1 Introduction

Shape-memory alloys (SMAs) are a class of alloys with an intrinsic ability to remember an original shape. At the macroscopic level, they feature the superelastic effect and the shape-memory effect, two uncommon properties not present in materials traditionally used in engineering. The former is related to the ability of the material to regain its initial shape upon removal of the external load, while the latter refers to the capacity of recovering imposed deformations through thermal cycles.

Due to these unique characteristics, SMAs have been successfully adopted for many different innovative applications [1,2], ranging from orthodontic arch-wires for the correction of teeth malpositions to biomedical devices such as stents and prostheses. For the increasing interest towards the use of the SMA technology, many constitutive models able to reproduce their macroscopic effects have been studied. However, the experimentally observed [3–5] dependence of their mechanical response on the loading rate and a characteristic material internal time, \( \tau \), has been partially neglected during these modelling efforts.

Accordingly, we present and compare two uniaxial constitutive models capable of reproducing the rate-dependent superelastic behavior and both able to simulate the different elastic properties between austenite and martensite.

2 A Rate-Dependent Viscous Constitutive Model

2.1 General Framework. The material crystallographic state is described through a scalar internal variable, \( \xi \), representing the martensite fraction. Next, we introduce another scalar internal variable, \( \xi_{ST} \), which represents the static martensite fraction that is related to the ability of the material to regain its shape and the static one, \( \xi_{ST} \), the latter representing the martensite fraction which would be obtained for static loading conditions or, equivalently, for a very small ratio between the loading rate and a characteristic material internal time, \( \tau \).

The evolution of the two internal variables is described through two different rate equations and the possible difference between \( \xi \) and \( \xi_{ST} \) should model either rate-dependent phenomena, such as heat exchanges with the surrounding, or transient thermal processes.

As a consequence, two different stresses can be introduced: the actual one, \( \sigma \), and the static one, \( \sigma_{ST} \), the latter representing the stress that would be obtained for the case of static loading conditions. Accordingly, by assuming the small deformation regime, the following stress-strain relationships are introduced:

\[
\sigma = E[\epsilon - \epsilon_s \xi \text{sgn}(\sigma)]
\]

\[
\sigma_{ST} = E_{ST}[\epsilon - \epsilon_s \xi_{ST} \text{sgn}(\sigma_{ST})]
\]

being \( \epsilon_s \) the recoverable strain, \( \text{sgn}(\cdot) \) the sign function, and \( E \) and \( E_{ST} \) the equivalent elastic moduli computed according to the Reuss rule [6,7].

\[
E = E_A E_S \frac{E_{ST}}{E_{ST} + (E_A - E_S) \xi}
\]

\[
E_{ST} = \frac{E_A E_S}{E_{ST} + (E_A - E_S) \xi_{ST}}
\]

with \( E_A \) and \( E_S \) representing the Young’s modulus of the SMA in its full austenitic and martensitic form, respectively.

2.2 Kinetic Rules. We assume to work with two processes that may both produce variations of the martensite fractions:

- the conversion of austenite into martensite (\( A \rightarrow S \)),

This paper presents and compares two different uniaxial constitutive models for superelastic shape-memory alloys (SMAs), suitable to study the dependence of the stress-strain relationship on the loading-unloading rate. The first model is based on the inclusion of a direct viscous term in the evolutionary equation for the martensite fraction and it shows how the material response is bounded between two distinct rate-independent models.

The second model is based on a rate-independent evolutionary equation for the martensite fraction coupled with a thermal balance equation. Hence, it considers mechanical dissipation as well as latent heat and includes the temperature as a primary independent variable, which is responsible of the dynamic effects. The ability of both models to reproduce the observed reduction of damping properties through the modification of the hysteresis size is discussed by means of several numerical simulations. Finally, the capacity of the constitutive equations to simulate experimental data from uniaxial tests performed on SMA wires and bars of different size and chemical composition is shown. [DOI: 10.1115/1.2204948]

Keywords: shape-memory alloys, superelastic effect, constitutive modelling, experimental tests
• the conversion of martensite into austenite ($S\rightarrow A$).

Following Refs. [6,8], we choose linear kinetic rules to model the evolution in time of the martensite fractions.

2.2.1 Conversion of Austenite Into Martensite. This process is modelled as

$$\dot{\xi} = (1 - \xi) \frac{[\sigma]}{[\sigma] - \sigma_f^A} \mathcal{H}^A - \frac{\xi - \xi_{ST}}{\tau} \mathcal{H}_0,$$  \hspace{1cm} (3a)

$$\dot{\xi}_{ST} = (1 - \xi_{ST}) \frac{[\sigma_{ST}]}{[\sigma_{ST}] - \sigma_{f,ST}^A} \mathcal{H}^A_{ST}$$  \hspace{1cm} (3b)

where $\mathcal{H}^A_{ST}$ and $\mathcal{H}_0$ are zero unless the conditions described in the following are satisfied:

\[
\begin{align*}
\mathcal{H}^A_{ST} &= 1 \text{ when } [\sigma] > 0 \text{ and } \sigma_f^A \leq [\sigma] \leq \sigma_s^A \\
\mathcal{H}_{ST} &= 1 \text{ when } [\sigma] > [\sigma_{ST}] \\
\mathcal{H}_0 &= 1 \text{ when } [\sigma] < 0
\end{align*}
\]

where $\sigma_f^A$, $\sigma_s^A$, $\sigma_{ST}$, and $\sigma_{f,ST}^A$ are material properties representing the stress levels at which the actual and static $A\rightarrow S$ transformations start and finish, respectively.

2.2.2 Conversion of Martensite into Austenite. This process is modelled as

$$\dot{\xi} = \xi \frac{[\sigma]}{[\sigma] - \sigma_f^S} \mathcal{H}^S - \frac{\xi - \xi_{ST}}{\tau} \mathcal{H}_0,$$  \hspace{1cm} (4a)

$$\dot{\xi}_{ST} = \xi_{ST} \frac{[\sigma_{ST}]}{[\sigma_{ST}] - \sigma_{f,ST}^S} \mathcal{H}^S_{ST}$$  \hspace{1cm} (4b)

where $\mathcal{H}^S_{ST}$ and $\mathcal{H}_0$ are zero unless the conditions described in the following are satisfied:

\[
\begin{align*}
\mathcal{H}^S_{ST} &= 1 \text{ when } [\sigma] < 0 \text{ and } \sigma_f^S \leq [\sigma] \leq \sigma_s^S \\
\mathcal{H}_{ST} &= 1 \text{ when } [\sigma] < [\sigma_{ST}] \\
\mathcal{H}_0 &= 1 \text{ when } [\sigma] > 0
\end{align*}
\]

where $\sigma_f^S$, $\sigma_s^S$, $\sigma_{ST}$, and $\sigma_{f,ST}^S$ are material properties representing the stress levels at which the actual and static $S\rightarrow A$ transformations start and finish, respectively.

2.3 Integration of Kinetic Rules. In order to simplify the notation, we define the actual and static martensite fraction increments, $\lambda$ and $\lambda_{ST}$, as

$$\xi = \xi_n + \lambda \text{ or } \lambda = \int_{t_n}^{t_{n+1}} \dot{\xi} dt$$  \hspace{1cm} (5a)

$$\xi_{ST} = \xi_{ST,n} + \lambda_{ST} \text{ or } \lambda_{ST} = \int_{t_n}^{t_{n+1}} \dot{\xi}_{ST} dt$$  \hspace{1cm} (5b)

where the subscript $n$ indicates a quantity that is evaluated at time $t_n$, while no subscript denotes a quantity that is evaluated at time $t_{n+1}$, with $t_n < t_{n+1}$. We then use a backward-Euler scheme to integrate the time-continuous evolutionary equations presented in the previous section. In particular, written in residual form and after clearing the fractions they specialize to:

Conversion of austenite into martensite

\[
\mathcal{R}^A = \lambda([\sigma] - \sigma_f^A) + (1 - \xi)([\sigma] - [\sigma_n]) \mathcal{H}^A + \frac{\Delta t}{\tau} \left(\xi - \xi_{ST}\right)([\sigma] - \sigma_f^A) \mathcal{H}_0 = 0.
\]

Conversion of martensite into austenite

\[
\mathcal{R}^S = \lambda([\sigma] - \sigma_f^S) + (1 - \xi)([\sigma] - [\sigma_n]) \mathcal{H}^S + \frac{\Delta t}{\tau} \left(\xi - \xi_{ST}\right)([\sigma] - \sigma_f^S) \mathcal{H}_0 = 0
\]

2.4 Solution Algorithm. The quantities $\lambda$ and $\lambda_{ST}$ can be computed by expressing the stresses using Eqs. (1a) and (1b) and by requiring the satisfaction of the active evolutionary equation. This can be performed both by adopting a typical iterative strategy, such as the Newton-Raphson scheme, and in closed form. For further information on the algorithmic solution, we address the reader to the work by Auricchio, Fugazza, and DesRoches [9].

3 A Rate-Dependent Thermomechanical Constitutive Model

3.1 General Framework. We consider that at each time instant a homogenized volume element is characterized by a set of external and internal variables. More precisely, we choose the uniaxial strain, $\varepsilon$, and the absolute temperature, $T$, as external variables and the martensite fraction, $\xi$, as the internal variable. In this respect, the thermodynamic state can be expressed by means of a free-energy, $\psi$, function of both sets of quantities.

3.2 Free Energy. Following Auricchio and Sacco [10] and still using the letters $A$ and $S$ to indicate, respectively, material parameters relative to the austenite and to the martensite, we consider the following free energy:

\[
\psi = \left(\varepsilon - \varepsilon_L\xi \text{ sgn}(\sigma)\right)^2 - (T - T_0)[\varepsilon - \varepsilon_L\xi \text{ sgn}(\sigma)]E\alpha
\]

where

- $\varepsilon_L$ and $\eta_L$ are the internal energy and the entropy of the austenite,
- $\Delta u$ and $\Delta \eta$ are the internal energy difference and the entropy difference between the austenite and the martensite,
- $C$ is the material heat capacity,
- $T_0$ is the natural or reference state temperature,
- $E$ is the elastic modulus,
- $\sigma$ is the stress,
- $\alpha$ is the thermal expansion factor.

Based on the free energy, it is possible to derive the consistent expressions for the dependent variables, such as the stress:

\[
\sigma = \frac{\partial \psi}{\partial \varepsilon} = E[\varepsilon - \varepsilon_L\xi \text{ sgn}(\sigma)] - E\alpha(T - T_0)
\]

As in the previous model, the quantity $E$ is computed according to the Reuss formula.

3.3 Heat Equation. The heat equation can be written as:
\[ C \dot{T} + \text{div} \mathbf{q} = b - \beta(T - T_{\text{ext}}) \]  
(10)

where \text{div} indicates the divergence operator, a superposed dot indicates a time-derivative, \( \mathbf{q} \) is the heat flux, \( b \) is the heat source, and \( \beta \) is a material constant linking the temperature difference between the element being considered and the surrounding. Since for this specific study we are considering elements with small size cross sections (i.e., wires and bars), in the previous equation we can neglect the contribution given by the heat flux. In particular, we have:

\[ C \dot{T} = b - \beta(T - T_{\text{ext}}) \]  
(11)

Finally, the heat source \( b \) can be written as:

\[ b = \mathcal{H}_{\text{inc}} + D_{\text{mec}} \]  
(12)

where:

- \( \mathcal{H}_{\text{inc}} \) represents the heat production associated with the thermomechanical coupling and it is defined as:

\[ \mathcal{H}_{\text{inc}} = T \frac{\partial^2 \psi}{\partial T \partial \dot{\epsilon}} + T \frac{\partial^2 \psi}{\partial \dot{\epsilon} \partial \dot{\xi}} \xi \]  
(13)

- \( D_{\text{mec}} \) represents the heat production associated to the dissipative mechanical processes and it is defined as:

\[ D_{\text{mec}} = \sigma \dot{\epsilon} - \left( \frac{\partial \phi}{\partial \dot{\epsilon}} + \frac{\partial \psi}{\partial \dot{\xi}} \right) \]  
(14)

Due to the specific form of the free energy chosen, we have:

\[ \mathcal{H}_{\text{inc}} = T \{ - E \alpha \dot{\xi} + [\Delta \eta + E \alpha \epsilon_L \text{sgn}(\sigma)] \dot{\xi} \} \]  
(15)

\[ D_{\text{mec}} = \Pi_1 \dot{\xi} \]  
(16)

with

\[ \Pi_1 = \Delta u - T \Delta \eta + \epsilon_L |\sigma| \]  
(17)

representing the thermodynamic force associated with \( \dot{\xi} \).

It is also important to note that the strain rate, \( \dot{\epsilon} \), is a known quantity since at each time instant, \( t \), we assume to know the deformation time history.

### 3.4 Kinetic Rules

For each process, the evolution of the martensite fraction is expressed in terms of the driving force which assumes the form:

\[ F = |\sigma| - TA \]  
(18)

where

\[ A = \frac{\Delta \eta}{\epsilon_L} \]  
(19)

Consistently with the model presented previously, and for comparison purposes, we choose linear kinetic rules for this constitutive equation as well.

#### 3.4.1 Conversion of Austenite Into Martensite.

\[ \dot{\xi} = (1 - \xi) \frac{\dot{F}}{F - R_{AS}} \mathcal{H}_{AS} \]  
(20)

The term \( \mathcal{H}_{AS} \) is the activation factor relative to the \( A \rightarrow S \) transformation and it is defined as:

\[ \mathcal{H}_{AS} = \begin{cases} 1 & \text{when } \dot{F} > 0 \text{ and } R_{AS}^S < F < R_{AS}^I \\ 0 & \text{otherwise} \end{cases} \]

where

\[ R_{AS}^S = \sigma_{s,ST} - T_{RA} \quad R_{AS}^I = \sigma_{f,ST} - T_{RA} \]

The quantities \( \sigma_{s,ST} \) and \( \sigma_{f,ST} \) are material parameters indicating, respectively, the initial and final stress value at which the \( A \rightarrow S \) transformation starts and finishes at temperature \( T_R \).

### 3.4.2 Conversion of Martensite Into Austenite

\[ \dot{\xi} = \xi \frac{F}{F - R_{SA}^S} \mathcal{H}_{SA} \]  
(21)

The term \( \mathcal{H}_{SA} \) is the activation factor relative to the \( S \rightarrow A \) transformation and it is defined as:

\[ \mathcal{H}_{SA} = \begin{cases} 1 & \text{when } F < 0 \text{ and } R_{SA}^S < F < R_{SA}^I \\ 0 & \text{otherwise} \end{cases} \]

where

\[ R_{SA}^S = \sigma_{s,SA} - T_{RA} \quad R_{SA}^I = \sigma_{f,SA} - T_{RA} \]

The quantities \( \sigma_{s,SA} \) and \( \sigma_{f,SA} \) are material parameters indicating, respectively, the initial and final stress value at which the \( S \rightarrow A \) transformation starts and finishes at temperature \( T_R \).

### 3.5 Integration of Heat Equation

Integration of Eq. (11) leads to

\[ \frac{C}{t - t_n} \left( T - T_n \right) - b_d + \beta(T - T_{\text{ext}}) = 0 \]  
(22)

where

\[ b_d = \frac{1}{t - t_n} [\Pi_1 \lambda + T \Gamma_1] \]  
(23)

with

\[ \lambda = \xi - \xi_n \]  
(24)

\[ \Pi_1 = \Delta u - T \Delta \eta + \epsilon_L |\sigma| \]  
(25)

\[ \Gamma_1 = -E \alpha (\epsilon - \epsilon_n) + [E \alpha \epsilon_L \text{sgn}(\sigma) + \Delta \eta] \lambda \]  
(26)

with \( \epsilon \) and \( \epsilon_n \) assumed to be known.

### 3.6 Integration of Kinetic Rules

We can obtain the time-discrete phase-transition rules by writing Eqs. (20) and (21) in residual form. After clearing the fractions we obtain

\[ R_{AS}^S \lambda (F - R_{AS}^S) + (1 - \xi) (F - F_n) \mathcal{H}_{AS} = 0 \]  
(27a)

\[ R_{SA}^S \lambda (F - R_{SA}^S) - \xi (F - F_n) \mathcal{H}_{SA} = 0 \]  
(27b)

The quantity \( \lambda \), already defined during the development of the viscous model, can be computed both numerically and in closed form from by requiring the satisfaction of the active evolutionary equation. Detailed information on the integration technique can be found in the work by Auricchio, Fugazza, and DesRoches [11].

### 4 Numerical Simulations

We consider two different strain-controlled loading patterns. The first one is a single loading-unloading cycle (Fig. 1) and the second one is made of multiple loading-unloading cycles (Fig. 2). Both patterns are also described in Table 1 in terms of strain versus adimensional time (named it as \( t_{\text{sim}} \)). We consider material parameters that are typical for commercial SMAs. In particular, for the viscous model we choose

\[ \sigma_{s,ST} = 200 \text{ MPa} \quad \sigma_{f,ST} = 300 \text{ MPa} \]

\[ \sigma_{s,SA} = 200 \text{ MPa} \quad \sigma_{f,SA} = 100 \text{ MPa} \]

\[ \sigma_{s,AS} = \sigma_{s,ST} \quad \sigma_{f,AS} = 1000 \text{ MPa} \]
Several analyses are performed by scaling the adimensional time by different time scale factors, \( t_{\text{fact}} \); accordingly, the change of the time scale factor corresponds to a change in the rate at which the loading history is applied.

The first two analyses are obtained by scaling the single loading-unloading pattern (Fig. 1) by time factors \( 10^3 \) and \( 10^{-3} \) s, corresponding, respectively, to very slow and very fast loading conditions (in particular, by indicating time with \( t \), it results: \( t = t_{\text{adim}} \times t_{\text{fact}} \)).

The results are expressed in terms of stress-strain relationships as well as material temperature-time and are presented in Figs. 3–5. It is of interest to observe that:

(i) The loading rate does influence the material response by increasing the final stress value for the conversion of austenite into martensite as well as by increasing the initial stress value for the conversion of martensite into austenite, as experimental tests display [3–5].

(ii) The thermomechanical model is able to simulate the evolution of the material temperature during the deformation process. Its variation is negligible when performing tests in very slow loading conditions but it is not when performing tests in very fast loading conditions.

The last analysis is obtained by scaling the multiple-cycle loading-unloading pattern (Fig. 2) by the time factor 1, corre-

\[
\sigma_s^{SA} = 900 \text{ MPa} \quad \sigma_f^{SA} = \sigma_f^{ST} \\
\tau = 0.5 \text{ s}
\]

while for the thermomechanical model we select

\[
\sigma_s^{AS} = 200 \text{ MPa} \quad \sigma_f^{AS} = 300 \text{ MPa} \\
\sigma_s^{SA} = 200 \text{ MPa} \quad \sigma_f^{SA} = 100 \text{ MPa} \\
\Delta u = 30 \text{ MPa} \quad \Delta \eta = 0.20 \text{ MPa K}^{-1} \\
C = 4 \text{ MPa K}^{-1} \quad T_{\text{ext}} = T_0 = T_k = 293 \text{ K} \\
\alpha = 0 \text{ K}^{-1} \quad \beta = 0.1
\]

Furthermore, the SMA under investigation has the following elastic properties:

\[
E_A = 40,000 \text{ MPa} \quad E_S = 20,000 \text{ MPa} \quad \epsilon_L = 5\%
\]
sponding to moderate loading conditions. Numerical results are shown in Figs. 6 and 7 and the following considerations apply:

(i) The cyclic response differs according to the model. In particular, the viscous model displays a smooth behavior, while the thermomechanical model shows subloops being described by straight lines.

(ii) As in the previous analyses, it is noticed that when considering the viscous model the material response is affected by material parameters $\sigma_s^{\text{ST}}$ and $\sigma_s^{\text{SA}}$.

(iii) The stress-strain relationships exhibited by both constitutive equations are bounded between two limiting behaviors corresponding to very slow and very fast loading conditions (see the results from the previous numerical tests for comparison).

5 Comparison with Experimental Data

We now want to validate the ability of the models to reproduce the rate-dependent superelastic behavior of SMA elements by comparing numerical results with experimental data.

We focus on a single strain-driven loading-unloading cycle up to a 5% strain performed at different frequency levels. In particular, we consider the following four sets of data.

- **Set 1.** The material is a commercial superelastic NiTi straight wire with circular cross section of diameter 0.76 mm provided by Memry Corp. (Menlo Park, USA).

Tests related to set 1, set 2, and set 3 were performed at the Parco Scientifico Tecnologico e delle Telecomunicazioni in Valle Scrivia (Tortona, Italy) by Fugazza [5], while tests related to set 4 were performed by DesRoches, McCormick and Delemont [3] at the Georgia Institute of Technology (Atlanta, GA). Both experi-

<table>
<thead>
<tr>
<th>Table 2 Mechanical parameters related to both constitutive models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$E_1$ (MPa)</td>
</tr>
<tr>
<td>$E_2$ (MPa)</td>
</tr>
<tr>
<td>$\varepsilon_1$ (%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3 Mechanical parameters related to the viscous model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$\sigma_s^{\text{AS}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{ST}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{SA}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{ST}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{SA}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{AS}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{SA}}$ (MPa)</td>
</tr>
<tr>
<td>$\sigma_s^{\text{ST}}$ (MPa)</td>
</tr>
<tr>
<td>$\tau$ (s)</td>
</tr>
</tbody>
</table>
mental investigations were aimed at studying the rate-dependent cyclic properties of superelastic NiTi SMA elements.

Tables 2–4 report the mechanical parameters (thermodynamic parameters are the same as those used in Numerical Simulations) of the considered SMA materials, while Figs. 8–15 show the comparison between the experiments and the model responses. Moreover, in Figs. 16 and 17 we provide the results coming from the computation of the equivalent viscous damping, which gives a measure of the damping potential of the material. It is defined as the dissipated energy per cycle divided by the product of $4\pi$ and the strain energy for a complete cycle [3]. From the resulting examination, we may draw the following conclusions:

- From the static tests (Figs. 8, 10, 12, and 14) we observe a good match between experiments and numerical results, especially when considering the first set of data. Also, both models well estimate the equivalent viscous damping (Fig. 16). Under these loading conditions, the presented constitutive equations exhibit the same mechanical response since

### Table 4 Mechanical parameters related to the thermomechanical model

<table>
<thead>
<tr>
<th></th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_s^{AS}$ (MPa)</td>
<td>360</td>
<td>310</td>
<td>390</td>
<td>270</td>
</tr>
<tr>
<td>$\sigma_f^{AS}$ (MPa)</td>
<td>385</td>
<td>380</td>
<td>485</td>
<td>520</td>
</tr>
<tr>
<td>$\sigma_s^{SA}$ (MPa)</td>
<td>165</td>
<td>100</td>
<td>200</td>
<td>350</td>
</tr>
<tr>
<td>$\sigma_f^{SA}$ (MPa)</td>
<td>135</td>
<td>55</td>
<td>140</td>
<td>100</td>
</tr>
</tbody>
</table>
the viscous term, in the viscous model, and the heat equation, in the thermo-mechanical model, provide negligible rate-dependent effects.

- From the dynamic tests (Figs. 9, 11, 13, and 15) we note that

the model performance strongly depends on the SMA material under investigation. In particular, for the second, third, and fourth set of data, the viscous model better fits the experiments, while for the first set of data the thermomechanical model provides better accuracy. Finally, the experimental stress-strain curves highlight how wires have bigger energy dissipation capabilities than bars (i.e., wider hysteresis loops) and higher values of equivalent viscous damping (Fig. 17).

- The two models, especially the viscous one, are able to capture with good approximation the maximum stress level attained at the end of the loading phase in both static and dynamic loading conditions. Also, they successfully reproduce the experimentally observed hysteresis size reduction [3–5] with the loading rate increase.

6 Conclusions

In this paper we presented and compared two uniaxial rate-dependent constitutive models capable of reproducing the superelastic effect of SMAs and able to take into account the different elastic properties between austenite and martensite.
In the following, we summarize the most important results based on both the numerical tests and the comparisons with experimental data.

- The viscous model is based on the inclusion of a direct viscous term in the evolutionary equation for the martensite fraction.
- The thermomechanical model is based on a rate-independent evolutionary equation for the martensite fraction coupled with a thermal balance equation.
- The formulation of the viscous model is based on two scalar internal variables, the actual martensite fraction and the static martensite fraction, the latter representing the martensite fraction which would be obtained in static loading conditions.
- The formulation of the thermomechanical model is based on one scalar internal variable which is the actual martensite fraction.
- Both models require a limited number of mechanical parameters which can be determined from typical uniaxial tests performed on either wires or bars. They are the Young’s modulus of austenite and martensite, the plateau length and the stress levels at which the phase transformations take place. On the other hand, the thermodynamic parameters needed for the thermomechanical model are not straightforward to determine experimentally but can be easily found in the literature for typical SMAs.

- The ability of the two constitutive equations to simulate experimental data relative to tests performed in static and dynamic loading conditions on superelastic SMA wires and bars has also been assessed. Static tests have provided a good comparison between experiments and numerical analyses while dynamic tests were strongly affected by the different material response exhibited by the considered SMAs for the same loading frequency, probably due to their different chemical composition.

In conclusion, the advantages of the presented models are the simplicity, the possibility of implementing a robust solution algorithm, and the ability to reproduce experimental data obtained at different frequency levels of excitation.

Acknowledgment

D.F. would like to thank Ms. Federica Onano of Parco Scientifico Tecnologico e delle Telecomunicazioni in Valle Scrivia (Torino, Italy) and Dr. Lorenza Petriti of Politecnico di Milano (Milano, Italy) for their help during the experimental tests as well as the financial support of the Italian National Civil Protection Department which, through its Servizio Sismico Nazionale section, provided a scholarship. Also, the financial support provided by the Progetto Giovani Ricercatori-anno 2002 of the Università degli Studi di Pavia is kindly acknowledged. Additional funding was provided by the PECASE Program of the National Science Foundation under Grant No. 0093868.

References