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XAFS: X-ray Absorption Fine-Structure

Summary

Basic Principles:

- X-ray absorption and fluorescence
- Simple theoretical description
- **Experiment Design:**
 - Transmission v. Fluorescence modes
 - Fluorescence detectors
- Data Analysis:
 - EXAFS Analysis: near neighbor *R*, *N*, and atomic species
 - XANES Analysis: formal valence and coordination chemistry

What is XAFS?

- X-ray Absorption Fine-Structure (XAFS) is the modulation of the x-ray absorption coefficient at energies near and above an x-ray absorption edge. XAFS is also referred to as X-ray Absorption Spectroscopy (XAS) and is broken into 2 regimes:
 - XANES X-ray Absorption Near-Edge Spectroscopy
 - EXAFS Extended X-ray Absorption Fine-Structure which contain related, but slightly different information about an element's local coordination and chemical state.
- Fe *K*-edge XAFS for FeO:
- XAFS Characteristics:
- local atomic coordination
- chemical / oxidation state
- applies to any element
- works at low concentrations
- minimal sample requirements



X-ray Absorption

- X-rays (light with wavelength 0.03<λ<12 Å or energy 1<*E* <500 keV) are absorbed by all matter through the *photo-electric effect*
- An x-ray is absorbed by an atom, promoting a core-level electron (*K*, *L*, or *M* shell) out of the atom and into the continuum. The atom is left in an *excited state* with an empty electronic level (a *core hole*). The electron ejected from the atom is called the *photo-electron*.



X-ray Fluorescence

- When x-rays are absorbed by the photo-electric effect, the excited core-hole will relax back to a "ground state" of the atom. A higher level core electron drops into the core hole, and a *fluorescent x-ray* or *Auger electron* is emitted.
 - X-ray Fluorescence: An x-ray with energy = the difference of the corelevels is emitted. Auger Effect: An electron is promoted into the continuum from another corelevel.



X-ray fluorescence occurs at discrete energies that are characteristic of the absorbing atom, and can be used to identify the absorbing atom.

The X-ray Absorption Coefficient: µ

The intensity of an x-ray beam as it passes through a material of thickness t is given by the absorption coefficient µ:

$$\mathbf{I} = \mathbf{I}_0 \mathbf{e}^{-\mu \mathbf{t}} \qquad \underbrace{I_0}_{I_0} \qquad \underbrace{I}_{I_0}$$



where l_0 is the x-ray intensity hitting the material, and / is the intensity transmitted through the material.

X-ray Absorption Measurements



XAS measures the energy dependence of the x-ray absorption coefficient $\mu(E)$ at and above the absorption edge of a selected element. $\mu(E)$ can be measured two ways:

Transmission: The absorption is measured directly by measuring what is transmitted through the sample:

$$I = I_0 e^{-\mu(E)t}$$
$$\mu(E)t = \ln(I/I_0)$$

Fluorescence: The re-filling the deep core hole, is detected. Typically the fluorescent x-ray is measured.

$$\mu(E) \sim I_f/I_0$$

X-ray Absorption Fine Structure

We're interested in the energy-dependent oscillations in $\mu(E)$, as these will tell us something about the neighboring atoms, so define the EXAFS as:

$$\chi(\mathbf{E}) = \frac{\mu(\mathbf{E}) - \mu_{\mathbf{0}}(\mathbf{E})}{\Delta \mu_{\mathbf{0}}(\mathbf{E}_{\mathbf{0}})}$$

We subtract off the smooth *"bare atom" background* $\mu_0(E)$, and divide by the *"edge step"* $\Delta \mu_0(E_0)$ to give the oscillations normalized to 1 absorption event:



EXAFS: $\chi(k)$

It's convenient to think of the XAFS in terms of *photo-electron wavenumber* k: $k=\sqrt{\frac{2m(E-E_0)}{\hbar^2}}$ rather than x-ray energy, so that we have $\chi(k)=\chi(k[E])$: 0.60.50.40.3 $\chi^{(k)}$ 0.20.10.0 -0.1-0.2120 $\mathbf{2}$ 6 8 1014 Δ k (Å⁻¹)

 $\chi({f k})$ is often shown weighted by ${f k}^2$ or ${f k}^3$ to amplify the oscillations at high-k:



The EXAFS Equation

To model the EXAFS, we use the EXAFS Equation:

$$\chi(\mathbf{k}) = \sum_{\mathbf{j}} \frac{\mathbf{N_j f_j(\mathbf{k}) e^{-2\mathbf{k}^2 \sigma_j^2}}}{\mathbf{k R_j}^2} sin[2\mathbf{k R_j} + \boldsymbol{\delta_j(\mathbf{k})}]$$

where f(k) and $\delta_j(k)$ are *scattering* properties of the neighboring atom. Knowing these, we can determine:

- R distance to neighboring atom.
- **N** coordination number of neighboring atom.
- σ^2 mean-square disorder of neighbor distance.

The scattering amplitude f(k) and phase-shift $\delta_j(k)$ depend on atomic number Z of the scattering atom.

XAFS Theory: X-ray Absorption by a Free Atom

An atom absorbs and x-ray of energy E, destroying a core electron with energy E_0 and creating a photo-electron with energy $(E - E_0)$. The core level is eventually filled, and a fluorescence x-ray or an Auger electron is ejected from the atom.



x-ray absorption needs an available state for the photo-electron to go into:

> No available state: No absorption

Once the x-ray energy is large enough to promote a core-level to the continuum, there is a sharp increase in absorption.

For an isolated atom, $\mu(E)$ has a sharp step at the core-level binding energy, and is a smooth function of energy above this absorption edge.

X-ray Absorption with Photo-Electron Scattering

With another atom nearby, the ejected photo-electron can *scatter* from a neighboring atom and return back to the absorbing atom.



When this happens, the photo-electron scattered back will interfere with it-self.

 μ depends on the presence of an electron state with energy $({\bf E}-{\bf E}_0)$ and at the absorbing atom.

The amplitude of the back-scattered photo-electron *at the absorbing atom* will oscillate with energy, causing oscillations in $\mu(\mathbf{E})$.

The XAFS oscillations are an interference effect due to the presence of neighboring atoms.

The EXAFS Equation: simple description

Starting with $\chi \sim \psi_{\rm scatt}(0)$, we can build a simple model for the EXAFS from the photo-electron as:

- 1. leaving the absorbing atom
- 2. scattering from the neighbor atom
- 3. returning to the absorbing atom



With spherical wave $e^{i\mathbf{k}\mathbf{r}}/\mathbf{k}\mathbf{r}$ for the propagating photo-electron, and a scattering atom at a distance $\mathbf{r}=\mathbf{R}$, we get

$$\chi(\mathbf{k}) = rac{\mathbf{e}^{\mathbf{i}\mathbf{k}\mathbf{R}}}{\mathbf{k}\mathbf{R}} \left[2\mathbf{k}\mathbf{f}(\mathbf{k})\mathbf{e}^{\mathbf{i}\boldsymbol{\delta}(\mathbf{k})}
ight] rac{\mathbf{e}^{\mathbf{i}\mathbf{k}\mathbf{R}}}{\mathbf{k}\mathbf{R}} + \mathbf{C}.\mathbf{C}.$$

where the neighboring atom gives the amplitude f(k) and phase-shift $\delta(k)$ to the scattered photo-electron.

Development of the EXAFS Equation

Combining terms (including the complex conjugate), we get

$$\chi(\mathbf{k}) = rac{\mathbf{f}(\mathbf{k})}{\mathbf{kR}^2} \mathbf{sin}[2\mathbf{kR} + \boldsymbol{\delta}(\mathbf{k})]$$

for 1 scattering atom.

For N neighboring atoms, and with thermal and static disorder of σ^2 , giving the *mean-square disorder* in R, we have

$$\chi(\mathbf{k}) = \frac{\mathbf{N}\mathbf{f}(\mathbf{k})\mathbf{e}^{-2\mathbf{k}^2\sigma^2}}{\mathbf{k}\mathbf{R}^2}\mathbf{sin}[2\mathbf{k}\mathbf{R} + \boldsymbol{\delta}(\mathbf{k})]$$

A real system will have neighboring atom at different distances and of different type. We add all these contributions to get a version of the EXAFS equation:

$$\chi(\mathbf{k}) = \sum_{\mathbf{j}} \frac{\mathbf{N}_{\mathbf{j}} \mathbf{f}_{\mathbf{j}}(\mathbf{k}) \mathbf{e}^{-2\mathbf{k}^2 \sigma_{\mathbf{j}}^2}}{\mathbf{k} \mathbf{R}_{\mathbf{j}}^2} \mathbf{sin}[2\mathbf{k} \mathbf{R}_{\mathbf{j}} + \boldsymbol{\delta}_{\mathbf{j}}(\mathbf{k})]$$

The Photo-Electron Mean-Free Path

Getting to

$$\chi(\mathbf{k}) = \sum_{\mathbf{j}} \frac{\mathbf{N}_{\mathbf{j}} \mathbf{f}_{\mathbf{j}}(\mathbf{k}) \mathbf{e}^{-2\mathbf{k}^2 \sigma_{\mathbf{j}}^2}}{\mathbf{k} \mathbf{R}_{\mathbf{j}}^2} \sin[2\mathbf{k} \mathbf{R}_{\mathbf{j}} + \delta_{\mathbf{j}}(\mathbf{k})]$$

we used a spherical wave for the photo-electron: $e^{i\mathbf{k}\mathbf{r}}/\mathbf{k}\mathbf{r}$. The photo-electron can also scatter *inelastically*, and may not be able to get back the absorbing atom. Using a damped wave-function: $e^{i\mathbf{k}\mathbf{r}}e^{-\mathbf{r}/\lambda(\mathbf{k})}/\mathbf{k}\mathbf{r}$ where $\lambda(\mathbf{k})$ is the photo-electron's *mean free path*, the EXAFS equation becomes:

$$\chi(\mathbf{k}) = \sum_{\mathbf{j}} \frac{N_{\mathbf{j}} f_{\mathbf{j}}(\mathbf{k}) e^{-2\mathbf{R}_{\mathbf{j}}/\lambda(\mathbf{k})} e^{-2\mathbf{k}^2 \sigma_{\mathbf{j}}^2}}{\mathbf{k} \mathbf{R}_{\mathbf{j}}^2} sin[2\mathbf{k} \mathbf{R}_{\mathbf{j}} + \delta_{\mathbf{j}}(\mathbf{k})]$$

The mean-free-path is < 25 Å for much of the EXAFS range. λ and the \mathbf{R}^{-2} term make EXAFS a *local atomic probe*.



Scattering Amplitude and Phase-Shift

The scattering amplitude $f(\mathbf{k})$ and phase-shift $\delta_{\mathbf{j}}(\mathbf{k})$ depend on atomic number.



The scattering amplitude $f(\mathbf{k})$ peaks at different \mathbf{k} values and extends to higher- \mathbf{k} for heavier elements. For very heavy elements, there is structure in $f(\mathbf{k})$.

The phase shift $\delta_{j}(\mathbf{k})$ shows sharp changes for very heavy elements.

These scattering functions can be accurately calculated, and used in the EXAFS modeling.

Z can usually be determined to with 5 or so. Fe and O can be distinguished, but Fe and Mn cannot be.

The EXAFS Equation

Finally we have an equation we can use to model and interpret EXAFS:

$$\chi(\mathbf{k}) = \sum_{\mathbf{j}} \frac{\mathbf{N_j f_j(\mathbf{k}) e^{-2\mathbf{R_j}/\boldsymbol{\lambda}(\mathbf{k})} e^{-2\mathbf{k}^2 \sigma_j^2}}{\mathbf{k} {\mathbf{R_j}}^2} \mathrm{sin}[2\mathbf{k} \mathbf{R_j} + \boldsymbol{\delta_j}(\mathbf{k})]$$

If we know the *scattering* properties of the neighboring atom: f(k) and $\delta_j(k)$, and the mean-free-path $\lambda(k)$ we can determine:

- R distance to neighboring atom.
- N coordination number of neighboring atom.
- σ^2 mean-square disorder of neighbor distance.

Since the scattering amplitude f(k) and phase-shift $\delta_j(k)$ depend strongly on atomic number, XAFS is also sensitive to Z of the scattering atom.

X-ray Absorption Measurements: Experimental Design

Important points to consider for measuring XAFS are:

- Monochromatic x-rays: Need x-rays with a small energy spread or bandwidth: ∆E<1 eV at 10keV.
- Linear Detectors: The XAFS (k) is small, so we need a lotof photons (ie, a synchrotron) and detectors that are very linear in x-ray intensity (ion chambers).
- Well-aligned Beam: The x-ray beam hitting the detectors has to be the same hitting the sample.
- Homogeneous sample: For transmission measurements, we need a sample that is of uniform and appropriate sample thickness of nearly 2 absorptionlengths. It should be free from pinholes. If a powder, the grains should be very finegrained (absorption length) and uniform.

X-ray Absorption Measurements: The Experiment

Energy Scanning The beamline needs to deliver monochromatic x-rays that are tunable so we can scan energy across the absorption edge. We'll scan from 200 eV below to 800 eV above the Fe K-edge, like this:

Region Start	ting Energy (eV)	Ending Energy (eV)	Step Size (eV)
Pre-edge	-200	-20	5.0
XANES	-20	+30	0.5
EXAFS	+30	800	0.05 Å ⁻¹

Notes: all values relative to a nominal value of the edge energy. In the EXAFS region, it's common to step in *k* rather than energy. *Counting Statistics:* The EXAFS is a fairly small signal, so for good data $\mu(E)$ should have a noise level of about 10⁻³. That means we need to collect at least 10⁶ photons.

- Transmission : Fluxes at synchrotrons are > 10⁸ photons/sec. Count rate is not much of an issue.
- Fluorescence : May be a concern, especially when concentrationsare very low.

X-ray Absorption Measurements: Transmission



For concentrated samples, XAFS is best measured in transmission. To do this, we need enough transmission through the sample to get a decent signal for I. With,

$$\mu(\mathbf{E})\mathbf{t} = \ln(\mathbf{I}/\mathbf{I_0})$$

We adjust the sample thickness t so that $\mu(E)t \approx 2.5$ above the absorption edge and/or the edge step $\Delta \mu(E)t \approx 1$.

For Fe foil, $t \approx 7 \,\mu$ m.

Also: the sample must be uniform, and free of pinholes. For a powder, the grain size cannot be much bigger that an absorption length.

If a transmission experiment can be done, this is an easy measurement and gives excellent data. It's usually appropriate for model compounds, and concentrations > 10%.

X-ray Absorption Measurements: Fluorescence



For thick samples or lower concentrations (down to the ppm level), monitoring the x-ray fluorescence is the preferred measurement.



The x-rays emitted from the sample will include the fluorescence line of interest (here, both Fe K_{α} and Fe K_{β}) as well as *scattered* x-rays, and other fluorescence lines (here, Ca, Ti, and V).

There are both *elastically scattered* (at the same energy as the incident beam), and *inelastically scattered* (Compton effect) x-rays.

In many cases the scatter or other fluorescence lines will dominate the fluorescence spectra.

Fluorescence Measurements: Solid-State Detectors

An alternative is to use a *solid-state detector* with Ge or Si as the x-ray absorber. This uses electronic energy discrimination.

This has an advantage of being able to measure the *Full XRF Spectra*, for identifying other elements.

This can be used for XAFS measurements with concentrations down to 10's of ppm.



Though this has many advantages, it has a few drawbacks:

- **Dead time** The electronic discrimination saturates at $\sim 10^5$ Hz or so. Ten (or more) detectors are often used in parallel, but XAFS measurements are still often limited by these detectors.
- Complicated Maintaining, setting up, and using one of these is more work than an ion chamber.

Data Reduction: Strategy

No matter how we measure $\mu(E),$ we'll use these steps to reduce the data to $\chi(k)$ begin the analysis:

- 1. convert measured intensities to $\mu({f E})$
- subtract a smooth pre-edge function, to get rid of any instrumental background, and absorption from other edges.
- 3. normalize $\mu(\mathbf{E})$ to go from 0 to 1, so that it represents the absorption of 1 x-ray.
- 4. remove a smooth post-edge background function to approximate $\mu_0(E)$ to isolate the XAFS $\chi.$
- 5. identify the threshold energy $E_0,$ and convert from E to k space: $k=\sqrt{\frac{2m(E-E_0)}{\hbar^2}}$
- 6. weight the XAFS $\chi(\mathbf{k})$ and Fourier transform from \mathbf{k} to \mathbf{R} space.

After we get this far, we'll model f(k) and $\delta(k)$ and analyze $\chi(k)$ to get distance R, coordination number N.

Data Reduction: Pre-Edge Subtraction, Normalization





Pre-Edge Subtraction

We subtract away the background that fits the *pre-edge* region. This gets rid of the absorption due to other edges (say, the Fe L_{III} edge).

Normalization

We estimate the *edge step*, $\Delta \mu_0(E_0)$ by extrapolating a simple fit to the above $\mu(E)$ to the edge. We normalize by this value to get the absorption from 1 x-ray.

Data Reduction: Normalized XANES and E_0



XANES

The XANES portion (below) shows a fairly rich spectral structure. We'll come back to this for XANES analysis.

Derivative

We can select E_0 roughly as the energy with the maximum derivative. This is somewhat arbitrary, so we'll keep in mind that we may need to refine this value later on.

Data Reduction: Post-Edge Background Subtraction



Post-Edge Background

We don't have a measurement of $\mu_0(E)$ (the absorption coefficient without neighboring atoms).

We approximate $\mu_0(E)$ by an adjustable, smooth function: a *spline*.

This can be somewhat dangerous – a flexible enough spline could match the $\mu(E)$ and remove all the EXAFS!

We want a spline that will match the *low frequency* components of $\mu_0(E)$.

Data Reduction: $\chi(k)$, k-weighting



$\chi(\mathbf{k})$

The raw EXAFS $\chi(\mathbf{k})$ usually decays quickly with \mathbf{k} , and difficult to assess or interpret by itself.

It is customary to weight the higher-k portion of the spectra by multiplying by k^2 or $k^3.$

${f k}$ -weighted $\chi({f k})$: ${f k}^2\chi({f k})$

 $\chi(k)$ is composed of sine waves, so we'll Fourier Transform from k to $R\-space$. To avoid "ringing", we'll multiply by a window function.

Fourier Transform: $\chi(R)$



$\chi(\mathbf{R})$

The Fourier Transform of $k^2\chi(k)$ has 2 main peaks, for the first 2 coordination shells: Fe-O and Fe-Fe.

The Fe-O distance in FeO is 2.14Å, but the first peak is at 1.6Å. This shift in the first peak is due to the *phase-shift*, $\delta(\mathbf{k})$: $\sin[2\mathbf{kR} + \delta(\mathbf{k})]$.

A shift of -0.5Å is typical.

$oldsymbol{\chi}(\mathbf{R})$ is complex:

The FT makes $\chi(\mathbf{R})$ complex. Usually only the amplitude is shown, but there are really oscillations in $\chi(\mathbf{R})$.

Both real and imaginary components are used in modeling.

EXAFS Analysis: two shell model of FeO



The Fe-Fe EXAFS extends to higher- $\!k$ than the Fe-O EXAFS.

Even in this simple system, there is some overlap of shells in \mathbf{R} -space.

The agreement in $\operatorname{Re}[\chi(\mathbf{R})]$ look especially good – this is how the fits are done.

Of course, the modeling can get more complicated than this!



Fit results (uncertainties in parentheses):

Shell	N	${f R}$ (Å)	σ^2 (Ų)	ΔE_0 (eV)
Fe-O	6.0(1.0)	2.10(.02)	0.015(.003)	-2.1(0.8)
Fe-Fe	11.7(1.3)	3.05(.02)	0.014(.002)	-2.1(0.8)

XANES: X-ray Absorption Near Edge Structure



Dipole selection rule: $\Delta \ell = \pm 1; \Delta J = 0, \pm 1$ At photoelectron kinetic energies with wavelengths near the first interatomic distance:

- Discrete transitions to unpopulated states
- XANES is a probe of the density of empty electron states projected onto the phtoabsorber

At higher XANES energies:

- The multiple scattering phenomenon dominates
- XANES is a probe for high order correlation functions

XANES: fingerprinting

- Despite large advances in the past years, predictive XANES calculations are not satisfactory.
- Interpretation is still qualitative by comparison to known,
 - reference spectra ("fingerprint") or
 - theoretically calculated spectra (e.g., multiple scattering or band structure calculations).
- XANES features and their intensity and energy position are dependent on a number of factors including
 - •absorbing atom valence
 - •coordination geometry
 - •orbital occupancy
 - •electronegativityof coordinating atoms
 - •interatomicdistances•orientation (for anisotropic samples)

Multiple scattering (MS) in the XAFS regime

- In addition to SS in EXAFS, oscillations from MS of the photoelectron can be observed in EXAFS - mostly at low k•
- MS is most likely along atoms with a collinear arrangement ("lensing" or "focussing" effect).
- MS peaks are never found in the FT spectrum at distances shorter than the second coordination shell
- A simple assumption is that the rel. mean square displacement is additive paths longer than triple scattering paths are rare

two-legged single scattering (SS)

three-legged or double scattering - most important when forward scattering angle is near 180°.

four-legged triangular triple scattering path- very important when the scattering angle is nearly 180°.

XANES Analysis: Oxidation State



XANES for Fe oxides and metal. The shift of the edge position can be used to determine the valence state.

The heights and positions of pre-edge peaks can also be reliably used to determine Fe^{3+}/Fe^{2+} ratios (and similar ratios for many cations).

XANES: orbital occupancy



ABSORPTION COEFFICIENT



XANES Analysis: Oxidation State and Coordination Chemistry



The XANES of Cr^{3+} and Cr^{6+} shows a dramatic dependence on oxidation state and coordination chemistry.

For ions with partially filled d shells, the p-d hybridization changes dramatically as *regular octahedra* distort, and is very large for *tetrahedral* coordination.

This gives a dramatic pre-edge peak – absorption to a localized electronic state.

Polarized XAFS -U L_{III} and L_I edge XANES features of oriented samples

- The linear uranyl (O=U=O²⁺) cation exhibits striking variations in its XAFS spectra as a function of orientation relative to the incident radiation's polarization vector.
 - ◆ Changes in U L_{III} XANES edge features: white line (2p→6d (6s) transition), 10eV feature (scattering on Oax), and 35eV feature (scattering on Leq)
 - U L_I XANES polarization dependency is more dramatic, due to the higher orientation of p-like final states.

Polarized XAFS -- U L_{III} and L_I edge XANES of oriented UO₂(CH₃CO₂)_{2*}2H₂O



Polarized XAFS -- U L_{III} and L_I edge XANES of oriented UO₂(CH₃CO₂)_{2*}2H₂O



XANES Interpretation

The EXAFS Equation breaks down at low-k, and the mean-free-path goes up. This complicates XANES interpretation:

We do not have a simple equation for XANES.

XANES can be described qualitatively (and nearly quantitatively) in terms of

coordination chemistry regular, distorted octahedral, tetrahedral, ...

molecular orbitals p-d orbital hybridization, crystal-field theory, ...

band-structure the density of available electronic states.

multiple-scattering multiple bounces of the photo-electron.

These chemical and physical interpretations are all related, of course:

What electronic states can the photo-electron fill?

XANES calculations are becoming reasonably accurate and simple. These can help explain what *bonding orbitals* and/or *structural characteristics* give rise to certain spectral features.

Quantitative XANES analysis using first-principles calculations are rare.









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E

Regole di sommazione

- Lo spin s viene eliminato se si sommano gli integrali del segnale dicroico sulle soglie L_{III} (I+s) ed L_{II} (I-s):
 l(L_{III})+l(L_{II})
- Per eliminare il momento angolare l occorre tenere conto della degenerazione: s = I(L_{III})-2I(L_{II})

Esempio: Mn12Ac



Secondo la teoria dei gruppi, il momento angolare orbitale è nullo (si dice che è quenchiato dal campo cristallino)

Esempio: Mn12Ac

