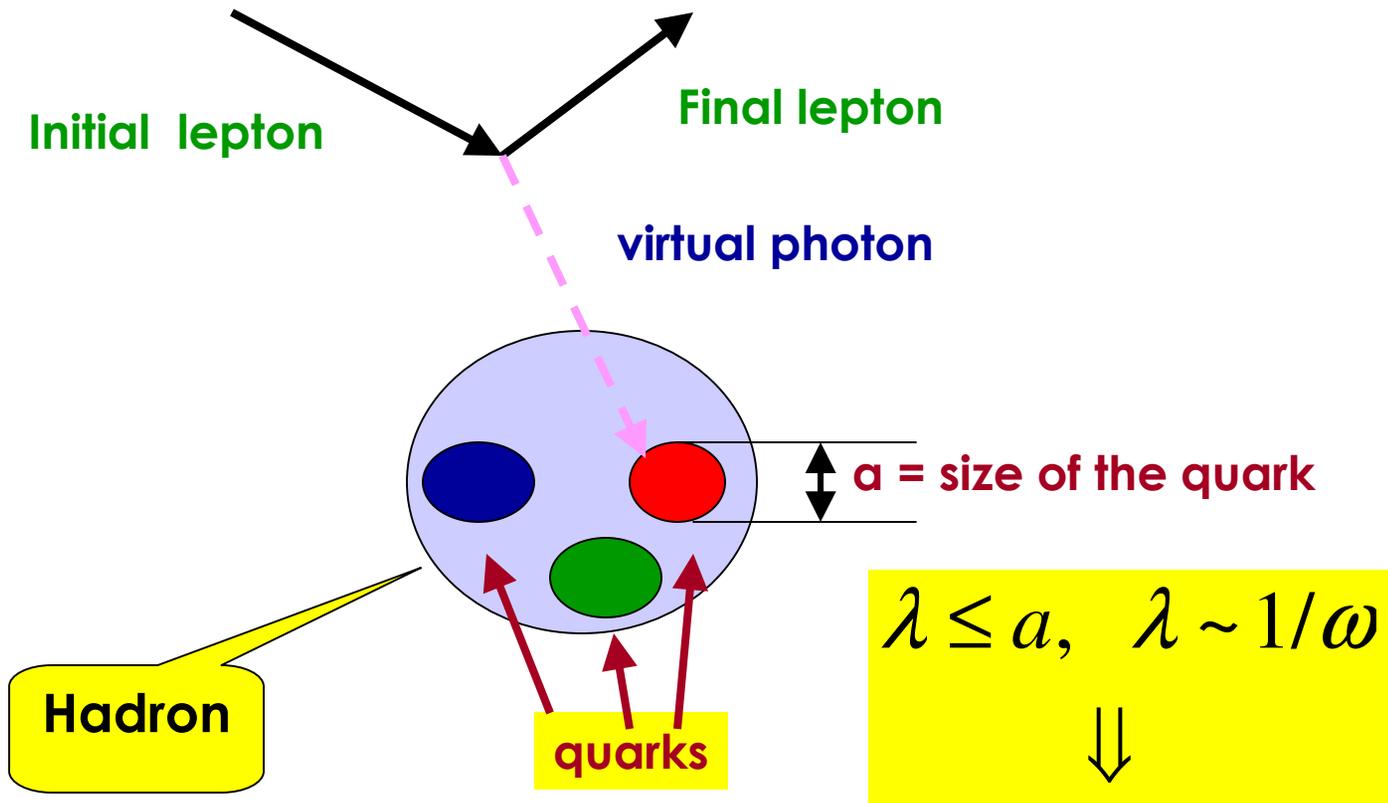


# **Review on the theory of the Deep-Inelastic Scattering at small $x$**

**B.I. Ermolaev**

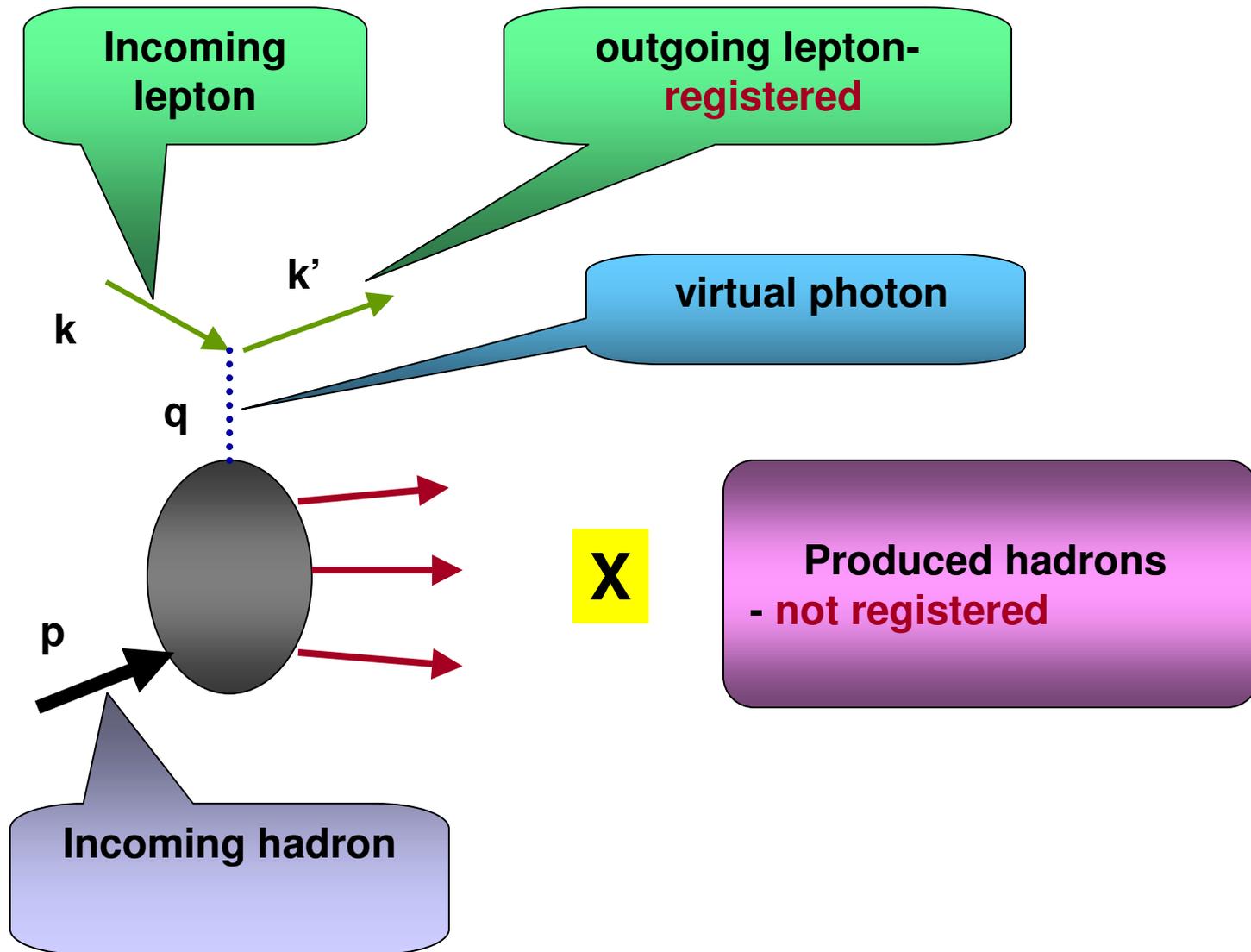
talk based on results obtained in collaboration with  
**M. Greco and S.I. Troyan**

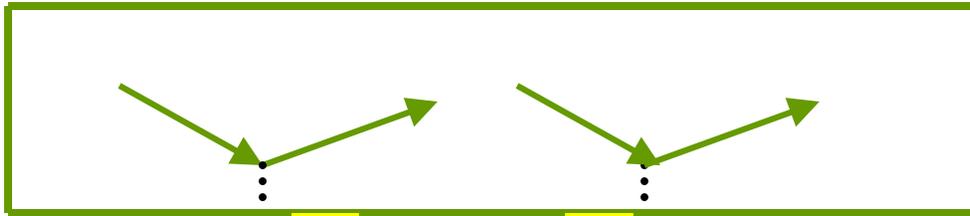
# Probing the hadron structure with lepton-hadron collisions



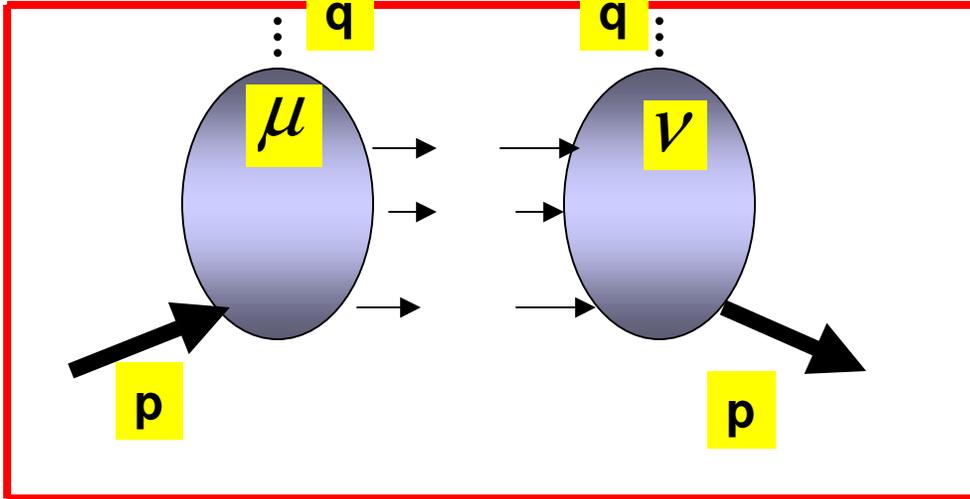
**Small  $a$   $\rightarrow$  high energies of the collisions**

# Deep-Inelastic Scattering





Leptonic tensor



hadronic tensor

$$W_{\mu\nu}$$

Does not depend on spin

Spin-dependent

$$W_{\mu\nu} = W_{\mu\nu}^{unpolarized}(p, q) + W_{\mu\nu}^{spin}(p, q)$$

symmetric

antisymmetric

The spin-independent part of  $W_{\mu\nu}$  is parameterized by two structure functions:

$$W_{\mu\nu}^{unpol} = \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \left( p_\mu - q_\mu \frac{pq}{q^2} \right) \left( p_\nu - q_\nu \frac{pq}{q^2} \right) \frac{F_2(x, Q^2)}{pq}$$

Projection operators respect Lorentz and gauge symmetries

$$q_\mu W_{\mu\nu} = q_\nu W_{\mu\nu} = 0$$

where  $\mathbf{p}$  is the hadron momentum,  $\mathbf{q}$  is the virtual photon momentum ( $Q^2 = -q^2 > 0$ ). Both of the functions depend on  $Q^2$  and  $x = Q^2 / 2pq$ ,  $0 < x < 1$ .

$$F_2 = 2xF_1 \quad \text{in the Born approximation and} \quad F_2 \rightarrow 2xF_1 \quad \text{at } x \rightarrow 0$$

In the QCD framework, the spin-dependent part of  $W_{\mu\nu}$  is also parameterized by two structure functions:

$$W_{\mu\nu}^{spin} = \frac{m}{pq} i\epsilon_{\mu\nu\lambda\rho} q_\lambda \left[ S_\rho g_1(x, Q^2) + \left( S_\rho - \frac{Sq}{pq} p_\rho \right) g_2(x, Q^2) \right]$$

where  $m$ ,  $p$  and  $S$  are the hadron mass, momentum and spin;  $q$  is the virtual photon momentum ( $Q^2 = -q^2 > 0$ ). Again both functions depend on  $Q^2$  and  $x = Q^2 / 2pq$ ,  $0 < x < 1$ . They measure asymmetries

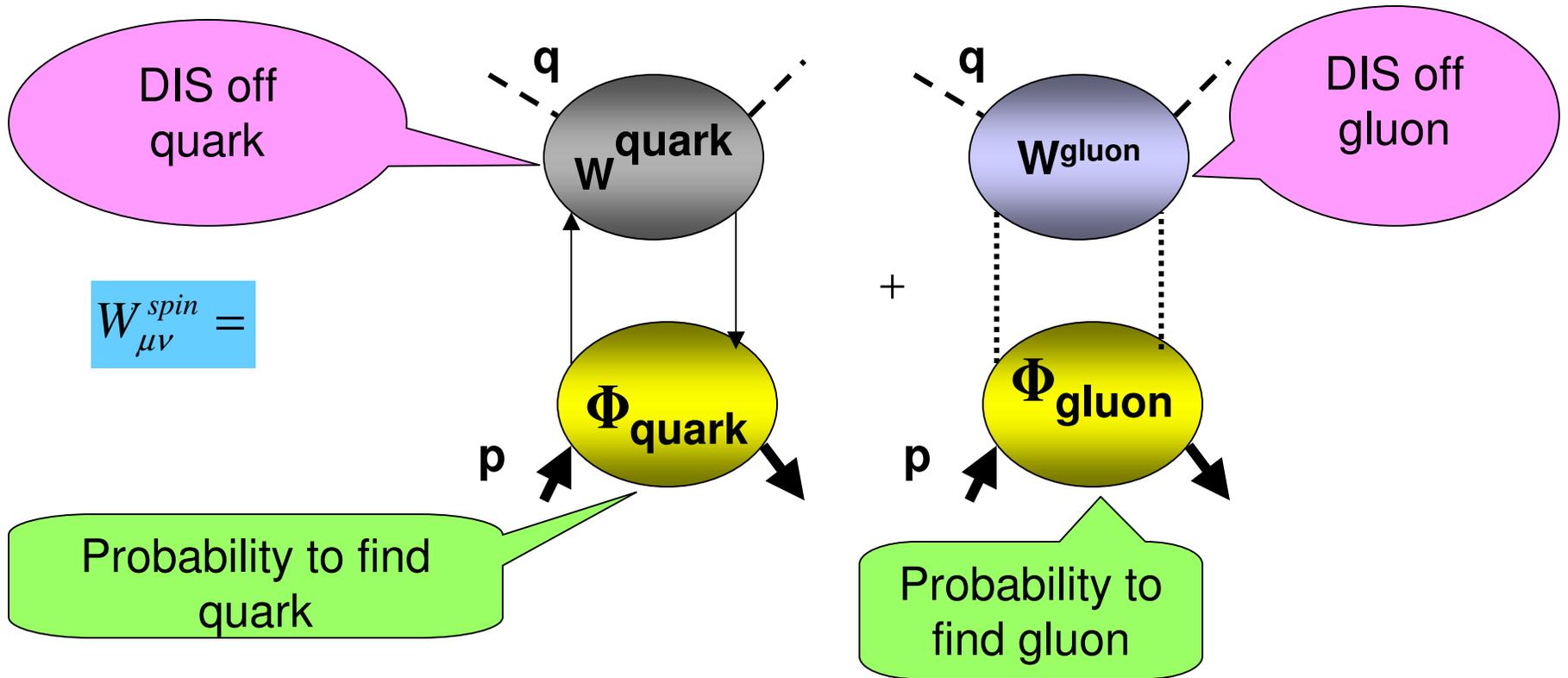
$g_1$  measures the longitudinal spin flip

$$g_1 \propto \sigma_{L\uparrow\uparrow} - \sigma_{L\uparrow\downarrow}$$

$g_1 + g_2$  measures the transverse spin flip

$$g_1 + g_2 \propto \sigma_{T\uparrow\uparrow} - \sigma_{T\uparrow\downarrow}$$

**FACTORISATION:**  $W_{\mu\nu}$  is a convolution of the  
the partonic tensor and probabilities to find a polarized parton  
(quark or gluon) in the hadron :



DIS off quark and gluon can be studied with perturbative QCD, with calculating involved Feynman graphs.

Probabilities,  $\Phi_{\text{quark}}$  and  $\Phi_{\text{gluon}}$  involve non-perturbative QCD. There is no a regular analytic way to calculate them. Usually they are defined from experimental data at large  $x$  and small  $Q^2$ , they are called the initial quark and gluon densities and are denoted  $\delta q$  and  $\delta g$ .

So, the conventional form of the hadronic tensor is:

$$W_{\mu\nu} = W_{\mu\nu}^{\text{quark}} \otimes \delta q + W_{\mu\nu}^{\text{gluon}} \otimes \delta g$$

Initial quark  
distribution

Initial gluon  
distribution

DIS off the quark,

DIS off the gluon

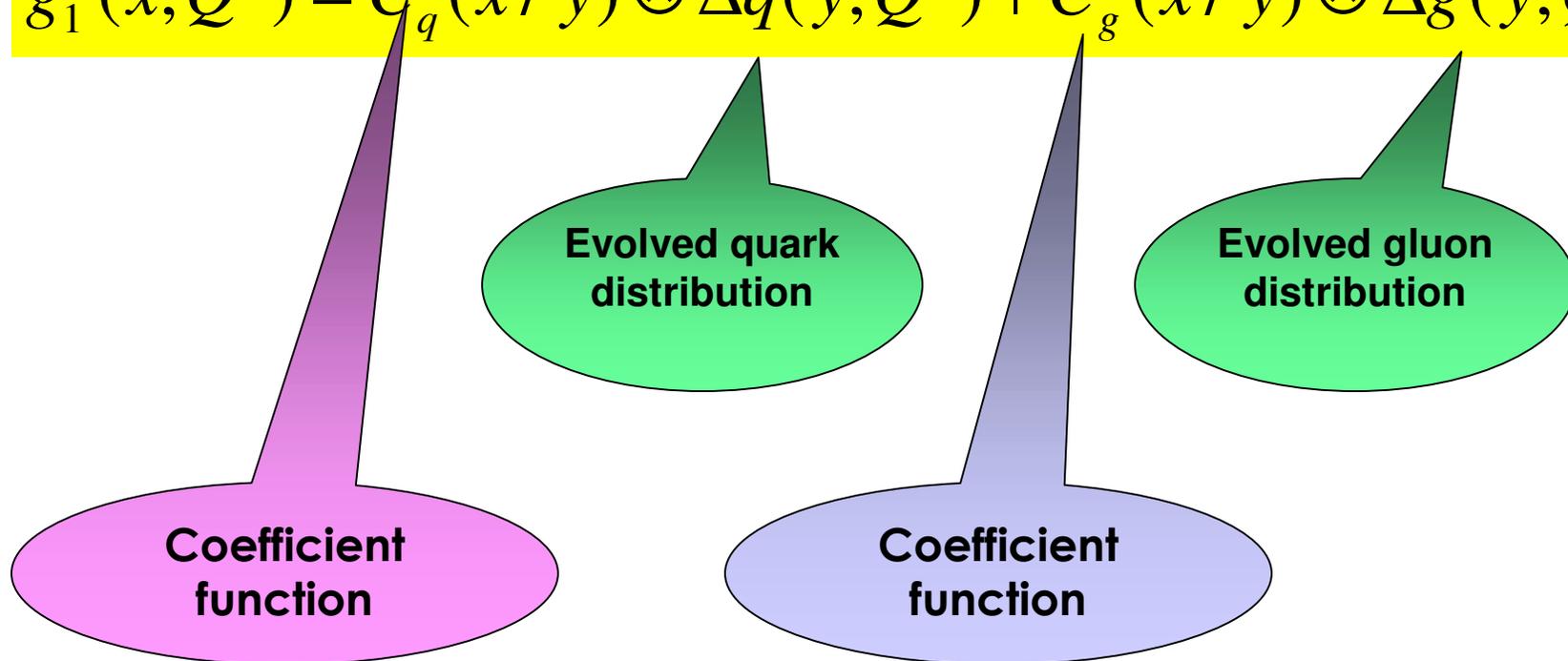
are calculated with methods of Pert QCD

The Standard Approach includes the Altarelli-Parisi alias DGLAP alias  $Q^2$ - Evolution Equations and the Standard Fits for initial parton densities

Evolution Equations: Altarelli-Parisi, Gribov-Lipatov, Dokshitzer

In particular,  $g_1$ :

$$g_1(x, Q^2) = C_q(x/y) \otimes \Delta q(y, Q^2) + C_g(x/y) \otimes \Delta g(y, Q^2)$$



## DGLAP evolution equations

$$\frac{d\Delta q}{d \ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g$$

$$\frac{d\Delta g}{d \ln Q^2} = \frac{\alpha_s}{2\pi} P_{gq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{gg} \otimes \Delta g$$

$P_{qq}, P_{qg}, P_{gq}, P_{gg}$  are splitting functions

**Mellin transform of the splitting functions = anomalous dimensions**

In DGLAP, coefficient functions and anomalous dimensions are known with LO and NLO accuracy, often at integer  $\omega = n$

LO

NLO

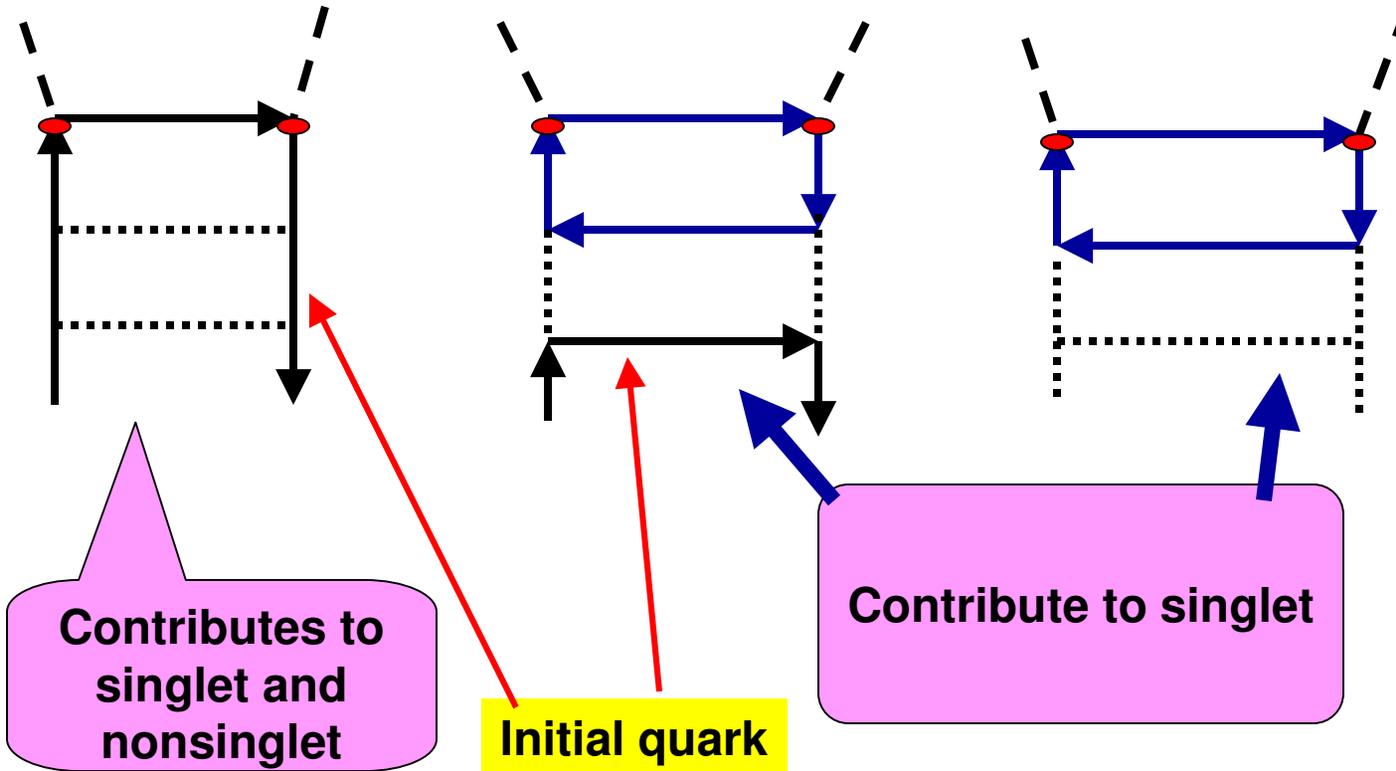
$$C(\omega) = 1 + (\alpha_s(Q^2)/2\pi) C^{(1)}(\omega) + \dots$$

$$\gamma(\omega) = (\alpha_s(Q^2)/4\pi) \gamma^{(0)}(\omega) + (\alpha_s(Q^2)/2\pi)^2 \gamma^{(1)}(\omega) + \dots$$

LO

NLO

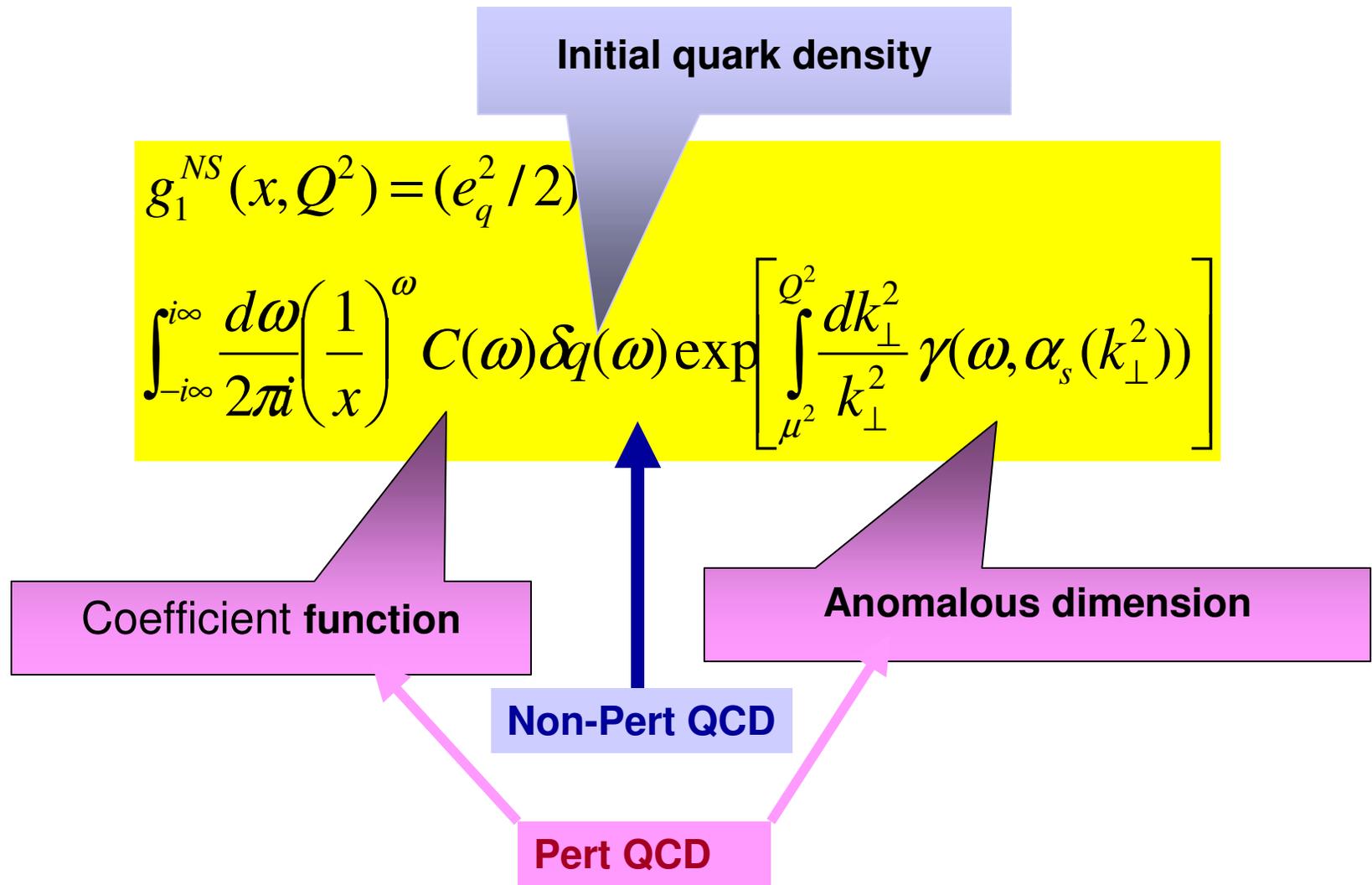
Piece of terminology

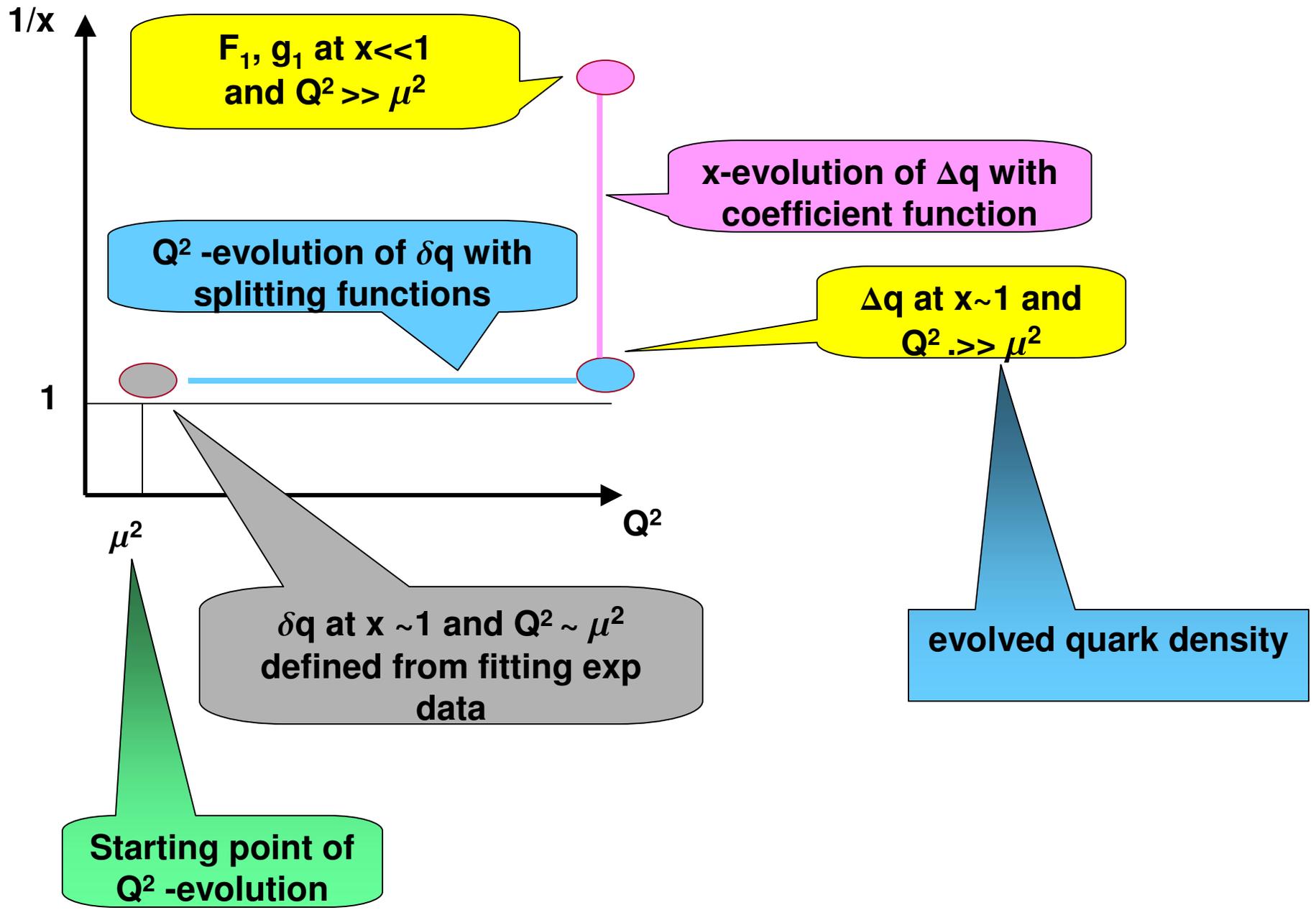


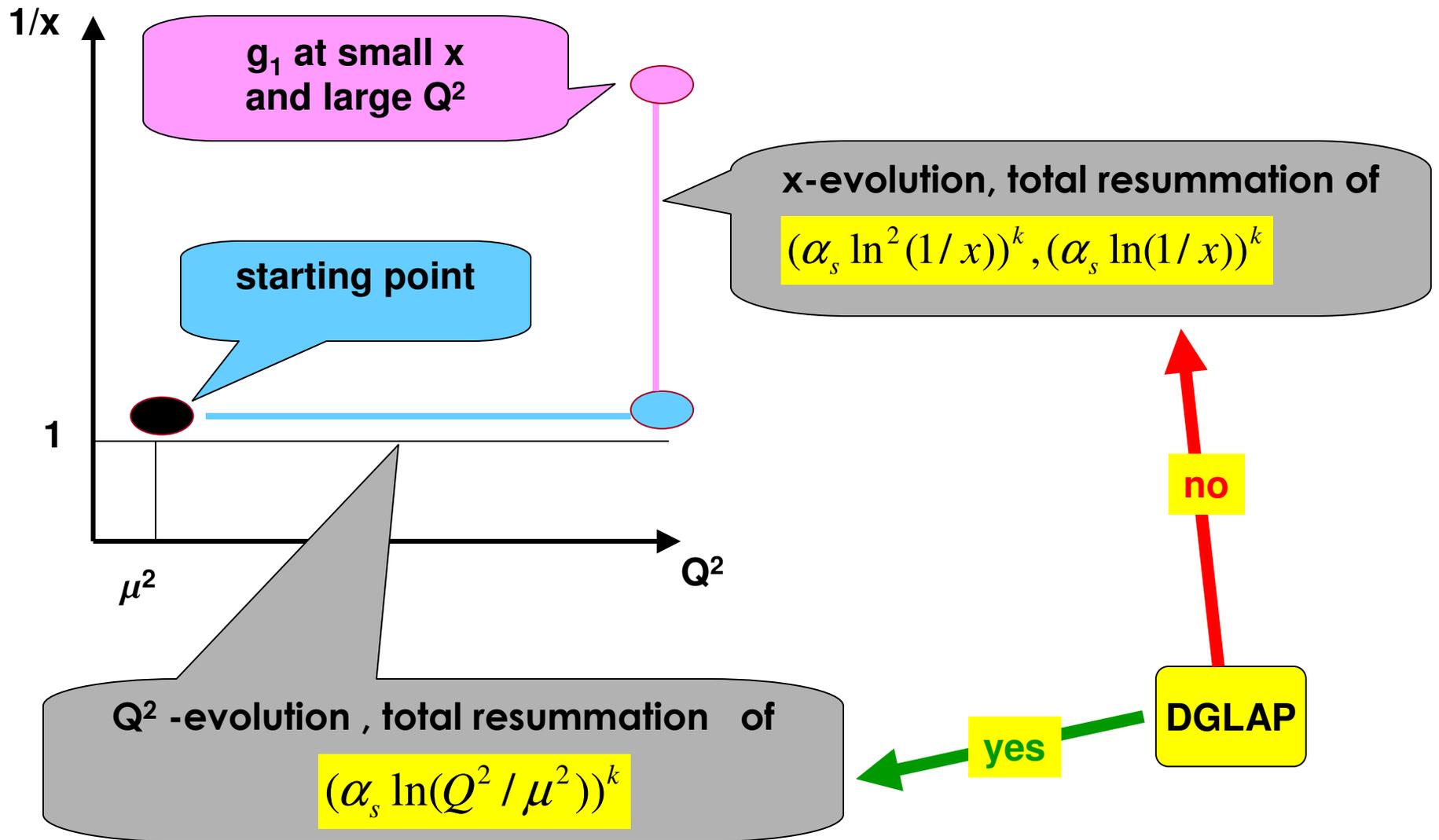
Each structure function has both the non-singlet and singlet components:

$$F_1 = F_1^{NS} + F_1^S \quad g_1 = g_1^{NS} + g_1^S$$

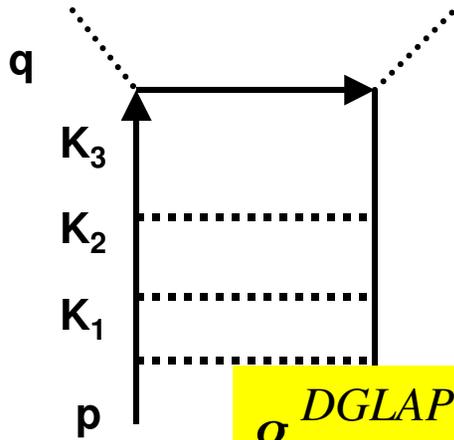
For example, for the simplest case of the non-singlets  $\mathbf{g}_1, \mathbf{F}_1$







**DGLAP cannot do total resummation of logs of  $x$  because of the DGLAP-ordering – KEYSTONE of DGLAP**



### DGLAP –ordering:

$$\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < k_{3\perp}^2 < Q^2$$

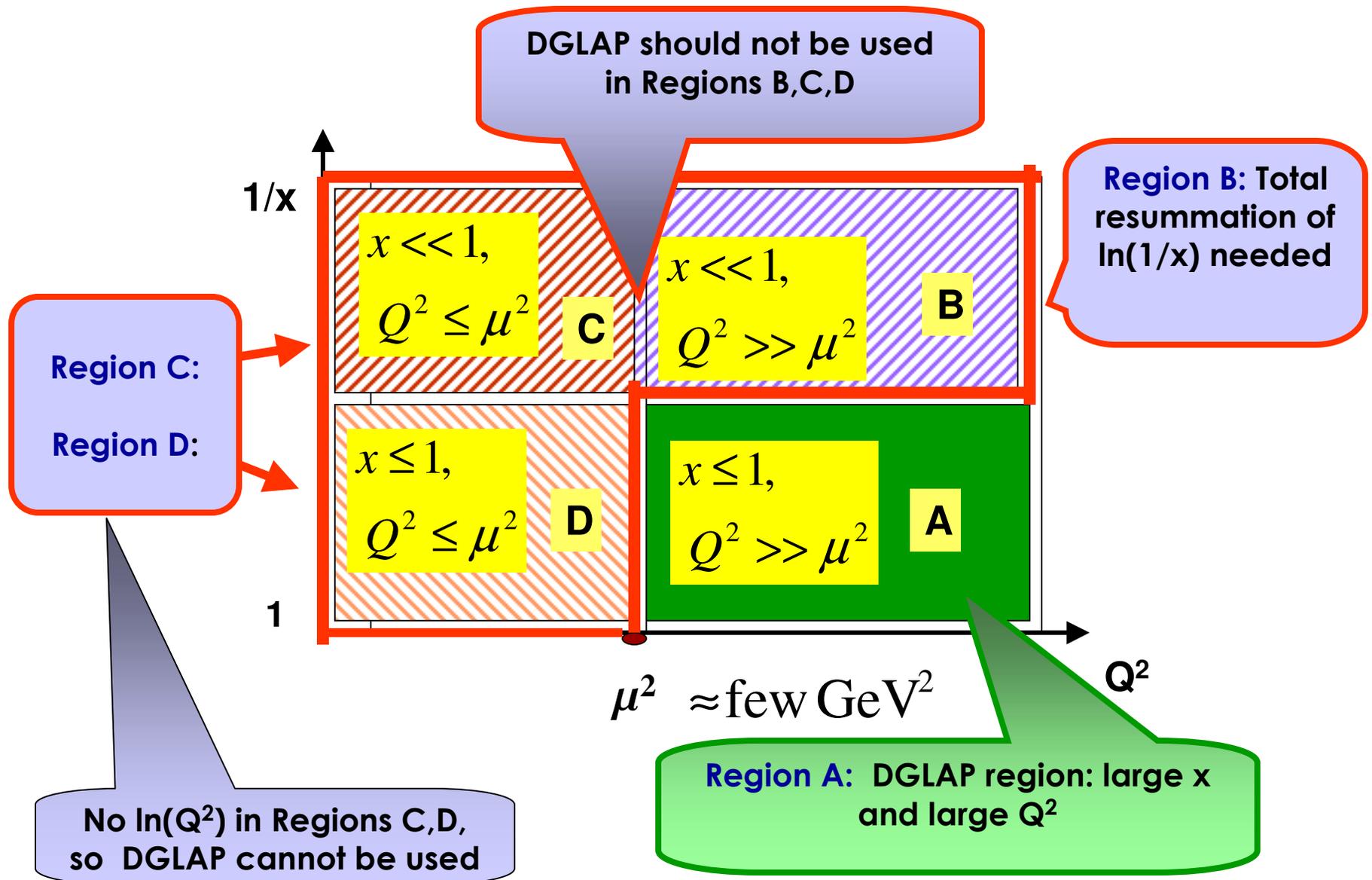
good approximation for large  $x$  when logs of  $x$  can be neglected. At  $x \ll 1$  the ordering has to be lifted

$$g_1^{DGLAP} \sim \exp [\ln(1/x) \ln \ln (Q^2 / \Lambda_{\text{QCD}}^2)]^{1/2}$$

$$F_1^{DGLAP} \sim \left( \frac{1}{x} \right) \exp [\ln(1/x) \ln \ln (Q^2 / \Lambda_{\text{QCD}}^2)]^{1/2}$$

providing the initial parton densities are not singular at small  $x$

from theoretical grounds:



**DL contributions**



$$(\alpha_s \ln^2(1/x))^k,$$

$$(\alpha_s \ln(1/x) \ln(Q^2/\mu^2))^k$$

$$k = 1, 2, \dots, \infty$$

**SL contributions**



$$(\alpha_s \ln(1/x))^k,$$

**These logarithms are important at small x but DGLAP does include the total resummation of such logarithms, so the small-x region is beyond the reach of the Standard Approach**

In practice SA solves this problem through introducing singular fits for initial parton densities, they cause a fast growth at small  $x$  and thereby mimic the resummation **Week point:** no theoretical grounds

Altarelli-Ball-Forte-Ridolfi, Blumlein- Botcher, Leader- Sidorov- Stamenov, Hirai et al

In the literature, there are different fits for initial parton densities. For example,

$$\delta q = Nx^{-\alpha} [(1-x)^\beta (1+\gamma x^\delta)]$$

$$\delta q = N [\ln^\alpha (1/x) + \gamma x \ln^\beta (1/x)]$$

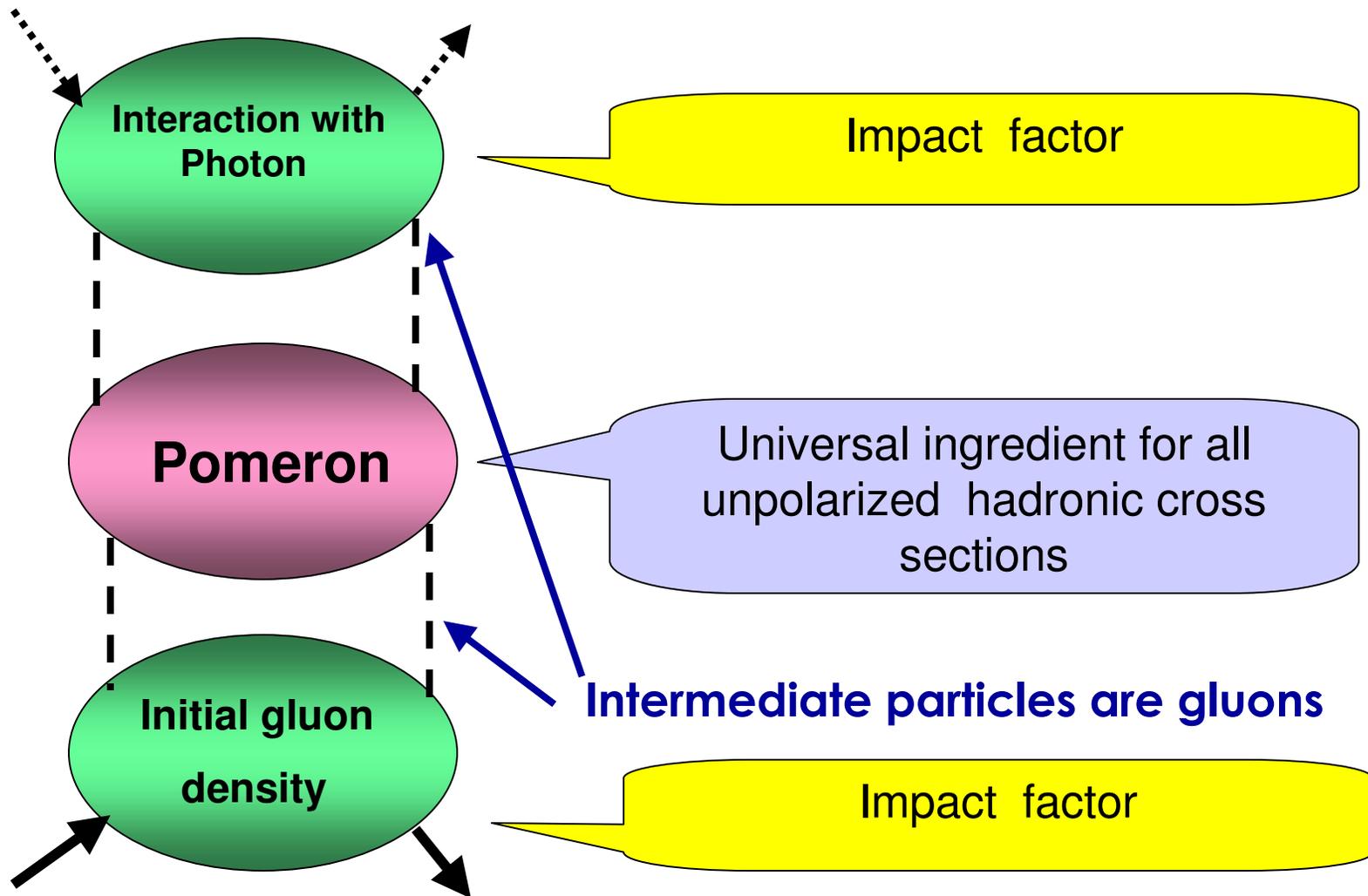
Altarelli-Ball-Forte-Ridolfi,

Parameters  $N, \alpha, \beta, \gamma, \delta$  should be fixed from experiment

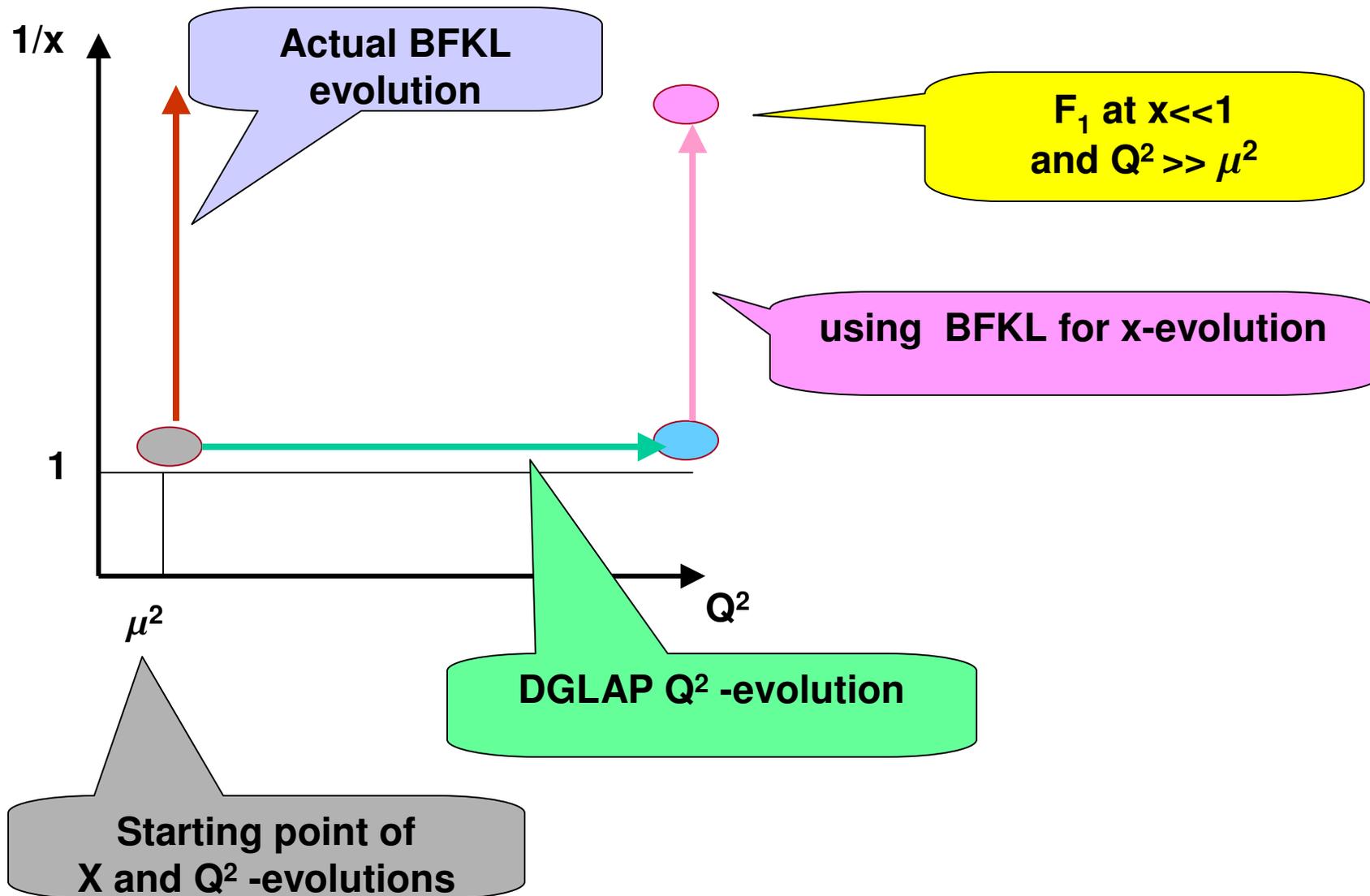
**Alternative, Straightforward Way:** Total resummation of leading logs of  $x$

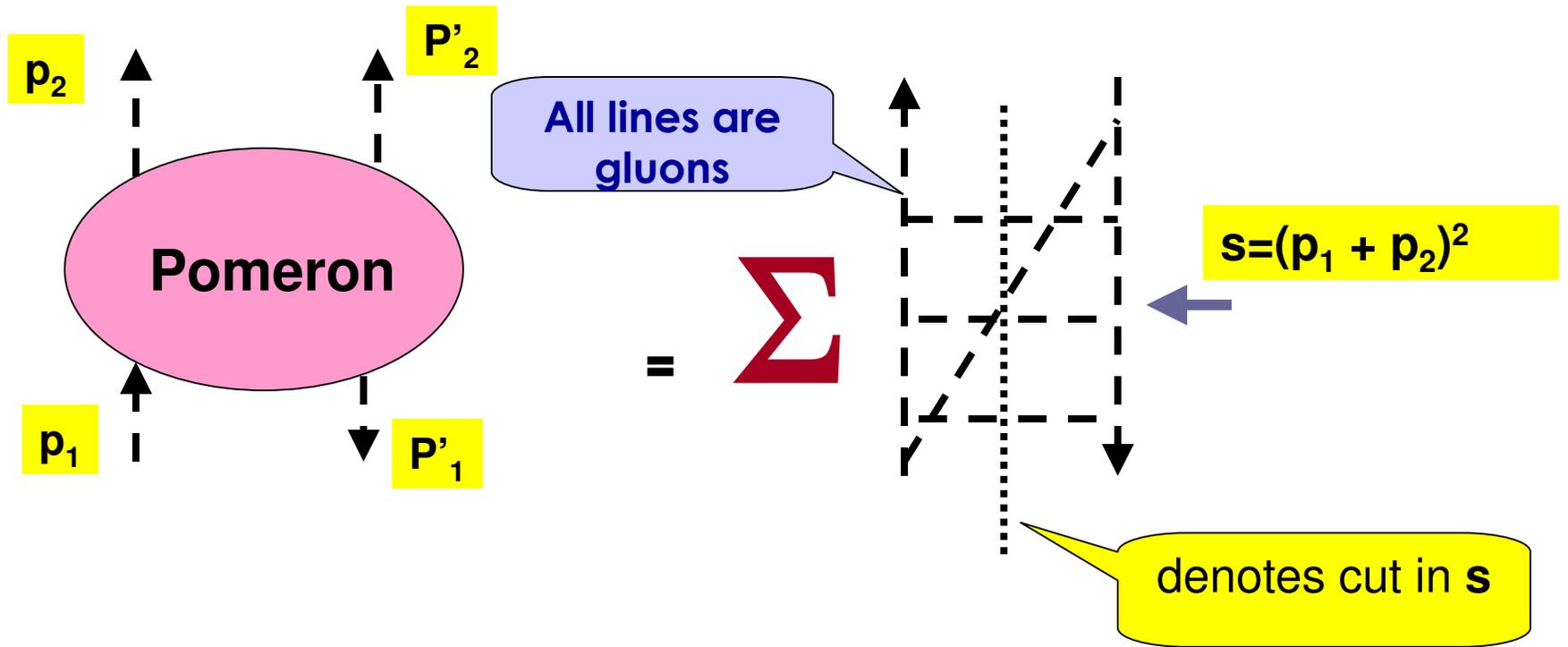
## Spin-independent structure functions at small $x$

no model-independent results. The most popular models are using BFKL Pomeron



# The way to use BFKL Pomeron





**BFKL pomeron = total resummation of leading logarithms of  $s$**

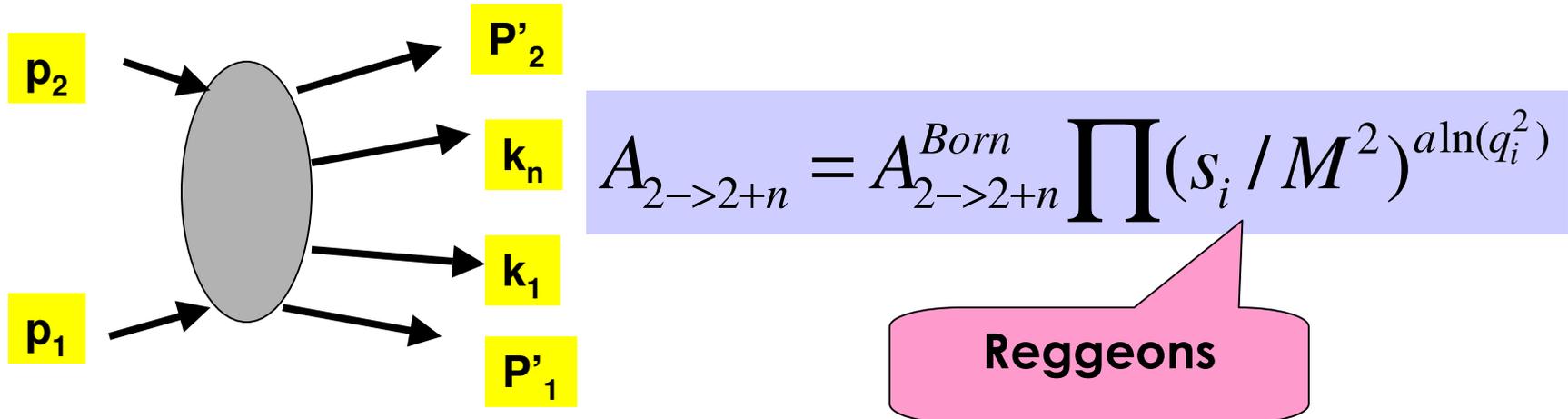
$$\left( \frac{s}{M^2} \right) \left[ 1 + c_1 \alpha_s \ln(s / M^2) + c_2 [\alpha_s \ln(s / M^2)]^2 + \dots \right]$$

**First loop=Born factor**

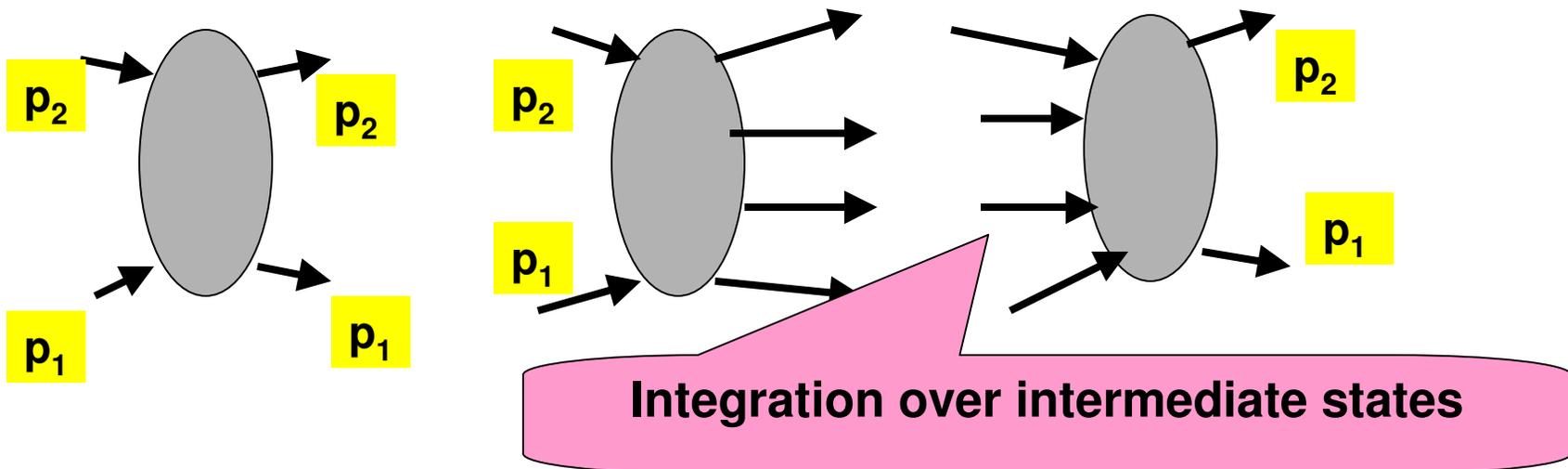
**Higher loops**

## Derivation of BFKL

**Step 1: amplitudes 2 -> 2 + in the multi-Regge kinematics**



**Step 2: BFKL Pomeron = | amplitudes 2 -> 2 + n |<sup>2</sup>**



**BFKL Pomeron sums up leading logarithms, although approximately**

**APPROXIMATIONS:**

- 1. Intermediate particles are in the multi- Regge kinematics (LO) or in the quasi multi-Regge kinematics (NLO)**
- 2.  $\alpha_s$  is fixed either at unknown scale (LO) or the scale is set in a model-dependent way (NLO)**

**NLO corrections are too great to neglect NNLO etc → NO END**

**BFKL Pomeron predicts the Regge behavior for  $F_1$  and  $F_2$  at  $x \ll 1$**

$$F_1 \sim \left(\frac{1}{x}\right)^{1+\Delta_P}, \quad F_2 \sim 2xF_1$$

**Pomeron intercept**

## Value of the BFKL intercept

$$\text{Leading order } \Delta_P^{LO} = (4N\alpha_s / \pi) \ln 2 \\ = 3.8\alpha_s \sim 0.5,$$

$$\text{Non-leading } \Delta_P^{NLO} = 0.08$$

Fadin –Kuraev-  
Lipatov

Fadin-Lipatov

LO and NLO intercepts violate the Unitarity and  
Froissar bound

$$\sigma_{tot}^{BFKL} \sim s^{\Delta_P}$$

$$\sigma_{tot} \leq c \ln^2 s$$

**Identical first-loop factors**

$$F_1^{BFKL} \sim \left(\frac{1}{x}\right) \exp[\Delta_P \ln(1/x)] \quad F_1^{DGLAP} \sim \left(\frac{1}{x}\right) \exp[\ln(1/x)]^{1/2}$$

**Difference comes from higher-loop factors**

**It is difficult to discriminate between BFKL and DGLAP at the present state of experiment, all hopes for LHC**

## Polarized DIS

$$g_1^{LL} \sim (1/x)^\Delta (Q^2/\mu^2)^{\Delta/2}$$

Bartels- Ermolaev-  
Manaenkov-Ryskin,  
Ermolaev-Greco-Troyan

Resummation of Leading Logarithms

whereas DGLAP predicts

$$g_1^{DGLAP} \sim \exp [\ln(1/x) \ln \ln (Q^2/\Lambda_{\text{QCD}}^2)]^{1/2}$$

Obviously  $g_1^{LL} \gg g_1^{DGLAP}$  when  $x \rightarrow 0$

## NEXT IMPORTANT STEP:

What is appropriate parameterization of  $\alpha_s$  at small  $x$  ?

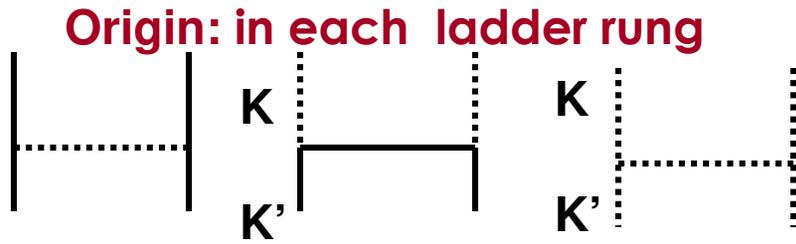
Standard parameterization

$$\alpha_s = \alpha_s(Q^2)$$

DGLAP-  
parameterization

Arguments in favor of the  
 $Q^2$ - parameterization:

Amati-Bassetto-Ciafaloni-Marchesini  
- Veneziano; Dokshitzer-Shirkov



$$\alpha_s = \alpha_s(k_{\perp}^2)$$

DGLAP-parameterization

Ermolaev-Greco-Troyan

However, such a parameterization is good for large  $x$  only. At small  $x$  :

$$\alpha_s = \alpha_s((k-k')^2) \approx \alpha_s((k_{\perp}^2 + k_{\perp}'^2)/x) \approx \alpha_s(k_{\perp}^2)$$

time-like argument

Participates in the Mellin transform

When DGLAP-ordering is used and  $x \sim 1$

space-like argument,

no Mellin transform

DGLAP -parameterization

$$\alpha_s = \alpha_s(Q^2)$$

Obviously, this parameterization and the DGLAP one converge when  $x$  is large but differ a lot at small  $x$

So, for studying  $g_1$  in the small- $x$  region, it is necessary to do:

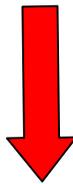
1. Total resummation of logs of  $x$
2. New parameterization of the QCD coupling

The basic idea: the formula

$$\alpha_s(k^2) = 1/(b \ln(k^2/\Lambda^2))$$

valid when

$$k^2 \gg \Lambda^2$$



it is necessary to introduce an infrared cut-off for  $k^2$

It is convenient to introduce the cut-off in the transverse space:

$$k_{\perp}^2 > \mu^2$$

Lipatov

As value of the cut-off is not fixed, one can evolve the structure functions with respect to  $\mu$   the name of the method:

## Infra-Red Evolution Equations (IREE)

### Highlights of the history of the method

- ★ Analyses of two-particle cuts in Regge kinematics **Gribov**
- ★ Factorization of photons with small transverse momenta **Gribov**
- ★ Infrared cut-off in the transverse momentum space **Lipatov**
- ★ Quark-quark scattering amplitudes **Kirschner-Lipatov**
- ★
- ★ QCD inelastic processes in Regge kinematics **Ermolaev-Lipatov**
- ★ Applications to Polarized Deep-Inelastic scattering **Bartels-Ermolaev  
-Manaenkov-Ryskin- Greco-Troyan**

Expression for the non-singlet  $g_1$  at large  $Q^2$ :  $Q^2 \gg 1 \text{ GeV}^2$

Initial quark density

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{x} \right)^\omega \left( \frac{\omega}{\omega - H(\omega)} \right) \alpha_q(\omega) \left( \frac{Q^2}{\mu^2} \right)^{H(\omega)}$$

Coefficient function

Anomalous dimension

New coefficient function and anomalous dimension sum up leading logarithms to all orders in  $\alpha_s$

Compare our non-singlet anomalous dimension to the LO DGLAP one:

expand C and H into series in  $1/\omega$

$$H = \frac{A(\omega)C_F}{2\pi} \left[ \frac{1}{\omega} + \frac{1}{2} \right] + \dots$$

coincide, save the treatment of  $\alpha_s$

$$\gamma_{NS}^{\text{LO DGLAP}} = \frac{\alpha_s(Q^2)C_F}{2\pi} \left[ \frac{1}{n(n+1)} + \frac{3}{2} - S_2(n) \right] \approx \frac{\alpha_s(Q^2)C_F}{2\pi} \left[ \frac{1}{n} + \frac{1}{2} + O(n) \right]$$

where

$$S_k(n) = \sum_{j=1}^n \frac{1}{j^k}$$

when  $n < 1$

small/large  $x$

small/large  $n$

Compare our coefficient function and the NLO DGLAP one

$$C = \frac{\omega}{\omega - H(\omega)} = 1 + \frac{A(\omega)C_F}{2\pi} \left[ \frac{1}{\omega^2} + \frac{1}{2\omega} \right] + \dots$$

LO

NLO

coincide, save the treatment of  $\alpha_s$

$$C_{NS}^{\text{DGLAP}} = 1 + \frac{\alpha_s(Q^2)C_F}{2\pi} \left[ \frac{1}{n^2} + \frac{1}{2n} + \frac{1}{2n+1} - \frac{9}{2} + \left( \frac{3}{2} - \frac{1}{n(1+n)} \right) S_1(n) + S_1^2(n) - S_2(n) \right]$$

when  $n < 1$

$$\approx \frac{\alpha_s(Q^2)C_F}{2\pi} \left[ \frac{1}{n^2} + \frac{1}{2n} + O(n) \right]$$

Expression for the singlet  $g_1$  at large  $Q^2$ :

$$g_1^S = \frac{\langle e_q^2 \rangle}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{x} \right)^\omega$$

$$\left[ \left( C_q^{(+)} \delta q + C_q^{(+)} \delta g \right) \left( \frac{Q^2}{\mu^2} \right)^{\Omega^{(+)}} + \left( C_q^{(-)} \delta q + C_q^{(-)} \delta g \right) \left( \frac{Q^2}{\mu^2} \right)^{\Omega^{(-)}} \right]$$

Large  $Q^2$  means

$$Q^2 > \mu^2; \mu \approx 5 \text{ GeV}$$

$$\Omega^{(+)} > \Omega^{(-)}$$

**Small  $-x$  asymptotics of  $g_1$ :** when  $x \rightarrow 0$ , the saddle-point method leads to

$$g_1^{NS} \sim \frac{e_q^2}{2} (1/x)^{\Delta_{NS}} (Q^2 / \mu^2)^{\Delta_{NS}/2} \delta q$$

**Nonsinglet intercept**

$$\Delta_{NS} = 0.42$$

At large  $x$ ,  $g_1^{NS}$  and  $g_1^S$  are positive

$$\delta q > 0$$



$$g_1^{NS} > 0$$

In the whole range of  $x$  at any  $Q^2$

Asymptotics of the singlet  $g_1$  are more involved

$$g_1^S \sim \frac{\langle e_q^2 \rangle}{2} S(\Delta_S) (1/x)^{\Delta_S} (Q^2 / \mu^2)^{\Delta_S / 2}$$

With intercept  $\Delta_S = 0.86$

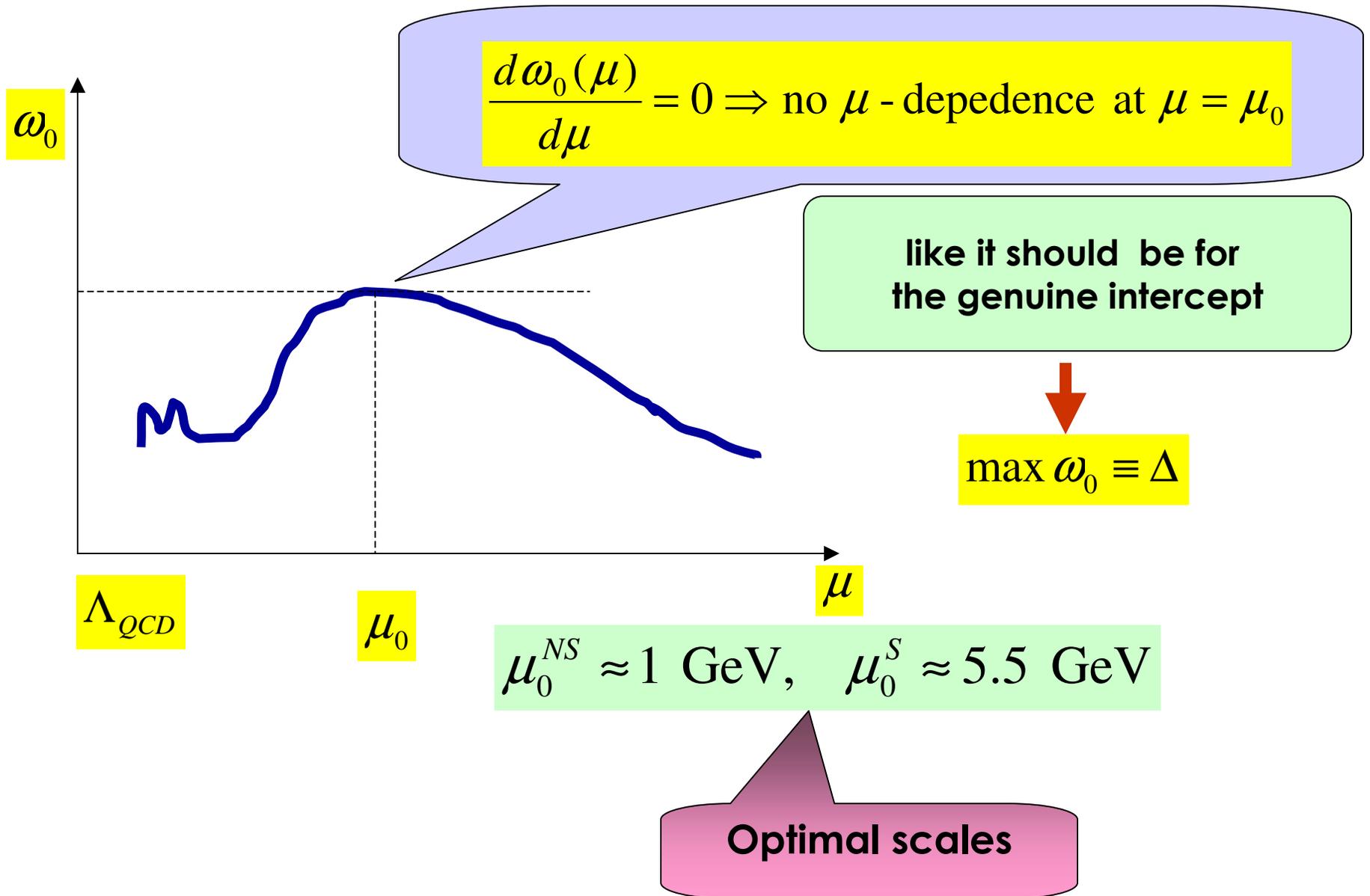
and

$$S(\Delta_S) = -\delta q - 0.064 \delta g$$

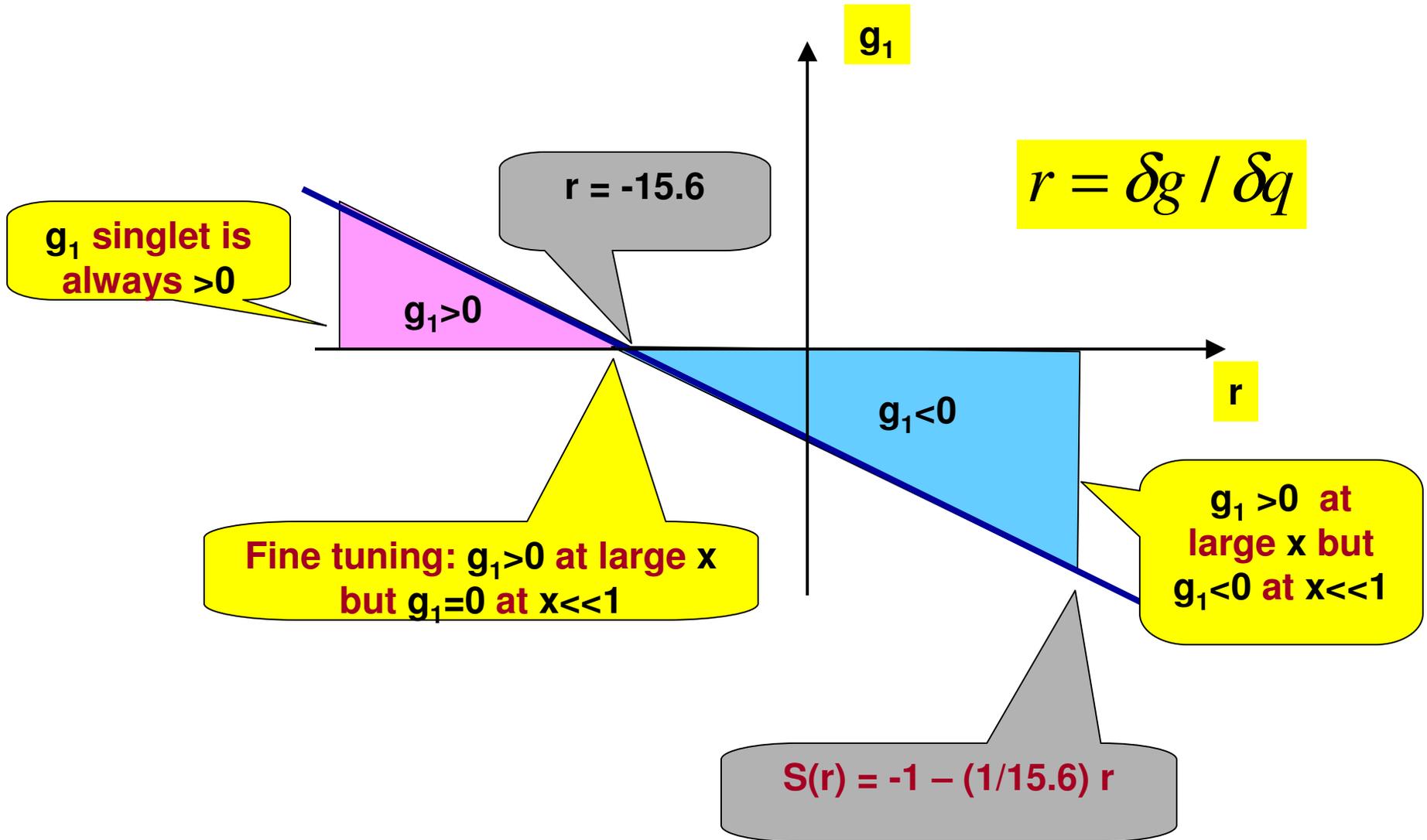
Interplay between the **quark** and **gluon** densities can lead to different sign of  $g_1$  singlet at  $x \ll 1$

**Warning: asymptotic expressions  $g_1 \sim (1/x)^\Delta$  are reliable at  $x < 10^{-5}$**

Obtaining intercepts: Plot  $\omega_0 = \omega_0(\mu)$



At large  $x$ ,  $g_1$  singlet is positive . When  $x \rightarrow 0$ , the sign of asymptotics of the singlet  $g_1$  depends on the ratio between the initial parton densities



Values of the intercepts perfectly agree with results of several groups who fitted experimental data.

non-singlet intercept

Soffer-Teryaev, Kataev-Sidorov-Parente, Kotikov-Lipatov-Parente-Peshekhonov-Krivokhijine-Zotov,

singlet intercept

Kochelev-Lipka-Vento-Novak-Vinnikov

### Anatomy of the singlet intercept

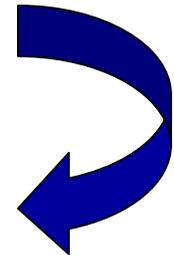
A. Graphs with gluons only:

$$\Delta_s = 1.1$$



violates unitarity

similar to LO BFKL



B. All graphs

$$\Delta_s = 0.86$$

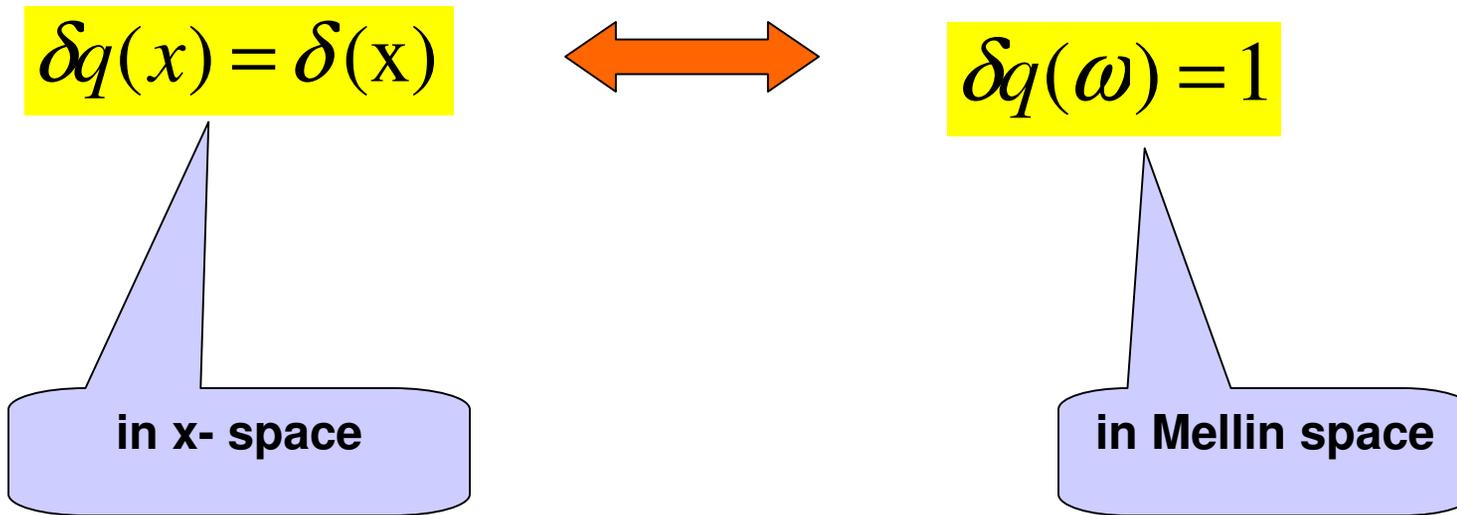


No violation of unitarity

Comparison of our results to DGLAP at finite  $x$  –no asymptotic formulae used

Comparison depends on the assumed shape of initial parton densities.

The simplest option: use the bare quark input



Numerical comparison shows that the impact of the total resummation of logs of  $x$  becomes quite sizable at  $x = 0.05$  approx.

**PUZZLE:** DGLAP should have Failed at  $x < 0.05$ .  
However, it does not take place.

**SOLUTION TO PUZZLE:** consider in more detail  
standard fits for initial parton densities

$$\delta q(x) = N x^{-\alpha} [(1 + \gamma x^{\delta})(1 - x)^{\beta}]$$

normalization

singular  
factor

regular factors

parameters  $\alpha \approx 0.58, \beta \approx 2.7, \gamma \approx 34.3, \delta \approx 0.75$

are fixed from fitting experimental data **at large**  $x$

In the Mellin space this fit is

$$\delta q(\omega) = N[(\omega - \alpha)^{-1} + \sum_{k=1}^{\infty} c_k ((\omega + k - \alpha)^{-1} + \gamma(\omega + k + 1 - \alpha)^{-1})]$$

Leading pole  
 $\alpha = 0.58 > 0$

Non-leading poles  
 $-k + \alpha < 0$

the small- $x$  DGLAP asymptotics of  $g_1$  is (inessential factors dropped)

$$g_1^{DGLAP} \sim (1/x)^\alpha$$

phenomenology

Comparison it to our asymptotics

$$g_1 \sim (1/x)^{\Delta_{NS}}$$

calculations

shows that the singular factor in the DGLAP fit mimics the total resummation of  $\ln(1/x)$ . However, the value  $\alpha = 0.58$  sizably differs from our non-singlet intercept  $= 0.4$

Although our and DGLAP asymptotics lead to the  $x$ - behavior of Regge type, they predict different intercepts for the  $x$ - dependence and different  $Q^2$  -dependence:

our calculations

$$g_1 \sim (1/x)^\Delta (Q^2 / \mu^2)^{\Delta/2}$$

whereas DGLAP predicts the steeper  $x$ -behavior and the flatter  $Q^2$  -behavior:

DGLAP

$$g_1^{DGLAP} \sim (1/x)^\alpha (\ln Q^2)^{\gamma(\alpha)}$$

$x$ -asymptotics was checked with extrapolating available exp data to  $x \rightarrow 0$ .  
It agrees with our values of  $\Delta$   
**Contradicts DGLAP**

our and the DGLAP  $Q^2$ -asymptotics have not been checked yet.

**Common opinion:** the total resummation is not relevant at available  $x$   
**Actually:** the resummation has always been accounted for through the standard fits, however without realizing it

**Common opinion:** fits for  $\delta q$  are singular but defined and large  $x$ , then convoluting them with coefficient functions weakens the singularity

$$C(x, y) \otimes \delta q(y) = \Delta q(x)$$

initial

x-evolved

**Obviously, it is not true:**  
They both are singular equally

Structure of DGLAP fit once again:

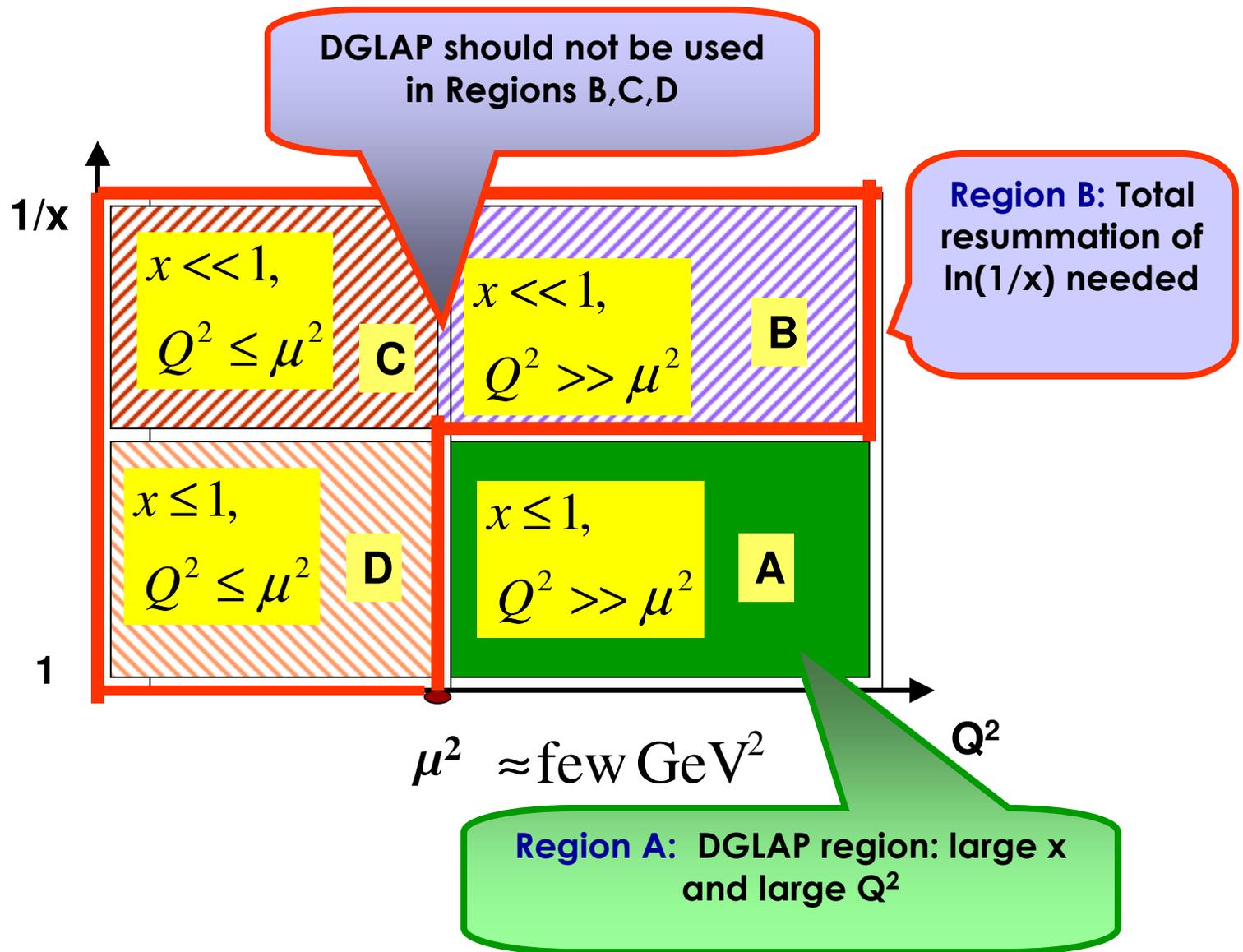
$$\delta q(x) = N x^{-\alpha} [(1 + \gamma x^\delta)(1 - x)^\beta]$$

Can be dropped when  
 $\ln(x)$  are resummed

x-dependence is weak at  $x \ll 1$  and can be  
dropped

Therefore at  $x \ll 1$

$$\delta q(x) \approx N(1 + ax)$$



**Unified description of  $g_1$  in Regions A&B:  
large  $Q^2$  and arbitrary  $x$ :**

**DGLAP**

**Good at large  $x$  because includes exact two-loop calculations but bad at small  $x$  as it lacks the total resummation of  $\ln(x)$**

**our approach**

**Good at small  $x$  , includes the total resummation of  $\ln(x)$  but bad at large  $x$  because neglects some contributions essential in this region**

**WAY OUT – interpolation expressions combining our approach and DGLAP**

- 1. Expand our formulae for coefficient functions and anomalous dimensions into series in the QCD coupling**
- 2. Replace the first- and second- loop terms of the expansion by corresponding DGLAP –expressions**

## Non-singlet $g_1$ : Our expressions

$$H_{LL}(\omega) = (1/2)[\omega - (\omega^2 - B(\omega))]^{1/2} \quad C_{LL}(\omega) = \omega / (\omega - H(\omega))$$

anomalous dimension

coefficient function

First terms of their expansions into the perturbation series

$$H_1 = \frac{A(\omega)C_F}{2\pi} \left[ \frac{1}{\omega} + \frac{1}{2} \right] \quad C_1 = \frac{A(\omega)C_F}{2\pi} \left[ \frac{1}{\omega^2} + \frac{1}{2\omega} \right]$$

New formulae combine Resummation and DGLAP:

$$H_C = H_{LL} - H_1 + H_{LO\ DGLAP} \quad C_C = C_{LL} - C_1 + C_{LO\ DGLAP}$$

formulae for the singlet anomalous dimensions and coefficient functions are written quite similarly

No singular parton densities are required

## Region C: small $x$ and small $Q^2$

At  $Q^2 \gg \mu^2$   $g_1$  depends on  $Q^2$  through logarithms:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^\omega \left(\frac{\omega}{\omega - H(\omega)}\right) \delta q(\omega) \left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$
$$= \sum_k \ln^k(Q^2 / \mu^2) C_k(x)$$

resummation of leading  $\ln(x)$

At  $Q^2 < \mu^2$   $g_1$  depends on  $Q^2$  through powers:

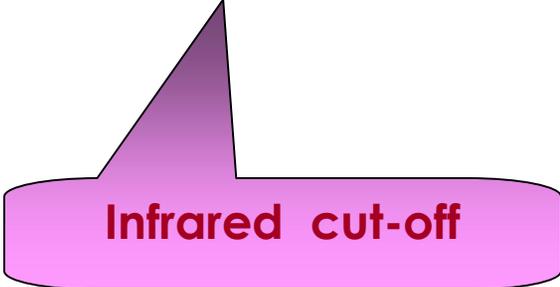
$$g_1^{NS} = \sum_k (Q^2 / \mu^2)^k C'_k(x)$$

again resummation of leading  $\ln(x)$

Description of  $g_1$  in Region C: small  $Q^2$  and small  $x$ :

Generalization of our previous results through the shift

$$Q^2 \rightarrow Q^2 + \mu^2 \quad \longrightarrow \quad x \rightarrow \bar{x} = (Q^2 + \mu^2)/2pq = x + z$$



Infrared cut-off

Similar shifts have been used for DIS structure functions by many authors, however from phenomenological considerations. **We do it from analysis of the involved Feynman graphs**

It leads to new expressions: **non-singlet  $g_1$  at small  $x$  and arbitrary  $Q^2$**

$$z = \frac{\mu^2}{2pq} \gg x = \frac{Q^2}{2pq}$$

**x -dependence**

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{z+x} \right)^\omega \left( \frac{\omega}{\omega - H(\omega)} \right) \delta q(\omega) \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\omega}$$

**Anomalous dimension**

**$Q^2$ -dependence**

**Coefficient function**

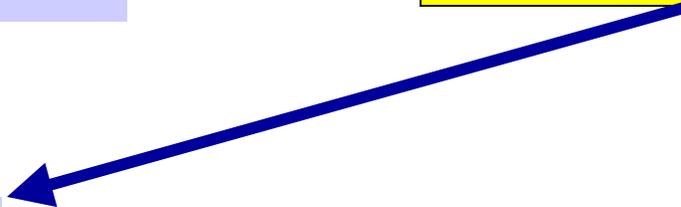
**Initial quark density**

# Technology of getting universal description of $g_1$ :

**Step 1:**  
Resummation of leading  
 $\ln(1/x)$  and  $\ln(Q^2)$



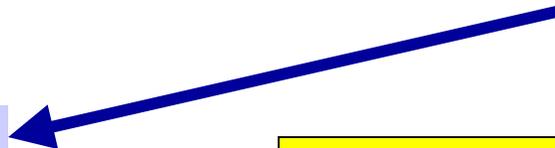
$g_1$  in Region B:  
large  $Q^2$  and small  $x$



**Step 2:**  
Combining above  
results and DGLAP



$g_1$  in Region A&B:  
large  $Q^2$  and arbitrary  $x$



**Step 3:**

Shift  $Q^2 \rightarrow Q^2 + \mu^2$



$g_1$  in Region C&D:  
small  $Q^2$  and arbitrary  $x$

Thus, we arrive at universal and model-independent description of  $g_1$  at arbitrary  $Q^2$  and  $x$  without singular fits:

$$g_1^{NS} = \frac{e_q^2}{2}$$

$$\int \frac{d\omega}{2\pi i} \left( \frac{1}{z+x} \right)^\omega C_C(\omega) \delta q(\omega) \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{H_C(\omega)}$$

$$z = \frac{\mu^2}{2pq}, \quad x = \frac{Q^2}{2pq}$$

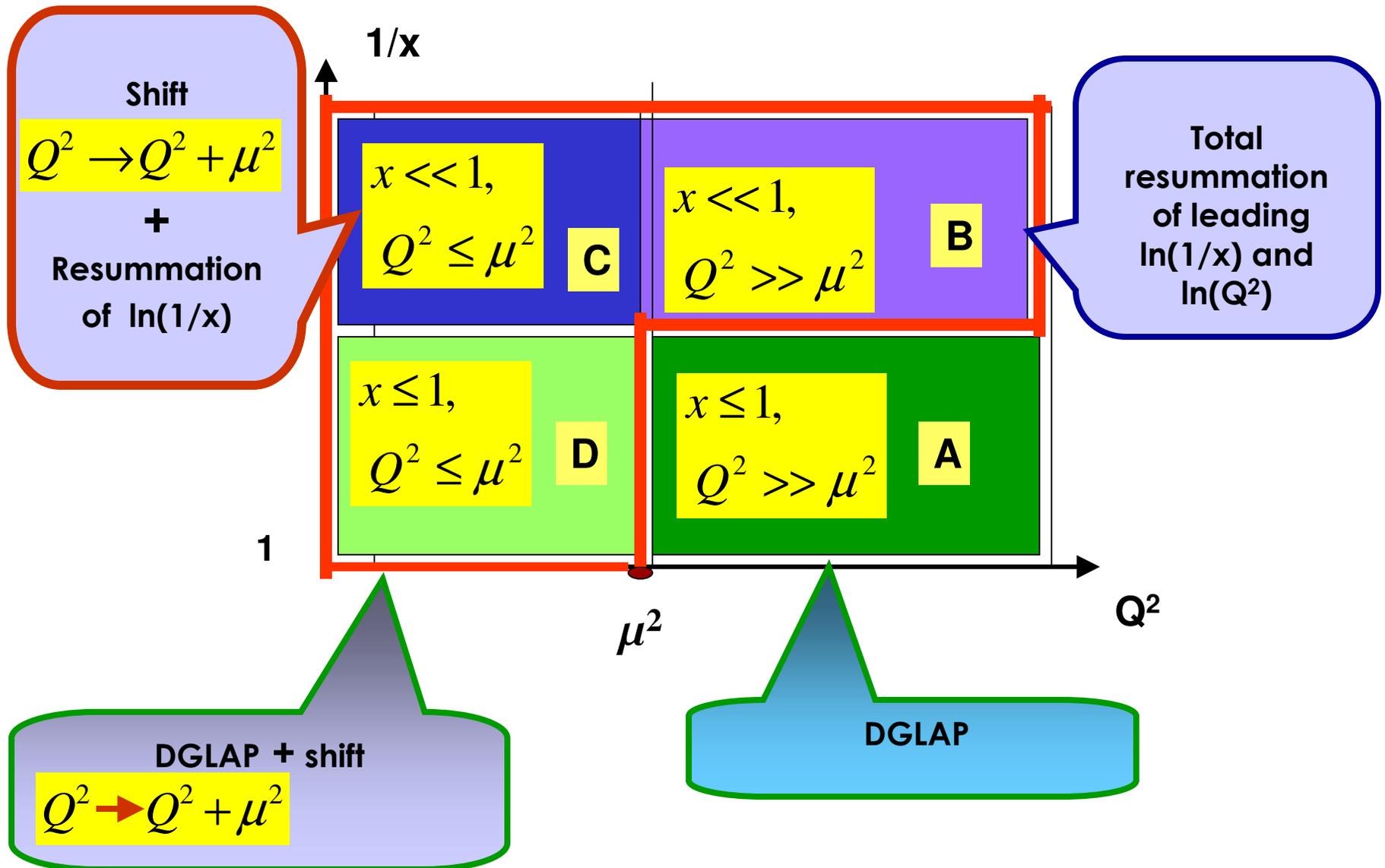
Combined coefficient function

Non-singular quark density

Combined anomalous dimension

expression for the singlet  $g_1$  is written quite similarly

Main impact on  $g_1$  in Regions A,B,C,D comes from:



## **Recent applications of our approach to:**

- 1. COMPASS results**
- 2. Power  $Q^2$ -corrections**

Taken from [www.compass.cern.ch](http://www.compass.cern.ch)



COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at [CERN](http://cern.ch) in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams. On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006. Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

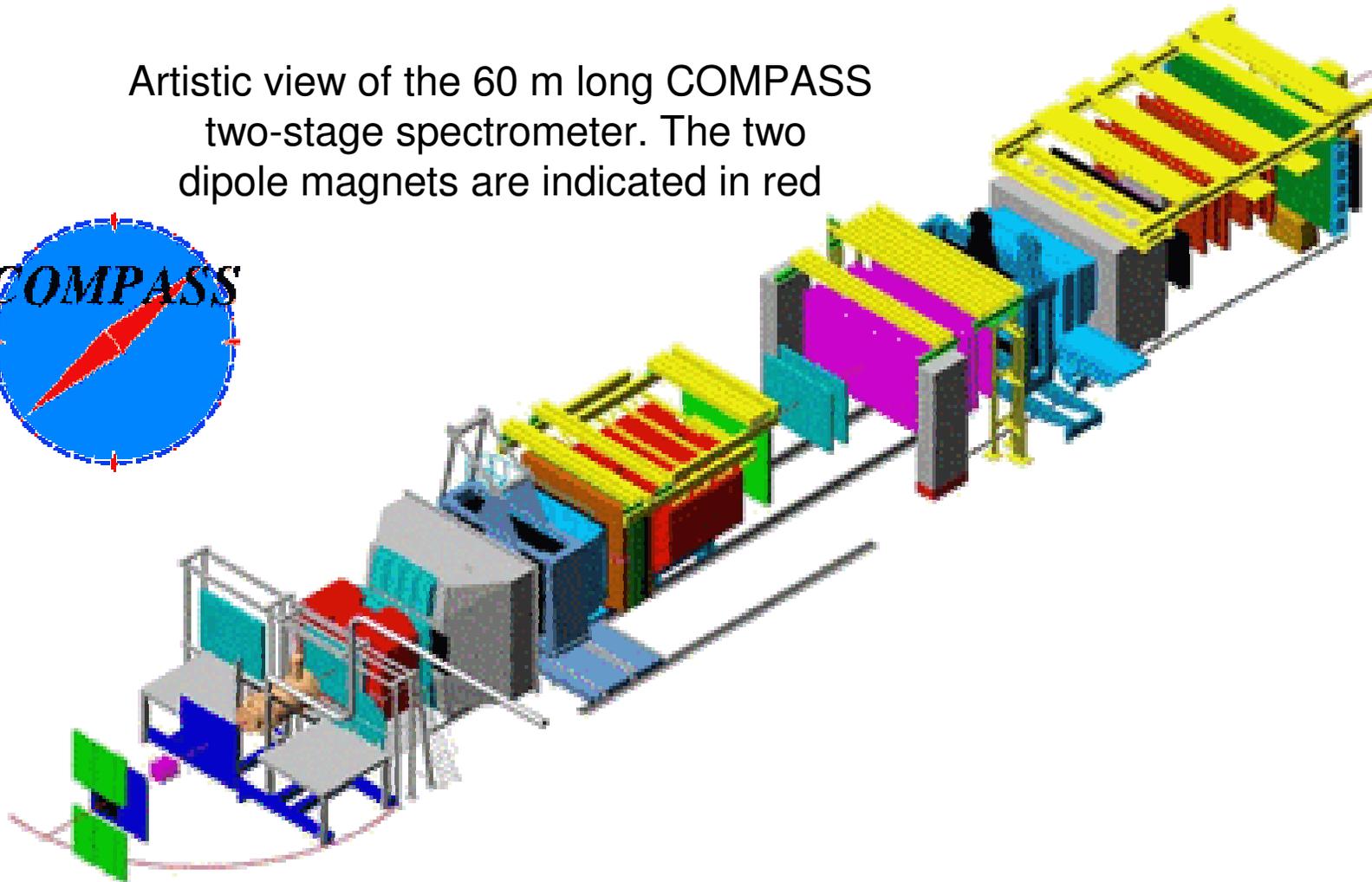
# COMPASS

Taken from [www.compass.cern.ch](http://www.compass.cern.ch)

## Common Muon Proton Apparatus for Structure and Spectroscopy



Artistic view of the 60 m long COMPASS two-stage spectrometer. The two dipole magnets are indicated in red



**Small  $Q^2$   
Singlet  $g_1$**

$$z = \frac{\mu^2}{2pq},$$

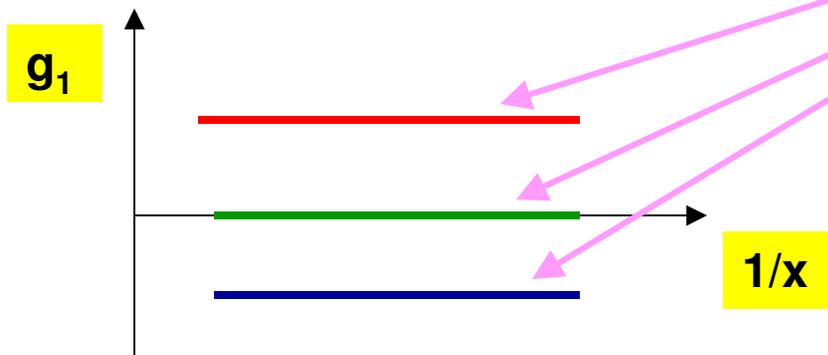
$$x = \frac{Q^2}{2pq}$$

$$g_1^S = \frac{\langle e_q^2 \rangle}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{z+x} \right)^\omega [C_q \delta q + C_g \delta g]$$

$$C_g = C_g^{(+)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$C_q = C_q^{(+)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

when  $Q^2 \ll \mu^2$  both  $x$ - and  $Q^2$ - dependences are flat, even for  $x \ll 1$ .



**Location of the line is determined by the  $z$ -dependence**

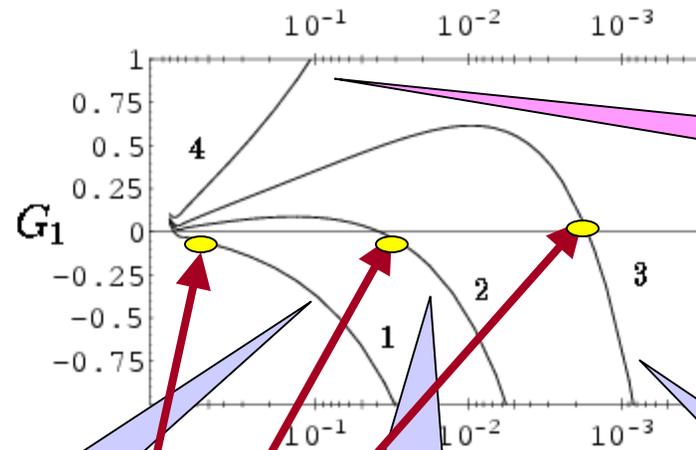
$$g_1(z) = \left( \frac{e_q^2}{2} \right) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left( \frac{1}{z} \right)^\omega \left[ C_q(\omega) \delta q + C_g(\omega) \delta g \right]$$

Approximating

$$\delta q \approx N_q, \delta g \approx N_g,$$

perform numerical calculations of  $G_1$

$$g_1 = (e_q^2 / 2) N_q G_1,$$



$$N_g/N_q = 0$$

$$N_g/N_q = -5$$

$$N_g/N_q = -8$$

$$N_g/N_q < -15.6$$

Position of the turning point is sensitive to  $N_g/N_q$ , so the experimental detection of it will allow to estimate  $N_g/N_q$

## Power Corrections to non-singlet $g_1$

Leading twist contribution

mass scale:  $Q^2 > M^2$

$$g_1(x, Q^2) = g_1^{LT}(x, Q^2) \left[ 1 + \sum_k C_k \left( \frac{M^2}{Q^2} \right)^k \right]$$

PC are supposed to come from higher twists.  
No satisfactory theory  
is known for the higher twists

Power corrections

Standard way of obtaining PC from experimental data at small  $x$ :

Leader-Stamenov-Sidorov

Compare experimental data to predictions of the Standard Approach and assign the discrepancy to the impact of PC

$$g_1^{LT} = g_1^{DGLAP}$$

## Counter-argument:

1. DGLAP, the main ingredient of SA, is theoretically unreliable at small  $x$ , so comparing experiment to it is not so productive: it proves nothing
2. SA cannot explain why PC appear at  $Q^2 > 1 \text{ GeV}^2$  only and predict what happens at smaller  $Q^2$

Our approach can do it:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{w}{\mu^2 + Q^2} \right)^\omega C(\omega) \delta q(\omega) \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

where  $w = 2pq$  and  $Q^2$  can be large or small,  $\mu = 1 \text{ GeV}$

$\mu = 1 \text{ GeV}$ , so when  $Q^2 < 1 \text{ GeV}^2$ , expansion into power series is:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{\omega}{\mu^2} \right)^\omega C(\omega) \delta q(\omega) \left[ 1 + \sum_{k=1} T_k(\omega) \left( \frac{Q^2}{\mu^2} \right)^k \right]$$

Power corrections

Leading contribution for  $g_1^{NS}$   
does not depend on  $Q^2$

At  $Q^2 > 1 \text{ GeV}^2$  expansion into series is different:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{\omega}{Q^2} \right)^\omega C(\omega) \delta q(\omega) \left( \frac{\mu^2}{Q^2} \right)^{H(\omega)} \left[ 1 + \sum_{k=1} T_k(\omega) \left( \frac{\mu^2}{Q^2} \right)^k \right]$$

Conventionally looking  
Power Corrections

Leading contribution for  $g_1^{NS}$

**These Power Corrections have perturbative origin and should be accounted in the first place. Only AFTER THAT one can reliably estimate a genuine impact of higher twist contributions**

# Conclusion

## Standard Approach

DGLAP was originally developed for operating at the region where both  $x$  and  $Q^2$  are large. Basic ingredients of the DIS structure functions – coefficient functions and splitting functions (anomalous dimensions) are calculated in DGLAP in the first and second loops. By construction, DGLAP describes the  $Q^2$ -evolution but cannot describe the  $x$ -evolution. Accounting for the  $x$ -evolution is especially important in the small- $x$  region.

In order to extend DGLAP to the region of small  $x$  and large  $Q^2$ , it has been complemented with rather complicated expressions for the initial parton densities  $\delta q$  and  $\delta g$  found from fitting experimental data.

**DGLAP + Standard fits form Standard Approach (SA). SA describes DIS at large  $Q^2$  and arbitrary  $x$ .**

## Alternatives to SA

**Unpolarized DIS:** No model-independent description. Models involve either phenomenological Pomerons or BFKL. Both LO and NLO BFKL intercepts are positive. They violate both the unitarity and Froissart bound. It is urgent to calculate NNLO corrections to the intercept

As the Pomeron intercept is small, it is difficult to discriminate between these approaches

### **Polarized DIS:**

Model-independent description of  $g_1$  combines total resummation of leading logarithmic contributions, DGLAP expressions, and shift of  $Q^2$ .  
It represents  $g_1$  at arbitrary  $x$  and  $Q^2$ .

DGLAP agrees with experimental data only when special expressions for initial parton densities are used. They include singular factors, though DGLAP offers no theoretical explanation of the origin of the factors

Actually, the singular factors mimic total resummation of leading logarithms  
When the resummation is accounted for, the expressions for initial parton densities can be simplified down to constants