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DOUBLE-STEP MIDPOINT METHODS FOR J_2 PLASTICITY WITH NONLINEAR HARDENING

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1 INTRODUCTION

We consider the J_2 elastoplastic constitutive model in the realm of small deformations. The model takes into account both linear isotropic hardening and nonlinear kinematic hardening in the form proposed by Armstrong and Frederick [1]. The aim of the work is to test and compare a set of two quadratically accurate integration algorithms based on a return mapping concept and adopting different midpoint integration rules. The considered algorithms are respectively labeled as DMPT1nl and DMPT2nl. The two algorithms are based on the idea of dividing each time step in two substeps and of updating the solution substep by substep. A wide testing of the considered methods in terms of accuracy and precision using different time discretizations is carried out by means of mixed stress-strain loading histories [2].

2 TIME CONTINUOUS MODEL AND INTEGRATION ALGORITHMS

Admitting a deviatoric/volumetric splitting of the stress tensor $\boldsymbol{\sigma} = \mathbf{s} + p\mathbf{1}$ and of the strain tensor $\boldsymbol{\varepsilon} = \mathbf{e} + (1/3)\theta\mathbf{1}$, the equations for the model under consideration are

$$p = K\theta \tag{1}$$

$$\mathbf{s} = 2G\left[\mathbf{e} - \mathbf{e}^p\right] \tag{2}$$

$$\boldsymbol{\Sigma} = \mathbf{s} - \boldsymbol{\alpha} \tag{3}$$

$$F = \|\boldsymbol{\Sigma}\| - \sigma_y \tag{4}$$

$$\dot{\mathbf{e}}^p = \dot{\gamma}\mathbf{n} \tag{5}$$

$$\sigma_y = \sigma_{y,0} + H_{iso}\gamma \tag{6}$$

$$\dot{\boldsymbol{\alpha}} = \dot{\gamma}H_{kin}\mathbf{n} - \dot{\gamma}H_{nl}\boldsymbol{\alpha} \tag{7}$$

$$\dot{\gamma} \ge 0, \quad F \le 0, \quad \dot{\gamma}F = 0 \tag{8}$$

where K is the material bulk modulus, G is the shear modulus, $\mathbf{e}^p$ is the traceless plastic strain,

Σ is the *relative stress*, $\boldsymbol{\alpha}$ is the backstress, F is the von Mises yield function, $\mathbf{n}$ is the normal to the yield surface, σ_y is the yield surface radius, $\sigma_{y,0}$ the initial yield stress, H_{kin}, H_{iso} and H_{nl} are respectively the linear kinematic and isotropic ad the nonlinear kinematic hardening moduli. Finally, Equations (8) represents the well known Kuhn-Tucker conditions. In what follows we give a brief sketch of the methods over a typical operative time step $[t_n, t_{n+1}]$.

The considered integration algorithms are based on the idea of dividing each time step in two substeps of equal amplitude: $[t_n, t_{n+\alpha}]$ and $[t_n, t_{n+1}]$ and of updating the solution substep by substep. Both algorithms use a standard backward Euler integration scheme along the first substep. In the second substep, the DMPT1nl algorithm adopts a return map update based on a projection along the midpoint normal-to-yield-surface direction onto the endpoint limit surface. In the second substep, the DMPT2nl scheme, instead, adopts an endpoint radial projection combined with a non standard trial state derived from the values of the history variables computed in the first substep and assuming a linear evolution in time over the whole time step. As a result both methods grant midpoint and endpoint yield consistency, which results in evaluating the plastic consistency parameter twice a time step. Figure 1 and Figure 2 provide a pictorial description of the two updating procedures in deviatoric stress space.

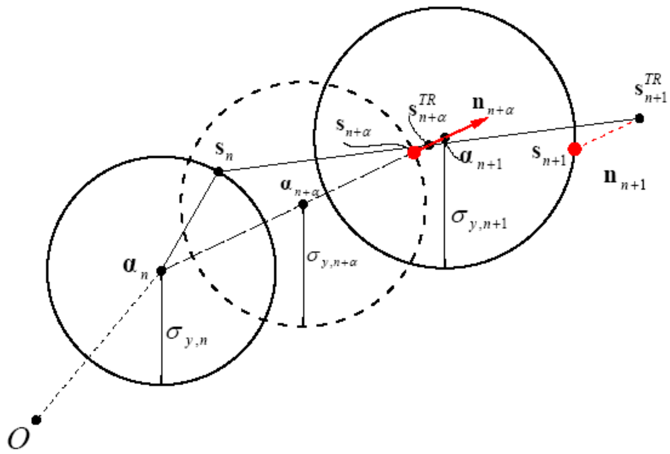

Figure 1: DMPT1nl updating procedure in deviatoric stress space

4 NUMERICAL TESTS

In this section the presented numerical methods are compared on a pointwise stress/strain loading history. The test is a biaxial non-proportional stress/strain loading history, obtained assuming to control ε_{11} and ε_{12},varied proportionally to the uniaxial yield strain

$$\varepsilon_{y,mono} = \sqrt{3/2}\,\sigma_{y,0}/E \tag{9}$$

The remaining stress components σ_{22}, σ_{33}, σ_{13}, σ_{23} are equal to zero (see Figure 3).

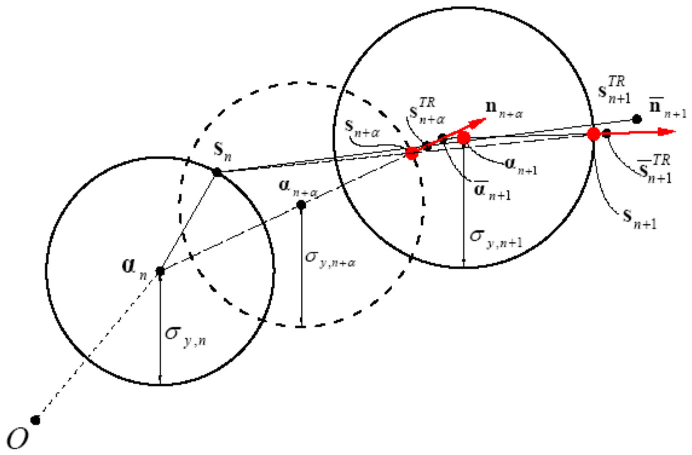

Figure 2: DMPT2nl updating procedure in deviatoric stress space

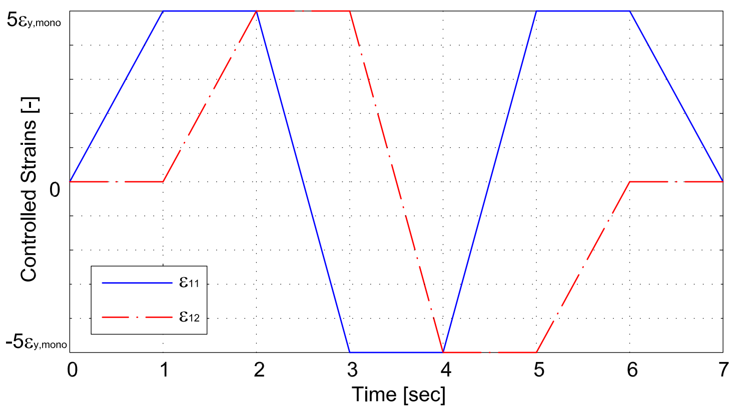

Figure 3: Pointwise stress-strain history

The material constants used in the test are E = 200000 N/m^2, ν = 0.3, $\sigma_{y,0}$ = 200 N/m^2, H_{iso} = 6000 N/m^2, H_{kin} = 20000 N/m^2, H_{nl} = 50.

The total error for the stress and the strain is the usual euclidean norm error scaled by the yield surface radius and summed over the entire time discretization. The reference solution is obtained applying the backward Euler integration rule combined with a return map enforcing yield consistency at the time interval endpoint and adopting a very fine time discretization (BEnl method). In Figure 4, the total stress and strain error curves are reported. For comparison purposes, the plots refer respectively to the considered methods DMPT1nl and DMPT2nl and to a couple of single step midpoint methods named SMPT1nl and SMPT2nl and to the exponential-based ESC^{2}nl method [2-3].

From inspection of Figure 4 it results that the double step midpoint methods under investigation possess second-order accuracy, i.e. the error is of the order of the square of the time step Δt. The double step algorithm DMPT2nl results as the most precise algorithm within the tested methods. A wider set of tests and a deeper analysis of the algorithm numerical properties can be found in [2].

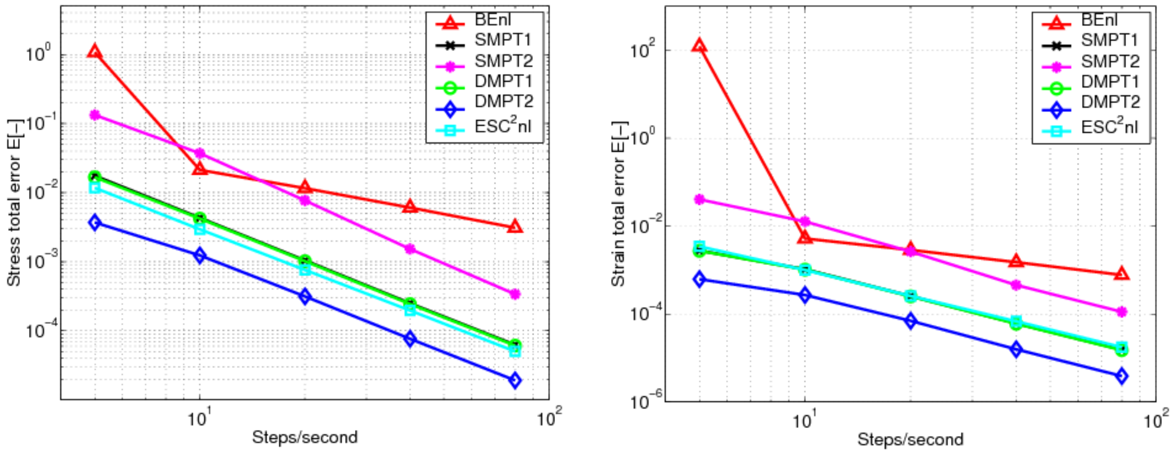

Figure 4: Pointwise stress/strain total error

REFERENCES

[1] F. Auricchio and R.L. Taylor, "Two material models for cyclic plasticity: Nonlinear kinematic hardening and generalized plasticity". *Int. J. Plast.*, **11**, 65–98 (1995).

[2] E. Artioli, F. Auricchio and L. Beirão da Veiga, "Generalized midpoint integration algorithms for J_2 plasticity with linear hardening", *Int. J. Num. Meth. Engng.*, *in press* (2007).

[3] E. Artioli, F. Auricchio and L. Beirão da Veiga, "Second-order accurate integration algorithms for von-Mises plasticity with a nonlinear kinematic hardening mechanism", *Comp. Meth. Appl. Mech. Engng.*, **196**, 1827-1846 (2007).