IGA collocation, aka "the ultimate reduced quadrature IGA method": Some results, applications, and open problems

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Isogeometric analysis (IGA) was introduced in [1] with the main aim of bridging Computer Aided Design (CAD) and Finite Element Analysis (FEA). In addition, thanks to the high-regularity properties of its basis functions, IGA showed a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard FEA. However, a well-known important issue of IGA is related to the development of efficient integration rules able to reduce the high array formation costs induced by standard Gaussian quadrature, in particular when higher-order approximations are employed. In an attempt to address the issue above taking full advantage of the special possibilities offered by IGA and in particular by the available higher regularity, isogeometric collocation (IGA-C) schemes have been first proposed in [2]. The main idea of IGA-C consists of the discretization of the governing partial differential equations in strong form, within the isoparametric paradigm, reducing the number of evaluations needed for array formation to only one per degree of freedom. The aim is to optimize the computational cost still relying on IGA geometrical flexibility and accuracy. In general, IGA-C features look particularly attractive when evaluation and formation costs are dominant, as in the case, e.g., of explicit structural dynamics. Detailed comparisons with both IGA and FEA Galerkin-based approaches were carried out, showing IGA-C advantages in terms of accuracy versus computational cost, in particular for higher-order approximation degrees. Since its introduction, many promising significant works on IGA-C were published in different fields, including, among others, phase-field modeling, linear and nonlinear elasticity, contact, as well as several interesting studies in the context of structural elements (see, e.g., [3] and references therein). In particular, IGA-C allows to reach new frontiers for mixed formulations, where methods that are known to be unstable in the Galerkin framework seem to be stable and very efficient when combined with IGA-C. Finally, it has been recently shown that IGA-C can be conveniently combined with many different spline spaces, able for example to be locally refinable or possessing other desirable properties not available with classical B-splines or NURBS (like, e.g., Hierarchical NURBS, T-splines, or Generalized B-splines). All these results naturally propose collocation as one of the most promising research directions in the field of IGA, able to combine simplicity and efficiency with an incredibly high potential. In this talk, a review of IGA-C results, applications, and open problems will be given.

Keywords: Isogeometric Analysis, Collocation, Mixed formulations, Local refinement

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