Crossing and Veering phenomena in a crank-mechanism

Marco Brino
Outline

Aim

- Investigate the dynamic behaviour of structures with close or even coincident eigenvalues

Dynamics with close eigenvalues

- Cyclic / symmetric structures
- Non-cyclic / non-symmetric structures
- Effects of boundary conditions
- From component to system dynamics
State of the art

➢ **Coincident or close modes structures:**

- **Symmetric structures**
  symmetric shells, bells, bladed disk, …

- **Cyclic structures**
  lumped systems, bladed disk (again), …

- **Uncoupled or slightly coupled systems**
  symmetric beams, bladed disk (again), …
Many references on theoretical dynamic systems with coincident or close eigenvalues

Young, Hwang, Chinese Proceed. 2007

Few references on experimental test-rigs

No experimental test-rigs with NO-SYMMETRY or NO-CYCLIC, with structures “near” to wing-shape

Adhikari, duBois, Lieven, JSV 2009
Most of them presents SYMMETRY and/or CYCLIC characteristics, or some UNCOUPLING parameter.

Chan, Ewins, MSSP 2010

Perkins, Mote, JSV 1986

Balmes, JSV 1993
Modal Analysis framework

Basic assumptions:

Equations of motion in matrix form: \( \mathbf{M} \ddot{\mathbf{x}} + \mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{f} \)

Considering the undamped system: \( \mathbf{M} \ddot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{f} \)

Solving eigenproblem: \( \det(\mathbf{K} - \omega^2 \mathbf{M}) = 0 \)

Solution: \( \omega_r^2 \) eigenvalues
\( \Phi^{(r)} \) eigenvectors
Structures with symmetry:

- Example: BELL

4th Mode

\( f_4 = 2860 \text{ Hz} \)

5th Mode

\( f_5 = 2860 \text{ Hz} \)
Coincident eigenvalues

Structural modifications:

- Modify lumped elastic parameters on symmetric structures to enforce coincident mode shapes
Coincident eigenvalues

**Structural modifications:**
- Modify lumped elastic parameters on symmetric structures to enforce coincident mode shapes
**Coincident eigenvalues**

**Structural modifications:**

- Modify lumped elastic parameters on symmetric structures to enforce coincident mode shapes
Crossing and veering phenomena

Cyclic structures

Deterministic analysis of crossing and veering phenomena:
- Crossing $k_{12} = k_{23} \div k_{31} = 90\% \div 110\% k_{12}$
- Veering $k_{23} = 102 \% k_{12} ; k_{31} = 90\% \div 110\% k_{12}$

Sensitivity to uncertainly and variability
- Choice of index to discriminate crossing and veering → MAC
- Orthogonality properties
- Sensitivity to eigenvalues AND eigenvectors changes
Curve Crossing: $k_{12} = 1 \text{ N/m}; k_{23} = 1 \text{ N/m}$

$K_{23}$: discriminant

$K_{31}$: variable

MAC index

**Before**

- Mode 1: $\lambda_1 = 0 \text{ rad}^2/\text{s}^2$
- Mode 2: $\lambda_2 = 2.98 \text{ rad}^2/\text{s}^2$
- Mode 3: $\lambda_3 = 3 \text{ rad}^2/\text{s}^2$

**After**

- Mode 1: $\lambda_1 = 0 \text{ rad}^2/\text{s}^2$
- Mode 2: $\lambda_2 = 3 \text{ rad}^2/\text{s}^2$
- Mode 3: $\lambda_3 = 3.02 \text{ rad}^2/\text{s}^2$

Path of modal properties

**Curve Crossing**

$D.o.F.$

$\lambda_1 = 0 \text{ rad}^2/\text{s}^2$

**Curve Veering**

$D.o.F.$

$\lambda_1 = 0.85 \text{ rad}^2/\text{s}^2$

Change of mode-shape order
**Crossing and veering phenomena**

**Curve Crossing:** $k_{12} = 1$ N/m; $k_{23} = 1$ N/m

**Curve Veering:** $k_{12} = 1$ N/m; $k_{23} = 1.02$ N/m

**K23**: discriminant

**K31**: variable

**LEISSA:** "figuratively speaking, a dragonfly one instant, a butterfly the next and something indescribable in between"

**Change of mode-shape order**

**Path of modal properties**

Before

After
Crossing and veering: “wing” structure

Non-cyclic structure

- LUPOS
- CAD model
- Experimental Test-rig

Beam 1
Mass 1
Beam 2
Mass 2
Beam 3
Mass 3
Crossing and veering: comparison LUPOS vs. 3D FEM

Parametric FEM: configuration parameter looped through the configurations to run families of FEA solutions

- **LUPOS model**

- **Solidworks CAE model**
Crossing and veering: FEM model updating

**Experimental Modal Analysis**

- 19 configurations (5° steps)
Crossing

Modal Assurance Criterion

\[ MAC_{i,j} = \frac{\left( \Phi_i^T \Phi_j \right)^2}{\left( \Phi_i^T \Phi_i \right) \left( \Phi_j^T \Phi_j \right)} = \cos^2 \alpha_{i,j} \]

- Crossing: modes 4/5 @ 68/69°
Crossing

Modal Assurance Criterion

➤ Crossing: modes 4/5 @ 68/69°

\[ MAC_{i,j} = \frac{\left(\Phi_i^T \Phi_j\right)^2}{\left(\Phi_i^T \Phi_i\right)\left(\Phi_j^T \Phi_j\right)} = \cos^2 \alpha_{i,j} \]
Veering

Modal Assurance Criterion

\[ MAC_{i,j} = \frac{\left(\Phi_i^T \Phi_j\right)^2}{\left(\Phi_i^T \Phi_i\right)\left(\Phi_j^T \Phi_j\right)} = \cos^2 \alpha_{i,j} \]

- Veering: modes 5/6 @ 72/81°
Veering

Modal Assurance Criterion

Veering: modes 5/6 @ 72/81°
Mode shapes plot

Information exchange without eigenvalues coincidence

3 interacting modes …
Crossing and veering: the crank-mechanism

Full 4c/4s crank-mechanism:

- CAD-CAE parametric modal simulations
Configuration definition in crank-mechanism

Config. 0°

Config. 45°

Config. 90°

Config. 135°

Config. 180°
Basic assumptions:

- Boundary Conditions
  - free-free
  - pinned-pinned
Crank-mechanism modelled in LUPOS

![Graphs showing crank-mechanism modelled in LUPOS with matrices M and K.](image)
Frequency loci w.r.t. crank angle in both conditions

Different behaviour with different BCs
Frequency loci w.r.t. stiffness of BCs

Configuration parameter:

- BC modelled as grounded springs

Config. 0°

Free-free

Freq. [Hz]

Pinned-pinned

Stiffness [%]

9th mode – 2 Torsional

6th mode - 1 Torsional

1st mode – Rigid body

1st-6th modes - Rigid body

25th mode - 1 Torsional
Frequency loci w.r.t. stiffness of BCs

Different configuration:

- Crank angle

Config. 90°
Influence of components coupling:

- From component to system dynamics
Comparison between component and system dynamics

Again different behaviour w.r.t. crank angle

➢ From component to system dynamics

Crankshaft

Crankmechanism

- 13\textsuperscript{th} mode - 1 Torsional
- 12\textsuperscript{th} mode - 4 Bending xy
- 11\textsuperscript{th} mode - 2 Bending yz
- 10\textsuperscript{th} mode - 3 Bending xy
- 9\textsuperscript{th} mode - 2 Bending xy
- 8\textsuperscript{th} mode - 1 Bending yz
- 7\textsuperscript{th} mode - 1 Bending xy
Modal Assurance Criterion

MAC index to compare mode shapes:

➢ Not an efficient index to compare component and assembly mode shapes

\[
MAC_{i,j} = \frac{\left(\Phi_i^T \Phi_j\right)^2}{\left(\Phi_i^T \Phi_i\right)\left(\Phi_j^T \Phi_j\right)} = \cos^2 \alpha_{i,j}
\]
Progressive influence of components interaction

Frequency w.r.t. crank angle

- Single component: crankshaft

\[
\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 0\%
\]

- 8th mode - 1 Bending yz
- 12th mode - 4 Bending xy
- 11th mode - 1 Torsional
- 10th mode - 3 Bending xy
- 9th mode - 2 Bending xy
- 7th mode - 1 Bending xy
Progressive influence of components interaction

Frequency w.r.t. crank angle

- Rods and pistons uncoupled

\[ \frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 0.01\% \]
Progressive influence of components interaction

**Frequency w.r.t. crank angle**

- Negligible coupling …

\[
\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 1\%
\]
Progressive influence of components interaction

**Frequency w.r.t. crank angle**

- Weak coupling …

\[
\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 10\%
\]
Progressive influence of components interaction

Frequency w.r.t. crank angle

- Significant coupling ...

\[
\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 50\%
\]
Progressive influence of components interaction

Frequency w.r.t. crank angle

- Full coupling

\[ \frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 100\% \]
Different use of MAC

Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 7 – 1 bend xy

Crankshaft mode 8 – 1 bend xz
Different use of MAC

Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 9 – 2 bend xy

Crankshaft mode 10 – 3 bend xy
Different use of MAC

Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 11 – 1 tors

Crankshaft mode 12 – 4 bend xy
Different use of MAC

Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?
Resume

Dynamics with close eigenvalues

- Cyclic / symmetric structures
- Non-cyclic / non-symmetric structures
- Effects of boundary conditions
- From component to system dynamics

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