X International Conference on Computational Plasticity
COMPLAS X
E. Oñate and D.R.J. Owen (Eds)

A DISCUSSION ON SMA BEAMS UNDER FLEXURE EXPLOITING THE SHAPE-MEMORY EFFECT

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Key words: Shape Memory Alloys, Beam model, Phase transformation, Shape memory effect, Superelasticity

1 INTRODUCTION

Shape Memory Alloys (SMAs) are materials able to change their christallographic configuration depending on the temperature and state of stress. As a natural consequence of their microscopic properties, SMAs show two different peculiar behaviors: *superelasticity*, i.e. the capability of recovering large stress-induced strains once the stress is removed, and *shape-memory effect*, i.e. the capability of recovering large stress-induced residual strains by heating the material. Consequently, thanks to such unique material features, SMA lend themselves to be successfully adopted in a broad set of advanced applications. Nowadays, the great and always increasing SMA interest involves different fields (aeronautical, biomedical, structural, earthquake engineering), so that research on constituitive laws, as well as attempts to fully understand SMA behaviors through experimental tests, are deeply stimulated.
The present work is devoted to the coupling of simple one-dimensional SMA constitutive model with a beam model, based on the classical Euler-Bernoulli theory. The goal is to develop an effective computational tool in order to reproduce some simple SMA experimental situations. In particular, the behavior of Nitinol wires subjected to shear force and bending is investigated. The paper is organized as follows. Firstly, a simple beam model is presented. The kinematics and equilibrium equations are given and the SMA constitutive law are briefly mentioned. Secondly, the attention is focused on the numerical results obtained when considering some Nitinol wires, carried out after the calibration of the model with experimental data.

2 THE BEAM MODEL

The model is based on the classical small deformation Euler-Bernoulli beam theory. According to such a model, plane sections normal to beam axis remain plane and normal to axis during the deformation. The beam has cross section A and length L. Let $\mathbf{x} = [x, y]$ be the position vector of a typical point in the cross section such that x lies on the center-line axis of the undeformed beam. Let u, v, θ denote the axis (generalized) displacements. The expression of the beam displacement field $\mathbf{s} = [s_x, s_y]^T$ is:

$$\begin{cases} s_x(x,y) = u(x) - y\theta(x) \\ s_y(x,y) = v(x) \end{cases} \tag{1}$$

Non trivial strain components can be calculated as follows:

$$\begin{cases} \varepsilon_{xx} = \frac{ds_x}{dx} = u' - y\theta' \\ \varepsilon_{yx} = \frac{1}{2}(\frac{ds_y}{dx} + \frac{ds_x}{dy}) = \frac{1}{2}(v' - \theta) \end{cases} \tag{2}$$

Introducing beam strain-like quantities, we can define the axial strain $\varepsilon = u'$, the curvature $\chi = \theta'$ and the shear strain $\gamma = v' - \theta$. The Euler-Bernoulli kinematic hypothesis discussed previously implies $\gamma = 0$ and $\theta = v'$, so that:

$$\varepsilon_{xx} = u' - yv'' \tag{3}$$

The equilibrium equations can be derived by introducing the beam kinematical assumptions into the principle of virtual displacement, $L_v^{\text{ext}} = L_v^{\text{int}}$, where L_v^{ext} and L_v^{int} are the external and internal virtual works, respectively.
The principle of virtual works leads to a system of non-linear equations that can be finally solved by means of a Newton-Rapson iterative method.

The set of constitutive equations capable of describing SMA macroscopic main behaviors are deduced on the basis of the constitutive model discussed in [1]. Assuming a small strain regime, the expression of the free energy function Ψ for a polycrystalline SMA material can be defined through the following convex potential:

$$\Psi(\varepsilon, \varepsilon^{tr}, T) = \frac{1}{2}E(\varepsilon - \varepsilon^{tr})^2 + \beta\langle T - T^*\rangle|\varepsilon^{tr}| + \frac{1}{2}h(\varepsilon^{tr})^2 + \mathcal{I}(|\varepsilon^{tr}|) \tag{4}$$

where: ε is the strain, E is the elastic modulus, T^* a reference temperature, β a material parameter related to the dependence of the critical stress on the temperature, while h defines the slope of the linear stress-transformation strain relation in the uniaxial case. $\mathcal{I}(|\varepsilon^{tr}|)$ is an indicator function introduced to satisfy the constraint on the transformation strain norm:

$$\mathcal{I}(|\varepsilon^{tr}|) = \begin{cases} 0 & if\, |\varepsilon^{tr}| \, \leq \, \varepsilon_L \\ +\infty & if\, |\varepsilon^{tr}| \, > \, \varepsilon_L. \end{cases} \tag{5}$$

where ε_L is the maximum transformation strain.
The constitutive equations may be then obtained exploiting standard arguments as detailed in [1].

3 NUMERICAL TESTS

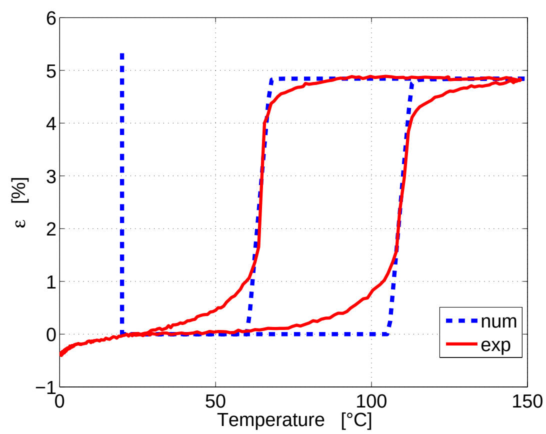
First of all, a model calibration procedure is performed [2]. Uniaxial experimental tensile tests with different applied loads are carried out heating and cooling a SMA wire (ϕ=0.5mm) in order to characterize the material. A *Matlab* interface is created to obtain material parameters comparing two different $\varepsilon - T$ curves. For instance, using experimental tests carried out at σ=100 MPa and σ=200 MPa, we get the material parameters reported in Table 1 (cf. [2] for more details).

Parameter	E	ε_L	β	T^*	h	$R^\dagger$
Value	4.2×10^4	4.9	6.8	60	1000	150
Unit	MPa	%	MPa/°C	°C	MPa	MPa

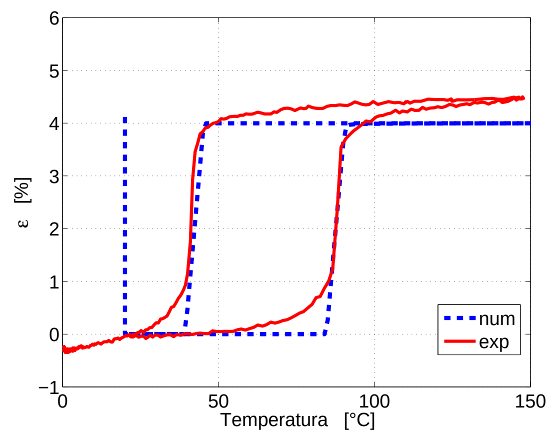
Table 1: Material parameters

† elastic domain radius at high temperatures

Figures 1a-b show a comparison between experimental and numerical $\varepsilon - T$ uniaxial tests.



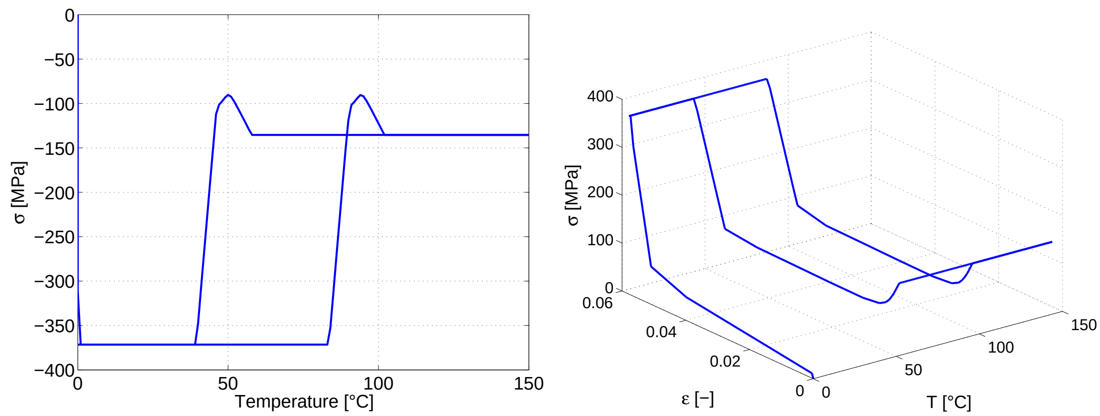
(a) Totally fitted $\varepsilon - T$ behavior



(b) Totally predicted $\varepsilon - T$ behavior

Figure 1: Numerical and experimental tests for two different uniaxial tensile loads

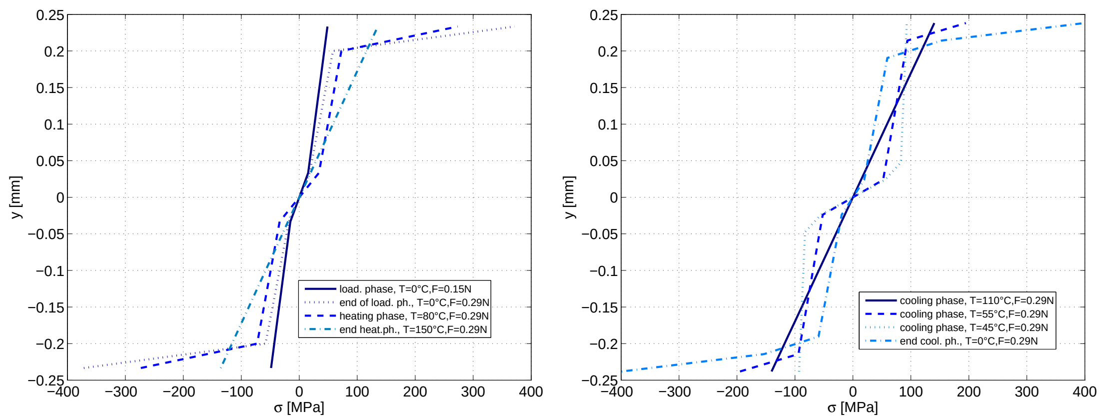
After implementing the Euler-Bernoulli beam model with cross section integration of the SMA one-dimensional constitutive law, we reproduce a simple experimental situation. A cantilever-beam problem is studied: a 5.5 mm long SMA wire (ϕ=0.5 mm) with a 0.29N shear force applied at the free end is the object of our numerical simulation. Temperature is cycled from 0 °C to 150 °C and then back to 0 °C; the results are reported in Figures 2a and 2b respectively, in terms of stress-temperature and stress-strain-temperature responses. In figure 3a-b the stress along the clamped end cross section is represented for different load-temperature conditions.



(a) σ vs T at the clamped end

(b) σ-ε-T at the clamped section related to the fibers subjected to the highest tension

Figure 2: σ and σ-ε at the clamped section as function of temperature



(a) σ at the clamped cross section during the loading and heating phases

(b) σ at the clamped cross section during the cooling phase

Figure 3: Stress responses along the clamped end cross section.

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