and good new data coming.

• Opinion: Either
  • All radii correct, and BSM—muonic specific force—is explanation despite problems, or
  • The electron based radius measurements will reduce to the muonic value.

• Comment: the theory for $(g-2)_\mu$ cannot be considered settled until the proton radius problem is settled. Further, there may be striking corrections to other processes that involve muons.

The end for now!