Statistical finite element analysis of the buckling behavior of honeycomb structures

Domenico Asprone\textsuperscript{a}, Ferdinando Auricchio \textsuperscript{b}, Costantino Menna\textsuperscript{a,*}, Simone Morganti \textsuperscript{b}, Andrea Prota \textsuperscript{a}, Alessandro Reali \textsuperscript{b}

\textsuperscript{a}Department of Structures for Engineering and Architecture (DiSt), University of Naples, Federico II, Via Claudio 21, 80125 Naples, Italy
\textsuperscript{b}Department of Civil Engineering and Architecture (DICAr), University of Pavia, Via Ferrata 3, 27100 Pavia, Italy

\textbf{A R T I C L E   I N F O}

Article history:
Available online 23 May 2013

Keywords:
Imperfections
Nomex honeycomb
Buckling
Finite element analysis
Stochastic approach

\textbf{A B S T R A C T}

The main key performance factors of honeycombs are represented by the ability to withstand through-thickness compression and to absorb energy by plastic deformation of the cell walls. The knowledge of the constituent material properties, including the sensitivity of these structures to material defects, and of the probability occurrence during the crushing mode represents a basic step to perform reliable finite element analyses able to accurately reproduce the behavior of such structures. The present paper reports a comprehensive study of the compressive response of hexagonal honeycomb structures made of phenolic resin-impregnated aramid paper (Nomex\textsuperscript{\textregistered}); the compressive response is numerically investigated and compared with experimental results. A shell model of a representative single cell made of expanded Nomex has been created using the implicit ABAQUS finite element solver. Imperfections due to the manufacturing process are taken into account including material imperfections (elastic modulus variability) and geometrical defects (thickness variability). Imperfections are included in the model by defining different material and thickness properties for each element according to a pre-defined statistical distribution. The effects of imperfections on the honeycomb structure behavior are investigated. The predicted structural response, numerically obtained using different sets of imperfections, shows a good correlation with experimental results.

\begin{thebibliography}{99}

1. Introduction

Nowadays composite sandwich structures are widely used in many different engineering fields especially in the aerospace and mass transport industry where weight reduction is one of the most significant design parameters. These materials offer many advantageous specific mechanical properties, such as stiffness-to-weight ratio, which make them suitable for component construction including secondary structural parts like flaps, wing-body fairings, engine cowls, spoilers, nacelles, and radomes [1–3]. Sandwich structures are generally made of two thin and stiff skins, separated by a thick, lightweight core, which increases the inertia of the resulting section without significantly increasing the mass. The skins primarily carry tensile and compressive loads whereas the core carries transverse shear loads and gives the panel high specific bending stiffness. In addition, the core influences the out-of-plane compressive behavior and the energy absorbing capability of the sandwich structure by its failure mechanisms. The skins are regularly made of lightweight materials such as aluminum or fiber reinforced composite laminates. Different types of sandwich core structures are commonly used in the aerospace and transportation industry including foam/solid core (ships, aircrafts), honeycomb (aircrafts, satellites), truss core (buildings, bridges), and web core [1,2], which can be manufactured by using a variety of base materials, e.g., metal foils, plastic foils, or resin impregnated paper-like materials made of synthetic or natural fibers.

The use of honeycomb is highly diffusion in many engineering applications due to the high density-specific performances they can offer. The main key performance factors of honeycombs are represented by the ability to withstand through-thickness compression and to absorb energy by plastic deformation of the cells. Generally, in the aerospace industry honeycomb structures are made of aluminum foils, phenolic-impregnated Nomex\textsuperscript{\textregistered} paper or carbon honeycomb, given their superior weight-specific mechanical properties in terms of stiffness and structural weight reduction [4–6]. Furthermore they offer design versatility and cost-efficient manufacturing [7]. However, the closed structure of honeycomb cells may lead to negative effects regarding condensed water trapped in the cells and trigger an increase in weight as well as a reduction of mechanical properties. To overcome this issue, new configurations for core structures, such as folded cores [8], have been recently introduced.
In particular, the manufacturing process is described in [9] and a wide range of experimental results can be found in [10].

The knowledge of core constituent material properties and the sensitivity of such structures to different sources of defects represent a fundamental concern for the development of a cellular-based finite element (FE) model able to capture the behavior of such structures, including the buckling limit and the folding mechanism occurring during the crushing mode.

Generally, the usual source of material imperfections is the manufacturing process of honeycomb structures. The most common manufacturing method is the adhesive bonding followed by an expansion process [7]: honeycomb starts out as flat strips of material, or ribbons. Strips of adhesive are placed on the ribbons in a staggered pattern; for metallic cores, a corrosive resistant coating is applied to foil sheets before printing the adhesive lines. The sheets are cut to the required thickness and stacked, and the adhesive is cured under pressure at elevated temperature. Once cured, the blocks are cut to the desired thickness, and then the ribbons are pulled apart or expanded to form honeycomb. Fully expanded honeycomb forms the classical hexagon shape whereas if the expansion is stopped before or after the hexagons are fully formed, an under-expanded or an over-expanded core can be obtained, respectively. When metallic cores are expanded, the sheets yield plastically and the node free wall joints thereby retain their expanded geometric shape. The procedure for nonmetallic honeycomb is slightly different. Here the honeycomb does not retain its shape after expansion and must be held in a rack. The block web material contains a small amount of resin which is heat-set in an oven. Most paper cores will retain their expanded shape. Then, the honeycomb block is dipped in liquid resin (usually phenolic or polyamide) and oven cured. The dipping curing cycle is repeated until the block is at the desired density.

Most honeycomb structures show a similar mechanical behavior under out-of-plane (i.e., through-thickness direction) compression. In particular, they are characterized by: (i) an initial linear-elastic regime up to the buckling limit and a subsequent compressive strength reduction; (ii) a plateau of constant stress, named crush strength, corresponding to a progressive degradation of cell walls; (iii) a final segment of densification characterized by the compression of the cell wall itself. However, honeycomb structures can exhibit different features in their compressive out-of-plane behavior depending on the nature of the honeycomb core constituent material. In particular, when the critical compressive stress is reached, the cells begin to collapse by elastic buckling, plastic yielding or brittle fracture, depending on the wall material type. Hanel et al. [11] investigated the influence of the different paper materials, i.e., Kevlar and Nomex, on the structural properties of wedge-shaped folded cores, whereas other authors investigated the structural behavior of a honeycomb structure made of Nomex paper material, making comparison with a honeycomb core made of aluminum [12]. They report that Nomex material is much more brittle than aluminum foil during the progressive failure mechanism and the crushing regime.

In dealing with the modeling of flat-wise compressive behavior, several approaches have been proposed, encompassing analytical models [13,14], macromechanical finite element models adopting equivalent solid formulations [12], as well as meso-mechanical models focusing on the cellular honeycomb structure, both analytical and numerical [8,15–18]. Nevertheless, the first two types of approaches present limiting drawbacks mainly related to the prediction of the shear behavior and the progressive collapse of the honeycomb cells. For these reasons, the problem is often tackled through detailed numerical models and/or through a virtual testing approach. In general, virtual testing consists of a step-by-step numerical procedure where a detailed model is employed on the basis of experimental tests. Usually, this procedure begins from a small scale (as an example, from coupon material tests); once the mechanical behavior is properly calibrated at that scale, more complex tests can either be simulated or carried out scaling up to the scale of the component/assembly. As a consequence of this process, experimental tests can be limited to a preliminary stage supporting numerical implementations in subsequent steps. This procedure represents a key tool especially for composite materials exhibiting different behavior at different scales. In fact, by reproducing the exact shape of the core structure, this numerical technique gives the possibility to reproduce not only the elastic behavior prior to cell wall buckling but also the complex crushing behavior characterized by cell wall folding mechanisms at micro/mesoscale, even though a large computational time may be required. Moreover, it allows to gather mechanical properties which are usually not available among experimental manufacturer databases. This kind of approach is usually pursued to model honeycomb or folded core materials by using different scales ranging from one single cell scale to large scale including the full honeycomb model. An example of this numerical approach can be found in recent studies performed by Heimbs and concerning virtual testing of sandwich panels manufactured with Nomex honeycomb and folded cores [8]. The author successfully simulated the cellular core behavior in different loading directions by adopting a very detailed finite element model and addressing numerical problems such as mesh and loading rate dependencies. In addition, Giglio et al. [19] investigated the compressive behavior of Nomex hexagonal honeycomb core (with 32 kg/m³ core density and a nominal cell size of 4.76 mm) through detailed finite element simulations; several aspects were investigated, including a comparison between the use of shell elements vs solid elements for the modeling of Nomex cell walls, mesh effects, glue and brittle phenolic resin effects as well as dimensional scale issues. The results showed that both solid and shell elements can evaluate with a reasonable error the compressive peak value, but shell elements fail to demonstrate reliable behavior in the plateau stress phase.

1.1. Research significance

In the present study, a statistical-based method to include imperfections in the simulation is presented and applied to predict the compressive behavior of hexagonal Nomex honeycomb structure. A shell model of a representative single cell made of expanded Nomex has been created using theABAQUS FE code (Simulia, Dassault Systèmes, Providence, RI, USA). The key aspect proposed in the study is the inclusion of imperfections in terms of both material (elastic modulus variability) and geometrical (thickness variability) defects. A stochastic approach is proposed by performing random sampling at each element of the FE mesh assuming different Young modulus values and different thickness properties according to a pre-defined statistical distribution. The modeled cell is used to address several aspects of the compressive response, particularly focusing on onset of buckling and collapse limit.

1.2. Imperfections – classification and modeling

Generally, different types of imperfections and irregularities may affect the ideal structure of a cellular honeycomb core and modify its mechanical behavior. These are inevitably generated by the manufacturing process and/or loading conditions. A general classification of imperfections can be provided as follows [20,21]:

- Geometrical imperfections: i.e., shape defects, curvatures, non perfect angular corners, surface roughness, wall thickness variability, uneven cells (global imperfections);
Material imperfections: i.e., variability of the material properties, for example due to the heterogeneity of the adopted composite system (e.g., Nomex paper or CFRP), resin accumulation in cell wall corners, pores, variation of fiber volume fraction (local imperfections);

- Initial imperfections: i.e., prestress and prestrain conditions;

- Loading imperfections: i.e., load misalignment, variation of loads.

Typically, the actual case originates as a combination of the kinds of imperfections listed above.

Defects are responsible for the initiation of damage and subsequent degradation (damage propagation, folding phase, tearing of the edges, etc.) and, therefore, greatly influence the global and local mechanical response of the structure. Several studies have been conducted addressing this issue. An ideal model without imperfections tends to overestimate the mechanical properties, in particular critical buckling load and initial stiffness [22, 23]. Combscure [20] reported some results showing how initial shape imperfections, thickness defects, and boundary condition imperfections can lead to a drastic decrease of the load carrying capacity of a structure under compression. Baranger et al. [22] analyzed the influence of geometrical defects in folded cores and their consequences on buckling behavior with the final aim of performing a numerical optimization of the core geometry. They reported that geometrical defects may play a major role in the response of the structure in terms of stability problem involving both local and global buckling.

In this context the modeling of imperfections represents a critical issue in the simulation process of cellular-based honeycomb structures. As a consequence of the variety of imperfections characterizing the manufactured core, different methods have been implemented to include imperfections into meso-scale honeycomb FE models for virtual testing simulations. Geometrical imperfections can be implemented on the basis of experimental observations or by the geometric scanning of the actual shape of a manufactured core [20, 23]. In addition, geometrical imperfections can be created in a model by performing random deviations from the ideal shape of nodal positions according to a fixed range of variability. This approach is generally called node shaking and is available in most of the commercial FE codes. Baranger et al. [22] proposed a new method which differs from the aforementioned procedures, based on the modeling of the manufacturing folding process in order to reproduce physical defects, such as the out-of-straightness of the edges. Li et al. [24] studied honeycomb cell structures having irregular cell shapes and non-uniform cell wall thickness by using a Voronoi tessellation technique and the FE method. Zheng and Yu [25] evaluated the influence of two types of material defects, i.e. caused by randomly thickening/removing cell walls, on the deformation modes and plateau stress of metal honeycomb structures by means of finite element simulations. They found that both types of defects were responsible for a significant weakening effect at low impact velocities during dynamic crushing of the investigated honeycomb structures. The effect of cell wall corrugation, curvature and missing cell walls was investigated by [26–28]. Moreover, Fan [29] investigated thermoplastic hexagonal honeycombs considering only that the vertical walls of the honeycombs were not perfectly straight (cell wall tilting angle of 0.2° from vertical position) for the nonlinear simulation of flatwise compression test, obtaining a slightly overestimated final strength.

Another technique used to model imperfections is to consider global modes of the cell on the basis of linear buckling analysis [30, 31]. More in detail, the ideal initial geometry (mesh) of the cell is distorted according to one of the computed global buckling modes (or eigenmodes) with an amplitude scaled down of about 1–5% of the wall thickness. Xue and Hutchinson [32] generated eigenmodes by quasi-static buckling analysis with ABAQUS/Standard and applied them to perturb the perfect geometry of square metal honeycomb and initiate cell wall buckling. Other examples can be found in [33, 34], where hexagonal honeycomb cores are investigated by means of ABAQUS/Standard and LS Dyna (LSTC, Livermore, CA, USA) FE codes. In all cases it is reported that the eigenmodes had a strong influence on the resulting compressive stress-strain curve with particular emphasis on the nonlinear part of the curve. The choice of one of the dominant modes and the magnitude of the scale factor may affect the global mechanical response, which sometimes can also be affected by the user’s sensitivity. In particular, different imperfections corresponding to different buckling modes may lead to different collapse modes, affecting the effectiveness in determining the initial stiffness and the compressive peak load [22, 32]. Moreover, the use of the first buckling mode not necessarily allows to describe all types of defects and it may be not appropriate when constituent cell materials are not homogeneous.

2. Materials and methods

The honeycomb sandwich material investigated within the present work is a 48 kg/m³ hexagonal honeycomb core with a nominal cell size of 3.175 mm, made of Nomex phenolic resin-impregnated aramid paper. The trade name is HRH 10-1/8-3.0 and it is manufactured by Hextel [35]. Hextel manufactures aramid-fiber reinforced honeycomb from three types of para-aramid substrates, including Nomex®, Kevlar®, and KOREX™ materials. The analyzed honeycomb consists of Dupont’s Nomex aramid-fiber paper dipped in a heat-resistant phenolic resin to achieve the final density. It provides high strength, toughness and fire resistance properties in a small cell size. It is widely used as core material for sandwich panels throughout the aerospace industry and also in several other commercial areas. According to the datasheet of the manufacturer, the code name is referred to its geometrical and mechanical features: HRH 10 indicates the product type, 1/8 is the cell size in fractions of an inch and 3.0 is the nominal density in pounds per cubic foot.

In Fig. 1a–c the geometry of the honeycomb is illustrated. All the honeycomb specimens considered in the following activities have been assembled with the “L” direction of the honeycomb core along the primary direction. The Out-of-plane crushing behavior of Nomex honeycomb has been investigated by flat-wise stabilized compressive tests according to ASTM C365M standard. The tests were run on 60 × 60 × 32.2 mm Nomex core coupons (five) bonded between two 1-mm-thick E-glass fiber reinforced phenolic resin skins (with a cured ply thickness of 0.25 mm), with a constant cross head velocity of 0.5 mm/min. The specimens were laminated with external skins in order to prevent local crushing at the edges of the honeycomb cores. It should be pointed out that the use of prepreg material and the autoclave molding process avoided the use of additional adhesive layer between the skin and the honeycomb core. Compressive modulus of the elastic phase, stabilized compressive strength and strain, crush strength, fully compacted compressive modulus and strain values at which densification occurs have been derived by these tests.

2.1. Model features

The elastic and failure behavior of the honeycomb structure is strongly influenced by cell wall mechanical properties and their modeling represents a crucial factor for the development of the FE honeycomb model. The manufacturing process generates an orthotropic material where aramid fibers embedded in the layers
are oriented almost randomly. The material is characterized by two principal directions, called machine direction and cross direction. The principal material direction corresponds to the thickness direction $T$ of the honeycomb structure.

Foil thickness can be highly variable whereas porosity and lack of matrix can affect the ideal smoothness of the foil. The overall thickness is very difficult to estimate and can be detected directly from micrographics or SEM images. It should be mentioned that it is not straightforward to gather experimental data of such paper material due to the thin configuration of the specimen. Mechanical characterization of Nomex paper material can be found in experimental works performed by Tsuji et al. [36] as well as Foo et al. [5] while Fischer et al. [37] performed experimental tensile and compressive tests on Kevlar based paper material used for folded cores showing a mechanical performance similar to the Nomex paper. These experimental tests (including tension, compression, and bending) provided stress–strain curves as well as stiffness and strength values reporting a nonlinear behavior that can be estimated by a bilinear elasto-perfectly plastic material law in compression. Moreover, differences in the material constitutive behavior were found in different loading directions, machine, and cross direction. For instance, Foo et al. [5] reported that the measured experimental Young's moduli (in tension) of the Nomex paper were equal to 3.13 GPa and 0.955 GPa in the machine and cross machine directions, respectively. Fisher et al. [37] derived the in-plane stress–strain curves of pre-impregnated aramid fiber paper used as standard base material for foldcore sandwich structures; in detail, they found that the axial tensile stress–strain curve was almost linear elastic with a failure stress of approximately 100 MPa, while the axial compressive stress–strain curve showed an elastic-perfect plastic like behavior with an ultimate stress ranging between 40 and 60 MPa.

The development of the Nomex honeycomb FE model is based on the definition of a representative cell geometry, its meshing, the definition of boundary and loading conditions, and the assignment of proper constitutive material laws including the implementation of imperfections. The periodicity of the idealized hexagonal microstructure allows to consider one representative hexagonal unit cell to simulate the buckling and compressive/crushing response of Nomex honeycomb observed in the experiments by assigning appropriate boundary conditions. The cell is extracted from the periodic microstructure of the honeycomb as highlighted in Fig. 1b (dashed line). Residual stresses deriving from the mechanical expansion process through which the honeycomb is manufactured are assumed to be negligible. Additionally, the small rounding of the corners of the actual cells is not taken into account. The final idealized cell geometry is shown in Fig. 1c in the $L$–$W$ plane and it is depicted in a

![Diagram of honeycomb sandwich](image)

**Fig. 1.** (a) 3D configuration of a honeycomb sandwich; (b) in plane periodical honeycomb microstructure for hexagonal cells, $W$–$L$ plane; (c) single representative periodic cell; (d) 3D finite element mesh rendering for a single Nomex honeycomb cell (left) and visualization of random material property assignment (right).
three-dimensional rendering in Fig. 1d. The representative cell is represented by a perfect hexagonal cell of diameter \( c = l\sqrt{3} \), paper thickness \( t \) and height \( h \). The corresponding projected area is equal to \( \frac{1}{2} \sqrt{3} l \times 3l \), i.e. the two cell dimensions in the \( W \) and \( L \) direction, respectively. The double wall thickness of both sides in the \( L \) direction is due to the manufacturing process (adhesive between strips) and a monolithic thickness of \( 2t \) is assumed in the model neglecting the bonding of the two walls. The hexagonal unit cell is discretized within the nonlinear FE code ABAQUS using 9000 S4 shell elements. The bottom edges of the unit cell are assumed to be fixed while the nodes of the top edges can only translate in the \( z \)-direction. To enforce periodicity, for the six lateral edges of the unit cell only displacements in the plane perpendicular to the wall direction and rotations around the radial direction are allowed. The cell is loaded by incrementally prescribing the \( z \) displacement of the top surface, indicated as \( \delta \).

According to the manufacturer, the cell walls are made of a 0.051 mm thick Nomex® T412 aramid paper with an additional phenolic resin coating, resulting in a total average wall thickness of roughly 0.063 mm. This value has been taken as a reference for the single wall thickness \( t \) in the honeycomb single cell model. The constitutive behavior of the implemented material model was assumed to be isotropic and linearly elasto-perfectly plastic until failure, on the basis of available experimental results for Nomex paper material [5, 36]; in detail, a value of 3.50 GPa was used as Young’s modulus whereas a value of 60 MPa was used as stress limit for plastic yielding of Nomex material. The assumption of this constitutive law is often used for this kind of detailed FE simulation; however, some limitations may arise when using this assumption mainly due to the fact that the true orthotropic material constitutive behavior is not taken into account and the real (plastic) failure strain of the material is not identified. These features may affect in someway Nomex damage prediction, such as tearing phenomena.

As previously mentioned, the actual cellular structure of an honeycomb core is non-uniform in geometry and characterized by imperfections and irregularities. This feature inevitably affects the critical buckling load of the single cell and the overall strength of the honeycomb structure which would be overestimated in a model without imperfections. In Nomex® honeycomb cores both global and local imperfections are present arising from the manufacturing process, i.e., the expansion of the hexagonal cells and the dipping into phenolic resin.

In the present work, since impregnated aramid-paper exhibits mainly variations in the material properties and paper thickness, two different sources of imperfections were investigated in the FE model of the Nomex cell:

1. foil thickness variability,
2. in-plane Young modulus variability.

In fact, there are significant variations in the material properties and paper thickness, especially in case of impregnated aramid paper. In particular, experimental tensile tests on aramid-impregnated paper revealed, through observations by optical strain measurement, that there was a strongly varying in-plane strain distribution over the paper area. This was attributed to the variation of paper thickness, fiber dispensation within the phenolic matrix, rough surfaces, resin accumulation [38]. In addition, Fischer et al. [37] observed displacements varying over the paper area as well, and microscopic analysis revealed irregular properties of paper thickness, fiber distribution/orientation with the presence of a quantity of voids in the phenolic resin. This evidence is important since the weakest or thinnest areas of a cell wall may initiate the global buckling, triggering damages. Other types of imperfections
described in the previous sections are not handled for the development of the model.

According to the current approach, the properties of Nomex paper are modified in terms of wall thickness and material constitutive properties by stochastically distributing thickness and elastic modulus values over the finite elements of the numerical model. Values of thickness and elastic modulus were chosen according to a uniform distribution, defined by the average value and the coefficient of variation (CV). In detail, the average values have been assumed equal to 0.063 mm and 3.50 GPa for the thickness and the Young modulus, respectively. In dealing with the CV, four different values have been considered, i.e., 10%, 15%, 20%, and 25%. All the adopted distributions are presented in combination with the results of the analysis (see Figs. 5–7).

![Graphs and images](image-url)
This approach has been implemented through a model generation tool by extracting, for each finite element, a value for the thickness and the elastic modulus according to the adopted distribution. Through this approach, three cases have been analyzed, using: (a) only thickness variation; (b) only Young modulus variation; (c) a variation of both parameters. Moreover, for each case five numerical samples have been generated. In Fig. 1d a single cell with randomly assigned material properties is depicted in a three dimensional rendering. It should be specified that some configurations of the samples did not reach convergence in the analysis; this condition was typically experienced for samples with the lowest values of CV, for both thickness and Young's modulus variability. In those cases, buckling was not able to initiate as a consequence of enough concentration of defects along the whole height of the cell. For this reason, new extractions were made until convergence was attained.

2.2. Approach validation

The proposed approach has been validated on the basis of the out of plane compressive/crushing experiments and analyses performed by Wilbert et al. [30] on Hexcel Al-5052-H39 (aluminum) honeycomb having nominal cell size \( c \) of 9.53 mm, wall thickness \( t \) of 95 \( \mu \)m and height \( h \) of 15.9 mm. The comparison in terms of experimental and estimated values of compressive strength, buckling limit and modes is reported in Fig. 3a–c where the compressive stress vs \( \delta/h \) curve and numerical buckling modes are provided. This case allowed to compare the current approach with a standard one where the critical buckling load was calculated by means of a linear buckling analysis, and post-buckling response (up to folding) was obtained by including some imperfections in the geometrical model according to the first buckling mode with a fixed amplitude of the displacement field.

The buckling and post-buckling analyses performed by Wilbert et al. [30] revealed that the out of plane mechanical response is initially stiff and linear elastic up to a level of stress close to 2.95 MPa. From this limit, the plate-like walls of the cell buckle into the first mode characterized by three half waves along the height of the cell and symmetric about mid-height. This generates a bifurcation in the elastic response that develops, at slightly higher stress, with the second buckling mode characterized by an anti-symmetric shape about mid-height with four half waves along the height of the cell. Further compression gives rise to plasticization in the cell wall concentrated around the cell middle height and, as a consequence, the compressive collapse load/stress of the structure is reached; particularly, in that case, it is equal to 4.93 MPa. The authors noticed that the predicted value of collapse stress was higher than the average value measured in the experiments, i.e.,

---

**Fig. 4.** Compressive stress–strain, \( \sigma \) vs \( \delta/h \), response for HRH 10-1/8-3.0 hexagonal Nomex honeycomb up to folding with FE results of cell deformations corresponding to A = critical buckling limit, B = shape up to compressive strength, C1–C5 = strain localization for the five cell samples, D = first fold of the cell.
3.72 MPa, and, for this reason, they performed imperfection sensitivity studies aimed at investigating various sources of imperfections which tend to reduce the collapse load.

Within the present approach, the mechanical properties of the honeycomb Al-5052-H39 foil were assigned on the basis of the experimental results gathered in that work. Accordingly, an isotro-
pic and linearly elasto-perfectly plastic until failure constitutive behavior has been adopted for the aluminum foil, with elastic modulus equal to 69 GPa, and yield stress as 248 MPa. The cell model was then generated considering only thickness variation and only Young Modulus variation into two limit cases for each one, corresponding to values of CV equal to 5%, and 20% (five samples for each case).

3. Results and discussion

3.1. Experimental behavior

The compressive stress–strain relationship (Fig. 2a) of the Nomex honeycomb core has been gained through an out-of-plane flat-wise compressive test previously illustrated. The nominal compressive stress has been derived as the reordered force divided by the projected area of the honeycomb specimen, i.e., \( \frac{L}{C^2} \) dimensions. As reported in different works dealing with the experimental compressive behavior of Nomex based sandwich structures, the constitutive behavior achieved from the present experimental tests, consisted of three characteristic stages: the elastic regime up to the stabilized compressive strength \( \sigma_c \), the crushing regime at nearly constant plateau stress (crush strength, \( \sigma_{\text{crush}} \)), and finally the densification regime, where the cellular structure is fully compacted resulting in a steep stress increase. Fig. 3a reports the mean curve of five replicate specimens, where the three characteristic deformation stages are noticeable. The average values of the compressive elastic modulus, compressive strength, crushing strength, and densification strain computed on five specimens are reported in Table 1 with the corresponding coefficient of variation.

Fig. 2b depicts some highlights (points A–E) on the deformation phases undergone by the Nomex structure under compressive stress. In details, after the initial linear elastic phase (with an average compressive elastic modulus of \( E_c = 137.7 \) MPa), the axial deformation develops into different waves along the total height of the cells for a short range of deformation values, leading to a short nonlinear elastic regime (postbuckling) that is not easy to capture in the experiment. In fact, for Nomex honeycombs, failure is due to a ‘crushing’ mechanism, initiated by elastic buckling and developing as a plastic buckling process \([39,40]\). By proceeding with axial deformation, the previous phase is rapidly followed by the localization of the deformation in correspondence to a certain number of cells of the honeycomb and at different positions along
Fig. 6. Effect of Young modulus variability on $\sigma$ vs $\delta/h$ curves for (a) CV = 10%, (b) CV = 15%, (c) CV = 20%, (d) CV = 25% with the corresponding PDF and focusing on the peak of compressive strength; (e) superposition of CV = 10% and CV = 25% cases.
the height of the cells, that triggers the attainment of the compressive collapse limit of the honeycomb, \( \sigma_c \); the experimental average value (computed on five specimens) is equal to \( \sigma_c = 2.08 \) MPa with a coefficient of variation \( CV_{exp} = 4.46\% \). It can be highlighted that the deformation does not localize into equal waves for all the cells of the honeycomb and does not develop symmetrically about the mid height of the cell. On the contrary, from Fig. 2b point B, it can be seen that the waves develop with different shapes and the collapse takes place randomly along the height of different cells of the specimen where the local stress concentration is attained. The collapse is due to the failure of the deformed foils of the cell which is represented by a drop in the compressive load carrying capacity approximately equal to one third of the collapse stress value. The load slightly increases after reaching the minimum of the stress due to the resistance offered by the deformed cells that start to fold on the plasticized points along the cell height. The folding process develops consecutive folds with small in length, rapidly collapsing one on each other. Within this stage the stress is not able to step up and is kept almost at the constant value of the crush strength, \( \sigma_{crush} = 1.31 \) MPa. When folding is completed throughout all the height of the cell, the densification regime takes place at an average value of strain of \( \varepsilon_d = 0.76 \). At this phase, the compressive resistance is offered by the compacted material leading to a step increase in the stress (point E Fig. 2b). It should be noticed that the free edge of the specimen may play a major role in the position of the concentration of the strain.

3.2. Numerical results

The results of the implicit analysis conducted with ABAQUS/Standard on the aluminum honeycomb cell are compared in terms of experimental and estimated values of compressive strength, buckling limit and modes obtained by Wilbert et al. [30]. Fig. 3a reports the compressive stress vs \( \delta/h \) for the different cases whereas the inset in the same figure depicts the aluminum cell with variable material properties represented by different colors. Due to high repeatability in the results, only 2 of the 5 sample curves have been reported in the graph. Dashed and solid lines correspond to \( CV \) equal to 5\% and 20\% respectively, whereas gray and black lines correspond to elastic modulus and thickness variability, respectively. Horizontal lines correspond to Wilbert’s result in terms of elastic buckling limit and compressive strength values from buckling analysis. It should be pointed out that the adopted thickness and Young’s modulus variation within the thin aluminum sheets is not intended to reproduce the physical pattern of imperfections affecting the aluminum material walls; in fact the range of variation could be reasonably lower in the actual case (where the cell material is homogeneous) with respect to the range of \( CV \) (i.e. 5\% and 20\%) adopted for the approach validation on aluminum honeycomb cell. However, for the scope of the study on Nomex material, for which a higher variability was expected, the implementation of this range of material imperfections allowed to evaluate the effect on the experimental compressive response and to appreciate the
differences with respect to a different type of approach applied for the buckling analysis. The variation of elastic modulus and thickness with CV equal to 5% generates approximately the same compressive response, with a compressive strength value reaching approximately 4.87 MPa. On the contrary, in case of CV equal to 20%, the thickness variation is characterized by a critical compressive stress lower than the one obtained with the elastic modulus variation: 4.50 and 4.73 MPa, respectively. In all cases, the elastic
stiffness, for both linear and nonlinear parts, is approximately the same since the curves appear almost superposed. Moreover, the response is characterized by an elastic critical buckling limit at a stress level of 3.0 MPa (A point in Fig. 3a and b), that is very close to the one obtained by Wilbert et al. [30] by means of elastic buckling simulation. At this point, a bifurcation takes place and a deformation shape corresponding to the first buckling mode (with three waves) shows up (point A in Fig. 3a and b); due to bifurcation, the post-buckling regime is held with a nonlinear trend, as found by Wilbert et al. [30] by adding shape imperfections to the cell. By slightly increasing the stress values, the cell deforms according to the second mode (with four waves point B in Fig. 3a and b) and the deformation continues with this shape up to the plasticization of the cell wall around middle height (point C in Fig. 3a and c). In Fig. 3a, the dashes blue line represents the response of an imperfect version of the unit cell obtained by Wilbert et al. [30] by assigning an initial deformation according to the first buckling mode with the point of maximum transverse deflection of amplitude equal to the cell thickness, t. In this case, the peak of the maximum compressive stress approximately corresponds to the one obtained in case of CV equal to 20% for thickness variation, whereas the elastic stiffness is not affected by a reduction as it happens in case of first buckling mode imperfection.

The results of the implicit analysis on the Nomex honeycomb cell are presented in the following in terms of compressive stress $\sigma$ vs $\delta/h$ curves for the different implemented imperfections: thickness variability, elastic modulus variability, and both thickness and elastic modulus variability. All the analysis have been run up to contact between cell walls, as a consequence of the folding process, i.e., approximately at a $\delta/h$ value equal to 4–5%. Each of the three cases of imperfection has been applied to the five samples, considering four different coefficients of variation, namely, CV = 10%, 15%, 20%, 25% (cf. Figs. 4–7). Unlike the range of compressive strain (i.e. $\delta/h$) adopted in Fig. 2a, for Figs. 4–7 this range is reduced to a maximum value of 5%, with the aim of focusing on the peak of compressive stress. For each graph, the corresponding Probability Density Functions (PDFs, black line in the figures) are reported for the variability of thickness only, elastic modulus only, as well as of both thickness and elastic modulus. Moreover, the region of the graph corresponding to the attainment of the peak of compressive stress is highlighted in Figs. 5–7 within the experimental statistical values (dashed lines in the figures), computed as:

$$\sigma_{\text{min}} = \sigma_{\text{exp}}^c (1 - CV_{\text{exp}}) \quad \text{and} \quad \sigma_{\text{max}}^c = \sigma_{\text{exp}}^c (1 + CV_{\text{exp}})$$

where $\sigma_{\text{exp}}^c$ and $CV_{\text{exp}}$ are the average experimental compressive strength and the corresponding experimental coefficient of variation, respectively. The compressive behavior reproduced by the analysis is practically the same for the investigated cases. For this reason, the detailed description of the $\sigma$ vs $\delta/h$ curve is conducted.
The five samples generated within the above-mentioned imperfections, are characterized by a very similar compressive behavior with small differences in the values of the peak of compressive stress. The linear elastic regime of the honeycomb cell is governed by the elastic modulus $E_3$. The theoretical prediction for the sandwich structure Young modulus $E_1$ [39,40] reported in Fig. 4, for normal loading into the through-the-thickness direction, simply reflects the cell material Young modulus $E_3$, scaled by the area of the load-bearing section:

$$E_1 = \frac{2}{\cos \alpha (1 + \sin \alpha)} t = \frac{\rho}{\rho_i},$$

where $\alpha$ is the angle between the inclined walls and the W direction and $\rho$ and $\rho_i$ are the density of the honeycomb structure and of the solid cell wall material, respectively. In the case of perfect hexagonal cells with $\alpha = 30^\circ$ and $c = l\sqrt{3}$, the relationship becomes:

$$E_1 = \frac{8}{3} \frac{t}{c} = \frac{\rho}{\rho_i}.$$

The linear elastic regime terminates when the cell walls of the honeycomb buckle elastically at a value of compressive stress of approximately 1.20 MPa (Fig. 4A). After reaching the critical buckling load a bifurcation takes place in the $\sigma$ vs $d/h$ curve giving rise to the nonlinear post buckling regime. At this point the cell bulges in a periodic way according to the shape depicted in Fig. 4B that evolves in a different manner at slightly higher level of stress. This is a common behavior among honeycomb structures; generally, the linear-elastic regime terminates when the cell walls of the honeycombs buckle elastically, or bend plastically, or fracture in a brittle manner [40], depending on the relative density $\frac{E}{E_s}$: in low density flexible structural honeycombs it is usually elastic buckling that first leads to nonlinear behavior, although it is found that in intermediate and high density honeycombs, fracture can occur under compression into the through-the-thickness direction. Commonly, the initiation of elastic buckling does not make the honeycomb lose all of its stiffness and load-carrying capacity; failure of structural plates is typically defined by the ultimate strength, considering post buckling strength [40]. The configuration at point B is kept unchanged up to the attainment of the compressive peak of stress $\sigma_c$, where yielding is reached in some elements of the mesh along the height of the cell leading to a sudden drop in the compressive stress. In terms of physical behavior, the sampling generation having different values of $E_i$ and $t$ within a statistical range of variation, is mainly reflected through different compressive strength values and different positions at which plasticization occurs rather than through different buckling mode sequences. Fig. 4C depicts five cells corresponding to the five different samples at the collapse deformation. It can be noticed that the position along the height at which compressive strain localizes is different for the five cases as well as the initial direction of folding, that can be even inclined with respect to the L direction. The folding mechanism begins from that point along the height of the cell leading to a sudden drop in the compressive stress. In dealing with the three cases of imperfections, the cell wall thickness variability generates a compressive response that is very similar among the five samples in case of $C = 10\%$ with an average compressive stress $\sigma_c$ equal to 2.084 MPa, that is well represented in the experimental range defined by $\sigma_{\text{min}}$, $\sigma_{\text{max}}$ (Fig. 5a). By increasing the thickness variability CV, the average compressive strength (computed on 5 samples) decreases almost linearly up to a value of 1.864 MPa corresponding to the case of $C = 25\%$ (Fig. 5b-d); in this case the $\sigma$ vs $d/h$ curves for the five samples appear very

<table>
<thead>
<tr>
<th>Compressive behavior</th>
<th>Average</th>
<th>Coefficient of variation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus (MPa)</td>
<td>137.7</td>
<td>7.74</td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>2.08</td>
<td>4.46</td>
</tr>
<tr>
<td>Crush strength (MPa)</td>
<td>1.31</td>
<td>2.27</td>
</tr>
<tr>
<td>Densification strain (-)</td>
<td>0.76</td>
<td>1.63</td>
</tr>
</tbody>
</table>

only for the case of both elastic modulus and thickness variability with $C = 10\%$, as depicted in Fig. 4.
different each other. In detail, from Fig. 5e, where the cases CV = 10% and CV = 25% are superposed, it appears that a significant variation of t reduces the compressive strength values, the slope of nonlinear elastic regime, and the residual stress after collapse.

A Young modulus variability gives rise to a compressive response that is very similar among the five samples and for all the CV adopted within the analysis. In particular, when the adopted Young modulus has a lower variability with respect to the initial one (i.e., 3.50 GPa), the five curves appear well superposed with an average compressive stress equal to 2.135 MPa that is slightly overestimated compared to the experimental one, but still within the experimental range defined by σ_{min} and σ_{max} (Fig. 6b and c). By increasing the CV of Young modulus (Fig. 6c and d), the average compressive strength decreases almost linearly up to the value of 2.036 MPa in case of CV = 25% that is still characterized by small statistical variation. The nonlinear elastic regime is stiffer in case of little variability whereas the post peak behavior is not affected by the different CV (see Fig. 6e).

When both foil thickness and Young modulus variability are used to model imperfection in the honeycomb cell, the average compressive strength decreases in a nonlinear manner from 2.072 MPa to 1.815 MPa for CV = 10% to CV = 25%, respectively (see Fig. 7a–e). From Fig. 8a it can be highlighted that, when the variability in the material properties is high, the effect of both types of imperfections is amplified and a greater loss in the compressive load carrying capacity is experienced (with respect to the previous cases). Moreover, in case of CV = 25%, the σ vs r/h curves show significant deviations for the five samples in terms of compressive strength and post peak behavior with a significant lower stiffness in the nonlinear elastic regime with respect to the case of CV = 10%.

Fig. 8 summarizes the results of the analysis in terms of average compressive strength σc (a), and average Young modulus Ec (b). The increase of CV on thickness variability and Young modulus triggers a linear decrease of the average compressive strength that is much more noticeable in case of thickness variability. In fact, in the range of CV = 10 and 25%, the compressive strength varies between 2.03 and 2.13 MPa when imperfection on Young modulus are included in the model; on the contrary, the compressive strength decreases up to 1.86 MPa when an high level of statistical deviation is considered for the thickness variability. The combination of both types of imperfections gives rise to a nonlinear decrease of compressive strength with increasing CV, with a sensible underestimation in case of CV = 25%, i.e., σc = 1.81 MPa, that is out of the experimental limits. It is interesting to notice (Fig. 8c) that only thickness variability leads to a wide range of statistical deviation on the computed compressive strength: between 1% and 4% values of CV. On the contrary, the Young modulus variability of the Nomex paper produces samples very close to each other in terms of compressive behavior, with a CV on the computed compressive strength always less than 1%.

The compressive elastic modulus of the honeycomb structure (Fig. 8b) appears slightly dependent on the CV when thickness and Young modulus imperfections are individually implemented in the cell model. However, in these cases the computed value of compressive elastic modulus of the honeycomb structure is overestimated compared to the experimental one. On the contrary, the combination of the two types of imperfections sensibly affects the compressive elastic modulus, especially in case of high CV.

4. Conclusions

In the present paper, a statistics-based method for virtual testing of cellular sandwich core structures using FE simulations has been presented. An hexagonal honeycomb core made of Nomex material has been investigated in order to cover some important aspects related to its out-of-plane compressive response, such as linear elastic response, onset of instability, collapse limit (and its localization and progressive folding). The proposed method deals with the random sampling of the elements of the mesh in a FE numerical analysis on the basis of a normal distribution for thickness and Young modulus of the cell wall material. This has allowed to incorporate material imperfections in the model and evaluate the influence of imperfection variability on the compressive response of Nomex honeycomb cells. All the simulations have been run through ABAQUS/Standard. The analysis have revealed that the compressive behavior of the Nomex honeycomb is more sensitive to thickness imperfections rather than Young modulus variation: a large CV on wall thickness tends to underestimate the compressive strength of the honeycomb if compared to the experimental values, whereas the Young modulus variation (within all the adopted CV) gives rise to compressive strength values always in the range of the experimental ones, even though the statistical variation on the numerical results is very limited. In general, when both the imperfections are included in the model in the range of CV = 10–15% a very good correlation to experimental results can be achieved with respect to compressive stress–strain relationships. Moreover, the detailed representation of the cell also allows for a deep investigation of the cell wall deformation patterns and failure modes to get a better understanding of the structural behavior, which can be expensive and sometimes difficult to study using only experimental observations. The present method can be potentially useful for the complete characterization of the mechanical behavior of honeycomb structures accounting for the influence of physical imperfections, especially when experimental characterization is not straightforward, such as in the case of tension and shear loading in both in-plane and out-of-plane directions. Future improvements applied to the proposed approach regard firstly the experimental characterization of Nomex material to have a more complete stress strain relationship up to failure; then, on the basis of the results, the implementation of more refined constitutive laws for such materials could be achieved.

Finally, we would like to emphasize that the approach could also be successfully extended and applied to other mechanical problems where different sources of imperfections play a major role, as well as when experimental testing is difficult and a virtual testing analysis is desirable.

Acknowledgements

The authors have been partially supported by the MIUR-PRIN project n. 2010BFXRHS. F. Auricchio, S. Morganti, and A. Reali have been also partially supported by the European Research Council through the Starting Independent Research Grants no. 259229. This support is gratefully acknowledged.

References